EPECs as models for electricity markets

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***Abstract*— We discuss two topics. The first is pragmatic; it concerns modeling and solving bilevel games in the form of equilibrium problems with equilibrium constraints, EPECS. Several applications come from modeling the behavior of generators and retail consumers in electricity markets. We explain how EPECs can be considered as complementarity problems — which are more familiar as stationary conditions in constrained nonlinear optimization — and hence software for solving complementarity problems can be applied.**

The second topic is more fundamental and raises perhaps more questions than it answers: Can we describe the meaning or value of models, e.g., EPECs, in economics when there is no obvious candidate for the "canonical" or "natural" model? This arises because while there are canonical models for the case of perfect competition, there seem to be a plethora of different approaches when players act strategically.

I. INTRODUCTION

Equilibrium problems with equilibrium constraints, EPECs, have been particularly useful in game theoretic models of electricity markets over a network of generators and consumers, for example [1]–[3]; see [4] for a brief review of these and related market models. EPECs arise when players face utility maximization problems in the form mathematical programs with equilibrium constraints, MPECs, [5], e.g., an electricity generator seeks to maximize profit where pricing and dispatch is represented a complementarity problem that includes the generator's strategy as a parameter.

Note MPECs have nonconvex constraints and therefore may have multiple local maxima. Thus the EPEC may not have any (pure strategy) Nash equilibria in some instances, see [1, Footnote 8, p.143], [6], [7]. On one hand, this motivates development computational methods for finding Nash points or their stationary conditions. On the other, we have to ask what is the meaning of a model without an equilibrium or with multiple equilibria? In particular, game theoretic models of imperfect competition seem inherently ambiguous.

We pursue the first notion in Section II, motivated by a simple three node example with one generator or consumer at each node. Our focus is the complementarity problem format [8] which is already well known in game theory and for which robust software exists [9]. We give an outline view of the second topic in Section III.

II. COMPLEMENTARITY MODELS OF PERFECT AND IMPERFECT COMPETITION

Here we derive two market equilibrium models, one under perfect competition and the other based on Cournot competition. Complementarity models appear to be natural for modeling perfect competition. See for example [10], [11]. In imperfect competition, EPECs seem natural; we explain how stationary conditions for EPECs can be formulated as complementarity problems, following [6], [7]. For completeness we mention that a popular alternative to Cournot competition, especially in electricity market models, is supply function equilibria [12]. Locational pricing in either paradigm leads to bilevel optimization and bilevel games, e.g. [1], hence MPEC and EPEC models [3], [4], [6], [7].

A. *Setting: triangular network with limited capacity on link 12*

We have a triangular network with a generator at each of nodes 1 and 2, and a consumer at node 3. Lines have identical transmission characteristics except that flow on line 12 is limited to f units, in either direction, while other links are uncapacitated. Any feasible output of q_i units at node i for $i = 1, 2$ will deliver the quantity

$$q_3 = q_1 + q_2 \tag{1}$$

to the consumer. The cost of generation at node $i = 1, 2$ is $c_i(q_i)$ where q_i denotes the output of generator i.

We will first consider the case when the generators act in a perfectly competitive manner, i.e., maximize their profit without consideration of the effect their strategy (quantity generated) has on the market. Then we will examine imperfect competition via a Cournot game. In each case, the consumer is not a strategic player, rather it represents elastic demand through an inverse demand function $p(q_3)$, which is the price at which the consumer will buy exactly $q_3 > 0$ units.

In each case we will reformulate or replace the Nash equilibrium as a set of stationary/optimality conditions in the form of a complementarity problem.

1) Cost of transmission in spot market: Using a lossless DC model of electrical flow [13], energy generation of q_1 units by generator 1 results in flow of $\frac{1}{3}q_1$ from node 1 to node 3 via node 2, and $\frac{2}{3}q_1$ direct from node 1 to node 3. Thus $\frac{1}{3}q_1$ flows on link 12. Similarly, q_2 units produced by generator 2 results in $\frac{1}{3}q_2$ units flowing, in the opposite direction, on the link 12. By specifying generator 1 as a more cost efficient generator, i.e., $0 \leq c_1(q) \leq c_2(q)$ for $q \geq 0$, we can expect it to generate greater flow than generator 2, hence congestion to only occur on line 12 if $\frac{1}{3}q_1 - \frac{1}{3}q_2 \geq f$.

To allow a different cost of transmission on link 12 for each generator we may use a constant marginal cost of transmission of λ_i for each generator $i = 1, 2$. In fact, rather than being a transmission price, λ_1 can be thought of as the opportunity cost to generator 1 for not receiving more transmission rights. For example when transmission is not priced in a real system, it may be rationed according to historical use ("grandfathered") or by another mechanism.

When the line is congested, the cost of transmission to generator 1 is $\frac{1}{3}\lambda_1 q_1$, whereas generator 2 receives income $\frac{1}{3}\lambda_2 q_2$ because its flow effectively relieves congestion. This is modeled via complementarity between the capacity slack on link 12 and the marginal cost of transmission for each generator:

$$0 \leq f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 \quad \perp \quad \lambda_i \geq 0, \quad i = 1, 2.$$

The symbol $\perp$ denotes orthogonality, which means that if line 12 is not saturated then each λ_i is zero, hence generators neither pay nor receive money to use the link.

We say transmission is priced if λ_1 and λ_2 are required to be equal. In this case we simplify the above, using the (single) price λ:

$$0 \leq f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 \quad \perp \quad \lambda \geq 0. \tag{2}$$

Henceforth we assume transmission on line 12 is priced.

B. *An explicit transmission pricing mechanism*

Here we develop an explicit pricing mechanism that addresses feasibility of players' quantities and also incentivises efficient generation. This will allow us to model strategic players who realise that their actions affect not only energy prices but also transmission costs. Nevertheless, later we will see that if there is perfect competition then equilibrium outcomes are almost identical for both pricing mechanisms. So, in principle, outcomes of perfect competition using implicit transmission pricing can be compared to those of imperfect competition using explicit pricing.

Under the explicit transmission pricing mechanism, the system is maintained by a system operator, SO, that has all generators' capacity at its disposal. Given the quantities q_1 and q_2 contracted by generators to the consumer, the SO computes a quantity adjustment ξ according to

$$\begin{array}{ll} \min\limits_{\xi} & c_1(q_1 - \xi) \, + \, c_2(q_1 + \xi) \\ \text{subject to} & f \geq \frac{1}{3}q_1 - \frac{1}{3}q_2 - \frac{2}{3}\xi. \end{array} \tag{3}$$

The idea is to preserve the total quantity $q_3 = q_1 + q_2 = (q_1 - \xi) + (q_2 + \xi)$ promised to the consumer, while ensuring feasibility of network flows in the most efficient way. If the proposed flow $\frac{1}{3}q_1 - \frac{1}{3}q_2$ on link 12 exceeds the capacity f, then the optimal ξ will be positive, i.e., generator 1 will produce less and generator 2 more (the latter to relieve congestion on link 12). If not, ξ will be negative or zero.

Let λ be the shadow price of the transmission capacity constraints, also called a Lagrange or Karush-Kuhn-Tucker or KKT multiplier in the optimization literature. The optimality conditions which specify ξ and λ are

$$0 \; = \; -c_1'(q_1 - \xi) + c_2'(q_2 + \xi) - \tfrac{2}{3}\lambda \tag{4}$$

and

$$0 \; \leq \; f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 + \tfrac{2}{3}\xi \quad \perp \quad \lambda \geq 0. \tag{5}$$

The price λ can only be positive when congestion is present after this transmission rebalancing, otherwise $\lambda = 0$.

To explain how the feasibility/pricing mechanism would be used in practice, after solving (3) to find ξ, the SO would require generator 1 to produce $q_1 - \xi$ rather than q_1 units. The generator would then transfer an amount of $c_1'(q_1 - \xi)\xi$ to the SO, which, if $\xi > 0$, represents marginal cost savings to the generator. This transfer can also be negative, e.g., the generator receives a payment from the SO if its production increases ($\xi < 0$). Likewise generator 2 must produce $q_2 + \xi$ units and transfers to the SO the net amount of $-c_2'(q_1 + \xi)\xi$. An important point is that this transmission pricing/feasibility mechanism is self funding as we now show. The total amount transferred to the SO from the generators, including the congestion charge, is

$$\begin{aligned} \tfrac{1}{3}\lambda(q_1 - q_2) \, + \, c_1'(q_1 - \xi)\xi \, - \, c_2'(q_2 + \xi)\xi & \\ = \; \tfrac{1}{3}\lambda(q_1 - q_2 - 2\xi) & \\ = \; \lambda f & \end{aligned}$$

where the first equality uses (4) and the second uses (5). Of course $\lambda \geq 0$ and $f \geq 0$ by assumption, so the total transfer to the SO is nonnegative.

1) Profits of agents: Generator 1 attempts to maximize its payoff, i.e. spot market profit, via its quantity q_1,

$$\max_{q_1 \geq 0} \; p_3 q_1 \, - \, c_1(q_1) \, - \, \tfrac{1}{3}\lambda q_1 \tag{6}$$

where the first term of the objective is revenue, in which p_3 is the price specified by the consumer at node 3, the second is cost of generation, and the last, cost of transmission. A similar problem is faced by generator 2.

The consumer's payoff is

$$\max_{p_3 \geq 0} \; -p_3 q_3 + \int_0^{q_3} p(\xi)d\xi \tag{7}$$

where the first term is the cost of consumption and the second is its benefit.

C. *Perfect competition*

1) Optimality of agents: Generator 1 solves (6) and generator 2 solves a similar problem. Here, in the situation of perfect competition, neither generator believes that it can affect the market price, i.e., generator i considers $\partial p_3/\partial q_i$ to be zero. Likewise, each generator does not consider the effect of its decision q_i on transmission pricing. Thus the generators' optimality conditions are the following complementarity conditions that are derived using negative gradients of the objective functions for each generator's problem:

$$\begin{array}{rcl} 0 \le -p_3 + c_1'(q_1) + \frac{1}{3}\lambda & \perp & q_1 \ge 0 \\ 0 \le -p_3 + c_2'(q_2) - \frac{1}{3}\lambda & \perp & q_2 \ge 0. \end{array} \tag{8}$$

Remark: We could have formulated these with a KKT multiplier [14], τ_i, corresponding to $q_i \ge 0$, e.g., the first complementarity condition is equivalent to $0 = -p(q_3) + c_1'(q_1) + \frac{1}{3}\lambda - \tau_1$ where $0 \le \tau_1 \perp q_1 \ge 0$. Instead, to ease notation, we have eliminated this multiplier.

For the consumer, the optimality condition is the complementarity problem

$$0 \le p_3 - p(q_3) \perp q_3 \ge 0. \tag{9}$$

That is, if q_3 is positive then $p(q_3) = p_3$.

2) Equilibrium conditions: The market equilibrium is described by the aggregate system consisting of conditions on flow balance, (1); feasibility and pricing of transmission, (2); and optimality for generators, (8), and the consumer, (9). This aggregate system is a single complementarity problem which we can hope to solve uniquely if there is a monotonicity property of the system [8]. In particular, there are five variables $(q_1, q_2, q_3, \lambda, p_3)$ which are matched by the number of equations and complementarity conditions.

Apparently different equilibrium conditions are obtained under the explicit pricing mechanism, for example, (2) in the equilibrium conditions is replaced by (4)–(5) which gives a complementarity problem with six variables, five complementarity conditions and one equation. A closer examination of this case shows that the payoffs of the generators also change: Generator 1's problem (6) becomes

$$\max_{q_1 \ge 0} \; p_3 q_1 - m_1 \xi - c_1(q_1 - \xi) - \tfrac{1}{3}\lambda q_1$$

where we use the notation $m_1 = c_1'(q_1 - \xi)$ to indicate that the competitive generator views the marginal cost of energy transfers to the SO as exogenous. The stationary conditions, which are only with respect to q_1, are given by the first complementarity condition of (8) with $q_1 - \xi$ replacing q_1. Likewise the second generator's profit maximisation problem yields stationary conditions that are the second complementarity condition of (8) with $q_2 + \xi$ replacing q_2. Thus, under (1), (2), (4)–(5), (8) and (9), the quantities $q_1 - \xi$, $q_2 + \xi$, q_3, energy price p_3 and transmission price λ satisfy the equilibrium conditions established previously for implicitly priced transmission. Conversely, an equilibrium $(q_1, q_2, q_3, \lambda, p_3)$ under implicit transmission pricing together with $\xi = 0$ satisfies all equilibrium conditions of the explicit case except possibly (4). This last condition follows automatically from (8) provided q_1 and q_2 are positive, and can be verified in some other cases by choosing $\xi \ne 0$.

An important practical point is that there are good algorithms and software available for solving complementarity problems such as the `PATH` package [9].

Of course a solution of the equilibrium conditions is a stationary point for each player's problem. In fact such a solution is necessarily a Nash equilibrium of the game — meaning each player has globally maximized its payoff provided the other players do not change their strategies — because each player faces a maximization of a concave objective subject to convex constraints, thus a stationary point must be a global maximum.

Remark: If transmission is not priced, we have one extra variable (each instance of λ may be replaced by λ_1 or λ_2) and the equilibrium system becomes under determined or "non-square". Therefore there may be infinitely many solutions, and we have to specify some other mechanism by which λ_1, λ_2 are selected.

D. Cournot competition

The idea of Cournot competition is that players compete by setting their quantity, as they did in the perfectly competitive case, and they are aware that changes in their quantity can affect the market. Generator 1's problem (6) holds here with two crucial differences: First, the generator recognizes that price has the form $p_3 = p(q_1 + q_2)$, which is affected by q_1. Second, the generator recognizes that q_1 affects the SO's (explicit) transmission pricing mechanism.

Optimizing generator 1's profit is a more complex problem than previously. It's profit function is

$$\begin{array}{l} \text{profit}_1(q_1, q_2, \lambda, \xi) \\ \quad = \; p(q_1 + q_2)q_1 - c_1'(q_1 - \xi)\xi - c_1(q_1 - \xi) - \frac{1}{3}\lambda q_1 \end{array}$$

and its profit maximisation problem is

$$\begin{array}{ll} \max\limits_{q_1, \lambda, \xi} & \text{profit}_1(q_1, q_2, \lambda, \xi) \\ \text{subject to} & 0 \le q_1 \\ & \text{(4)–(5) hold.} \end{array} \tag{10}$$

This problem is an MPEC, or mathematical program with complementarity constraints, MPCC, to be more precise. Note that although λ and ξ are not directly controlled by the generator, rather are determined by the SO's transmission pricing mechanism, the generator can influence them via q_1.

Generator 2 has profit function

$$\begin{array}{l} \text{profit}_2(q_1, q_2, \lambda, \xi) \\ \quad = \; p(q_1 + q_2)q_2 + c_2'(q_2 + \xi)\xi - c_2(q_2 + \xi) + \frac{1}{3}\lambda q_2 \end{array}$$

and its profit maximization is the MPEC,

$$\begin{array}{ll} \max\limits_{q_2, \lambda, \xi} & \text{profit}_2(q_1, q_2, \lambda, \xi) \\ \text{subject to} & 0 \le q_2 \\ & \text{(4)–(5) hold.} \end{array} \tag{11}$$

The consumer is not directly affected by whether the generators are perfectly competitive or play a Cournot game. The consumer is, of course, implicitly affected by the choices the generators make in terms of quantities.

The game between generators is now called an EPEC, since each strategic player faces an MPEC. EPECs are generally unsolvable, e.g., see [7, Example 12] which is in the setting of supply function equilibria. This unfortunate circumstance is hinted at by the fact that MPECs generally have nonconvex feasible sets, so stationary points need not be globally optimal. Nevertheless EPECs have convenient optimality conditions under reasonable conditions, as we will next explore.

1) Optimality of agents' MPECs: Here and in the next subsection, we apply the approach of [6], [7] for formulating stationary conditions for EPECs as complementarity problems.

A simple, if naive, way to approach MPECs is via nonlinear programming: replace the complementarity condition (5) by

$$0 \;\leq\; f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 + \tfrac{2}{3}\xi \qquad (12)$$

nonnegativity of λ, and a bilinear equality

$$0 \;=\; (f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 + \tfrac{2}{3}\xi)\lambda. \qquad (13)$$

A result of MPEC theory [15], [16] is that a locally optimal point (q_1, λ) of (10) must be stationary, i.e., has KKT multipliers, provided that the gradients of the active constraints, *except the bilinear function*, are linearly independent. ("Active" means the constraint holds with equality.) This condition is called the MPEC linear independence constraint qualification, MPEC-LICQ.

Regarding (10), a necessary condition for the MPEC-LICQ at a point (q_1, ξ, λ), where q_2 is a given parameter, is that we do not simultaneously have $q_1 = 0$, $f = -\frac{1}{3}q_2 + \frac{2}{3}\xi$ and $\lambda = 0$. For instance, we expect the MPEC-LICQ to hold at the optimal strategy q_1 provided $q_1 > 0$. A similar statement holds for generator 2.

Let us assume that maxima of the generators' MPECs are stationary and denote the KKT multipliers corresponding to (4) by τ_i^λ, (12) by $\tau_i^f \geq 0$, and (13) by μ_i. We will avoid the need for further multipliers, for nonnegativity of variables q_i and λ, by using complementarity conditions. We introduce the Lagrangian of generator i's profit maximization problem:

$$\begin{aligned} &L_i(q_1, q_2, \lambda, \xi; \tau_i^\lambda, \tau_i^f, \mu_i) \\ &\quad = \; -\text{profit}_i(q_1, q_2, \lambda, \xi) \\ &\qquad - \tau_i^\lambda[-c_1'(q_1 - \xi) + c_2'(q_2 + \xi) - \tfrac{2}{3}\lambda] \\ &\qquad - \tau_i^f(f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 + \tfrac{2}{3}\xi) \\ &\qquad - \mu_i(f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 + \tfrac{2}{3}\xi)\lambda. \end{aligned}$$

The stationary conditions for the generators are

$$\begin{aligned} &\text{for } i = 1, 2: \\ &0 \leq \frac{\partial L_i}{\partial q_i} && \perp\; q_i \geq 0 \\ &0 \leq \frac{\partial L_i}{\partial \lambda} && \perp\; \lambda \geq 0 \qquad (14) \\ &0 = \frac{\partial L_i}{\partial \xi} \\ &0 \leq f - \tfrac{1}{3}q_1 + \tfrac{1}{3}q_2 + \tfrac{2}{3}\xi && \perp\; \tau_i^f \geq 0 \end{aligned}$$

coupled with the equation constraints (4) and (13).

2) Equilibrium conditions for the Cournot EPEC: The aggregate system consists of flow balance, (1), and several sets of optimality conditions: (4) for transmission pricing, (13)–(14) for the generators, and (9) for the consumer. Note that this system is again a single complementarity problem for which the number of variables — there are twelve comprising four for each generator, $(q_i, \tau_i^f, \tau_i^\lambda, \mu_i)$, two for the consumer, (q_3, p_3), and two for the SO, (ξ, λ) — is matched by the number of equations and complementarity conditions. This is achieved by sharing, in the generators' optimality conditions, the transmission variables (ξ, λ) and the equality constraints (4) and (13).

A difficulty beyond the potential lack of equilibria for the EPEC is redundancy in the associated complementarity problem, notably in the functions appearing in the optimality conditions (14). Practically this need not be an obstacle to numerical solution by `PATH` as seen in [4], [7], or by other methods such as diagonalization (see [7] and references therein), an iterative scheme in which players take turns in optimizing their profits, and then repeat the process until, hopefully, convergence occurs.

An important computational limitation to date is that methods that hold any promise of taking reasonable computational time on problems (networks) of realistic size, such as those mentioned, are essentially heuristic. We lack practical methods that are guaranteed to identify a Nash equilibrium when there is one, or give a certificate of insolubility when there is not.

III. EPECs AND THE ECONOMICS OF ELECTRICITY MARKETS

MPEC and EPEC problems raise a lot of interesting questions that can all be traced to the incomplete or imperfect nature of the competition. They also raise practical questions.

A. Market architecture: bounds on inefficiency of incomplete electricity markets

A considerable attention has been devoted in the restructuring of electricity markets to what is commonly referred to as the market architecture. Following [17] we refer to the architecture as the blueprint that lists the different energy markets and ancillary submarkets to create and the interactions to establish between them for guaranteeing the good functioning of the electricity market. Wilson [18] gives an in depth discussion of this subject. We summarize his discussion there for our purpose by noting that Wilson refers to energy and transmission that he considers should exist as real time and forward markets. There is now a common view on this question in the US, at least in the zones which adopted restructuring, but other markets such as capacity markets are still under discussion [19]. The question of the architecture therefore remains relevant. In contrast, this common view of the organisation of the energy and transmission markets does not exist in Europe. The question of understanding the set of markets that should be created (energy, transmission, reserve, capacity in real time and forward) therefore is still at least partially unsolved today. The relevant aspect of it is what happens in terms of degradation of efficiency when these markets are not created. This question can be stated in terms of market completeness.

We refer to a complete market as one where energy and all constrained services are priced in real time and forward. The relevant question is thus to assess what happens when this is not the case, in other words when the market is incomplete. Wilson was the first one to argue that restructured electricity markets are massively incomplete, that is, that we are bound to find inefficiencies as a result of the difficulty of implementing a complete set of prices. Needless to say, some missing prices are more important than others.

Referring to the three-node example presented in section 2, we shall say that the real time market is incomplete when for instance transmission is not priced (this case is now settled in the US, it is still controversial in European where existing proposals only allow for a very partial pricing of transmission). This is an incomplete market [20]. Other incompleteness arises when pricing occurs with different granularity in the real time and forward markets. This is for instance the case when energy and transmission are both priced in real time but there is no forward transmission price. This is also true if there exist electricity market futures on energy with 3 year maturity but the maturity of transmission contracts is limited to one year. Because prices do not exist on all services and goods one is now unable to price all transactions (e.g. a delivery from node 1 to node 3) whether in real time (when transmission is not priced in real time) or forward (when transmission is priced in real time but not forward), [21]. Finance theory tells us that, even though it is impossible to value the transmission one can still find bounds on the value of that transaction. The question is how.

We conjecture (and are currently working on this conjecture) that the problem of finding bounds in incomplete electricity markets (and therefore to assess the inefficiency of an incomplete electricity market) can be formulated as an MPEC or in more complicate cases as a minimization problem subject to EPEC constraints. Here is our reasoning. Consider the three node problems of the preceding section and assume that transmission is not priced. We did represent this case by letting λ_1 and λ_2 take different values. Both λ_1, λ_2 are zero when the transmission constraint is slack (there is no need to price transmission in that case). But they may be non-zero and different when the line is saturated. According the remark at the end of §II-C, the complementarity model is no longer square and we have an infinite set of possible solutions. The question is then to find, for instance, the minimum welfare or the maximal welfare among these solutions.

This is an MPEC problem, possibly a quite difficult one. We argued in section 2 that the equilibrium constraints that represent the electricity market define a non-convex set and hence make the MPEC problem non-convex. We expect it to be even more difficult to minimize a concave welfare function over equilibrium constraints, which is necessary to assess the loss of efficiency that arises from market incompleteness. The problem would be similarly non convex if we were interested in the minimal profit that a generator can make as a result of the incompleteness of the market.

Except for Europe, nobody discusses the lack of a transmission market in real time anymore. But the problem of an incomplete pricing of transmission in the forward market remains, for reasons of liquidity. The construction of the underlying market model that represents the incomplete forward market is more complex but the principle is the same. One can thus again construct a problem that tries to assess the loss of welfare that can result from this incompleteness.

B. What do we really know about market power in electricity markets?

The question of market power is much more often addressed than the issue of market incompleteness. Still its treatment remains largely ambiguous and this ambiguity appears at different levels.

Consider first the Cournot version of the three-node problem presented in section 2 and assume that there is no transmission constraint. The problem reduces to a pure single stage energy model. Market power in the energy market is the standard question addressed in the numerous papers that were written in the aftermath of the Californian crisis. This model is well specified but its economic foundations are ambiguous. There are indeed several paradigms of imperfect competition, Cournot being only one of them (this contrast with perfect competition that represents an unambiguous reference case). There is in fact a whole range of degrees of market power that goes from collusion to a perfect competition, passing through the Cournot assumption in the middle. Economic theory provides no structural reason to select one or another with the result that any assumption of imperfect competition is generally loosely founded. This ambiguity does not prevent writing the model and treating it mathematically provided one accepts this degree of arbitrariness: the trick is to use conjectural variations that indeed allow one to model the range of possible assumptions of competition. The approach is discredited as a theoretical explanation of the degree of market power but it allows representing it. Even though lacking a fundamental basis (the true solution, namely via dynamic games, is not really an operational solution) conjectural variations can still be used in practice at some condition: in the same way as models are often used without precisely knowing the value of all relevant parameters of a problem (e.g. fuel prices), one may want to represent the uncertainty about what imperfect competition really means by parameters of conjectural variations (or almost equivalently conjectured supply functions). Suppose we do this. Did we solve all our difficulties?

Consider the two level models described in section 2 where generators exercise market power both on the energy and transmission markets. This may be a quite reasonable assumption of market power but it is not the sole one. It indeed all depends on what is commonly referred to as the relevant market for that service, that is the extent of the market where an agent is constrained by competitive forces. And, even though the questions remains largely unexplored, relevant market can be different for energy and transmission. Transmission being a relatively complex issue, consider the energy and CO2 allowance markets that probably constitute the clearest case of what is at stake: while one might conceive that the market power of a generator is high in energy because for instance lack of interconnection capacities, its market power on CO2 allowances is likely to very weak if not null as this is a global (in our case European) market where may more agents than electricity generators intervene. The introduction of market power therefore requires making differentiated assumptions of market power on different markets; can one really do this with a sufficient degree of confidence? Suppose again than this has been done, did we eliminate our difficulties?

Ancillary services that are those services that are determined by the architecture of the market make it possible to select different assumptions of leader follower in the construction of the EPEC model. The model of section 2 assumes that the generators are the leader and the transmission system operator acts passively. This is by far not an obvious assumption when transmission system operators are ruled by incentive contracts. Transmission system operators are indeed true monopolies that are regulated in one way or another. Some regulations do not incentivise for efficiency, which allows one to suppose that TSO act passively as in our model of section 2. Other organisations allow for some profit maximizing behavior, which implies that TSO can exert some market power. Depending on the assumptions made, the upper level of the EPEC can consists of generators or TSOs. More generally market power allows one to construct EPEC of quite different hierarchical structures depending on the assumptions made on the leaders and the followers. But suppose this is done, can one assume that all problems are solved.

Consider the simple Cournot problem of section 2 and assume that generators are price takers with respect to transmissions. Yao et al [3] show that even this simple assumption leads to different representation of market power on the generation side.

To sum up, market power is a complex issue that requires a lot of assumptions to be modeled. The effect of this multiplicity of assumptions on the final result clearly requires careful elaboration.

C. And some important technical problems

Notwithstanding our very imperfect knowledge of market power in restructured electricity markets, the analysis of market power cannot be avoided. While one cannot ascertain the extent to which market power is exercised, the different possibilities mentioned above can give us some insight on how it could be exercised and what possible "proportional" (in European law parlance) remedies one can devise. The question is particularly crucial when it comes to investment: agents can indeed exercise market power by restrictive investment and this type of abusive practice is impossible to detect. Suppose that one is willing to accept some mixes of perfect and imperfect competition as realistic paradigms to describe existing situations, that we construct an EPEC model of this situation and then solve it. Are all our difficulties over?

We already alluded to the fact that EPEC are difficult to solve. The problem is more serious; we are today not in position, when an algorithm does to solve an EPEC, to detect whether it is because the algorithm failed or because there is truly not pure strategy equilibrium of the EPEC problem. We mentioned before both the absence of pure strategy equilibrium and the existence of multiple equilibrium as real issues for which economic theory should provide more help than what it actually does. Alternatively one could request that the mathematical programming community makes it possible to detect for sure these cases from the failure of the algorithm. Economic theory provides some limited help in case of absence of pure strategy equilibrium. This can sometimes be overcome with a reasonable interpretation by mixed strategy equilibrium. Today, computation of mixed strategy equilibria is often attempted for problems where agents have finitely many strategies, or by discretizing continuous strategy sets. Is this suitable for EPEC models of markets?

REFERENCES

[1] C. A. Berry, B. F. Hobbs, W. A. Meroney, R. P. O'Neill, and W. R. Stewart Jr, "Understanding how market power can arise in network competition: a game theoretic approach," *Utilities Policy*, vol. 8, pp. 139–158, 1999.

[2] J. B. Cardell, C. C. Hitt, and W. W. Hogan, "Market power and strategic interaction in electricity networks," *Resource and Energy Economics*, vol. 19, pp. 109–137, 1997.

[3] J. Yao, I. Adler, and S. Oren, "Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network," Department of IEOR, University of California, Berkeley, California," Manuscript, 2005.

[4] X. Hu, D. Ralph, E. Ralph, P. Bardsley, and M. Ferris, "Electricity generation with looped transmission networks: bidding to an iso," University of Cambridge, The CMI Electricity Project, Department of Applied Economics, Cambridge, UK, working paper CMI EP 65, revised 2004.

[5] Z. Q. Luo, J.-S. Pang, and D. Ralph, *Mathematical Programs with Equilibrium Constraints*. Cambridge, UK: Cambridge University Press, 1996.

[6] X. Hu, "Mathematical programs with complementarity constraints and game theory models in electricity markets," Ph.D. dissertation, Department of Mathematics and Statistics, The University of Melbourne, Parkville, Victoria, Australia, 2003.

[7] X. Hu and D. Ralph, "Using EPECs to model bilevel games in restructured electricity markets with locational prices," University of Cambridge, Judge Business School, Cambridge, UK, Manuscript, 2005.

[8] F. Facchinei and J. Pang, *Finite-Dimensional Variational Inequalities and Complementarity Problems I, II*. Springer, 2003.

[9] S. Dirkse and M. C. Ferris, "The PATH solver: A non-monotone stabilization scheme for mixed complementarity problems," *Optimization Methods and Software*, vol. 5, pp. 123–156, 1995.

[10] B. Hobbs and U. Helman, "Complementarity-based equilibrium modelling for electric power markets," in *Modeling Prices in competitive electricity markets*, D. Bunn, Ed. John Wiley and Sons, 2004, ch. 3.

[11] Y. Smeers, "How well can one measure market power in restructured electricity systems?" CORE and INMA, Universite catholique de Louvain, Louvain-La-Neuve, Belgium, CORE discussion paper 2005/50, 2005.

[12] R. J. Green and D. M. Newbery, "Competition in the British electricity spot market," *Journal of Political Economy*, vol. 100, pp. 929–953, 1992.

[13] H.-P. Chao and S. Peck, "An institutional design for an electricity contract market with central dispatch," *The Energy Journal*, vol. 18, pp. 85–110, 1997.

[14] R. Fletcher, *Practical Methods of Optimization*. Wiley, 1987.

[15] M. Anitescu, "On using the elastic mode in nonlinear programming approaches to mathematical programs with complementarity constraints," *SIAM J Optimization*, vol. 15, no. 4, pp. 1203–1236, 2005.

[16] R. Fletcher, S. Leyffer, D. Ralph, and S. Scholtes, "Local convergence of SQP methods for mathematical programs with equilibrium constraints," University of Dundee, Department of Mathematics, University of Dundee, Dundee, UK, Numerical Analysis Report NA 209, 2002, to appear in *SIAM J Optimization*.

[17] S. Stoft, *Power System Economics: Designing Markets for Electricity*. Wiley-IEEE Press, 2002.

[18] R. Wilson, "Architecture of electric power markets," *Econometrica*, vol. 70, pp. 1299–1340, 2004.

[19] C. P. and S. Stoft, "The convergence of market design for adequate generation capacity — with special attention to the caiso's resource adequacy problem," Department of IEOR, University of California, Berkeley, California," A White Paper for the Electricity Oversight Board, April, 2006.

[20] Y. Smeers, "Market incompleteness in regional electricity transmission, Part I: the forward market," *Network and Spatial Economics*, vol. 3, pp. 151–174, 2004.

[21] ——, "Market incompleteness in regional electricity transmission, Part II: the forward and real time markets," *Network and Spatial Economics*, vol. 3, pp. 175–196, 2004.