Electricity generation with looped transmission networks: Bidding to an ISO*

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June 15, 2004

Abstract

This paper uses a bilevel game to model markets for delivery of electrical power on looped transmission networks. It analyzes the effectiveness of an independent system operator (ISO) when generators (and, in some cases, retailers) have market power. Generators (respectively retailers) bid a single parameter of their quadratic supply (demand) functions to the ISO, which taking these bids at face value, maximizes welfare subject to transmission constraints. Our numerical results suggest that transmission limits can have important impacts on market efficiency and the delivery of consumer welfare, but that market power may not significantly reduce economic efficiency. We also find that equilibrium outcomes are highly sensitive to firms' strategy spaces, and, contrary to the results of simpler models and/or published intuitions, that:

1. In the presence of transmission congestion and loop flows, supply function equilibria (SFE) are not bounded from above by Cournot equilibria, so Cournot equilibria may be more efficient than SFE;
2. Allocation of transmission rights to generators can reduce efficiency;
3. Uncertainty appears to only weakly reduce market power; and
4. Countervailing power on the part of buyers can lower efficiency.

*The authors would like to thank Andreas Ehrenmann, David Newbery and Karsten Neuhoff, as well as an anonymous referee, for discussions and suggestions that have improved the paper.

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1 Introduction

This paper, which is derived from the first author's doctoral thesis [28], focuses on pool-type electricity markets as operated in Australia, England and Wales, New Zealand, and some parts of the United States, including the Pennsylvania-New Jersey-Maryland (PJM) electricity market [42]. In these markets, an independent system operator (ISO) determines prices by maximizing "social welfare" based on generators' supply function bids and on known or possibly bid demand functions, subject to transmission constraints. Limits in transmission capacity mean the ISO potentially sets a different electricity price at each node of the transmission network (sometimes referred to as locational marginal prices [9, 26]). That is, there are multiple interlinked nodal markets, rather than a single market.

The transportation of electrical power on a transmission network from net generation nodes to net consumption nodes is governed by the Kirchoff Laws [45]. Due to power flow and counterflow, the actual transmission capacity of any link in the network depends on the load flow pattern, that is, the location and quantity of any injection or withdrawal of power on the entire network. As a consequence, transmission of power is different from the transportation of an ordinary commodity in a spatial market. This difference is particularly marked when the network contains loops, and there are transmission capacity limits.[1] Reliability and security constraints, power loss, and factors can also be important [2].

In these circumstances, the process by which electricity dispatches are determined has a critical impact on economic efficiency (see, for example, [30, Chapter 6] and [32]).

There are many models of markets for electricity generation, where an ISO coordinates generators', and sometimes retailers', bids, that do not account for loop flows and transmission constraints.[2] Papers that model these features of electricity networks include [5, 8, 15, 24, 48, 51]. For strategic generators, a significant difference between the former and latter models is that, in the latter, generators must solve a bilevel optimization problem,[3] because their profit functions depend on the dispatch resulting from the ISO's (nontrivial) optimization problem. In general, bilevel optimization problems are difficult to solve (see, for example, [35]).

This paper presents, for two different looped-networks, numerical examples on the effec-

[1] On the tractability of networks without loops, see, for example, [10].

[2] For example, [1, 3, 12, 17, 23, 27, 44, 46, 49] as well as some of those previously mentioned.

[3] A typical bilevel problem is an optimization whose constraints require that certain of its variables, called the "lower-level" variables, solve an optimization sub-problem that depends parametrically on the remaining, "upper-level", variables. An example from economics is the leader's problem in a Stackelberg game.

tiveness of the ISO when generators (and, in some cases, retailers) have market power. A simple loss-less model of electricity transmission is used—see [9, 20]. Following [5], it is assumed that cost and utility functions are quadratic, that generators non-cooperatively bid a single parameter of their supply function (the other parameter is known), and that their bids are constrained by the ISO to lie within a conjectured range. Two-parameter bids, which would allow market participants full freedom in specifying their supply function, generate multiple equilibria. This is, in the absence of network constraints, a phenomenon known when more general supply functions bids are allowed ([34]; see discussion in Subsection 2.2 and Subsection 3.4 below). Following others, including [24], we call the equilibria of the examined games, where they exist, conjectural supply function equilibria (CSFE). Demand-side bidding is allowed in some cases. The paper's results are obtained using novel (at least to economics) optimization techniques and numerical methods [29], and [28].[4]

The paper compares different CSFE with each other, and in some cases, to Cournot equilibria, and to the case where market participants reveal their actual supply/demand information to the ISO, the "truth-telling" outcome (call it TT). While TT is relatively easy to determine, and reporting it is not uncommon (for example, see [12, 39, 44, 47, 50]), TT should not always be taken as a normative benchmark. Rather, the relevant contrasts are between different realistic situations, which may, but need not, include TT (see discussion in Section 4). Moreover, the paper's results should be understood as counter examples—demonstrations of the possibility of certain outcomes.

The paper is organized as follows. In Section 2, the bilevel game (due to [5]) is formulated and two networks are defined: the simplest network loop, and a stylized model of the PJM. Section 3 gives a brief description of the numerical method used to solve the CSFE. The main results of the paper appear in Section 4. Four comparisons are made, often providing counter examples to standard conclusions. The comparisons and results are:

§4.2. A comparison between (1) an ISO with separate generators, and (2) Cournot competition, showing that changes in market conditions, including players' strategy spaces, can significantly impact on market outcomes. For example, in the presence of transmission congestion, SFE need not be bounded from above by Cournot equilibria, (contrary to the non-bilevel case [34, 23]). Similarly, a comparison of SFE shows that an increase in competition upstream of a congested node can raise or lower consumer welfare, depend-

[4]Algorithms for the gaming scenarios of the models in [5, 51] are developed in [24, 48].

ing on the allowed bidding strategy. The same increase in competition in the Cournot case, which only had a minimal impact on economic efficiency in the examined SFE, brought price close to TT, dramatically raising economic efficiency and consumer welfare. Moreover, the efficiency comparisons between the SFE and Cournot equilibria are all the more striking since the cost, utility and bid-space assumptions greatly favor the ISO (the ISO is assumed to know everything about the problem except the point value of a single parameter from each participant).

§4.3. A comparison between (1) an ISO with separate generators, (2) a vertically integrated generation and transmission monopolist (so there is no need for an ISO), and (3) an ISO and a generator-only monopolist. Several examples show that the granting of transmission rights to a generation monopolist reduces economic efficiency (contrary to, for example, a suggestion by Berry *et al.* [5, p. 157]).[5]

§4.4. A comparison between the cases of generator certainty and uncertainty about transmission constraints and demand. In our examples uncertainty reduces market power, but only weakly.[6]

§4.5. A comparison of bidding with market power by (1) generators only, and (2) generators and consumers, showing that bidding from consumers with market power can harm efficiency, and hence may not be an attractive means of curbing market power on the generation side, contrary to [50, 33] and suggestions by [39].[7]

The numerical results also suggest that (1) transmission constraints can have important and difficult to predict impacts on economic efficiency, and consumer and producer surplus; and (2) the effect of market power on economic efficiency tends to be small, even in the case of monopoly, with a more pronounced impact on consumer welfare.

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[5]Specific analysis of transmission rights, such as incorporating Transmission Congestion Contracts (TCCs) [26] into participants' profit functions, is found in, for example, [20]; [40, pp. 13 ff]; [31]; and [21]. For a bilevel example see [8].

[6]Oren, 1997, [39] recommended exploration of the issues in §4.4 and §4.5 below.

[7]Berry *et al.* [5, p. 156] raise the issue; Oren *et al.* [40, p. 12] demonstrate there may be no equilibrium with bilateral contracts.

2 Problem formulation

This section models the bilevel game (following [5]). Subsection 2.1 presents the optimal power flow problem, in our case a quadratic program, which is solved by the ISO to determine pricing and dispatch of electricity. The optimal power flow problem is determined by the players' bids and the network structure, the latter being known and fixed. Therefore pricing and dispatch depends implicitly on all players' actions; this dependency is the basis of each participant's bilevel profit[8] maximization. This bilevel programming model is developed in Subsection 2.2. The final subsection gives the data specifying the two networks that our subsequent computational experiments will be based on.

2.1 The ISO's pricing and dispatch problem

We consider a loss-less DC version of power flow and a simplified version of the ISO's problem as set up in [9, 20]. Suppose we are given a transmission network with $N+1$ nodes (sometimes called buses) labelled as $0, 1, \ldots, N$, and a set $\mathcal{L}$ of links, where the link between Node i and j is written ij. Node 0 is taken as the swing bus. In the DC formulation of a load flow problem, the swing bus has zero "distribution factors", that is, no contribution to flow on any link (see the optimal power flow problem (1) below).

The transmission network is managed by an ISO, and market participants (generators and consumers) have complete information about the ISO's operation procedure and all other participants' cost/utility functions. The market broadly works as follows:

> Generators and consumers submit bids representing their respective supply and demand functions to the ISO. The ISO, taking account of the network conditions, solves a social welfare maximization (or a social cost minimization) problem assuming the bids are truthful, and announces a dispatch for each player and possibly distinct prices at each node. This is called a nodal pricing scheme. Moreover, the existence of different prices at any two nodes implies network congestion, namely, one or more of the transmission lines is carrying electricity at its maximum capacity, where capacity is a physical attribute of a line that we assume is fixed and known. [9] Generators and consumers then clear the market according to the

[8] In the case of a consumer, allow profit to refer to the consumer's surplus.

[9] In reality, line capacity depends on the heat generated in transmission, air temperature and other factors.

dispatch. Finally, the ISO collects payment from consumers and pays generators according to the scheduled dispatch and nodal prices.

We assume for simplicity that there is a single[10] generator or consumer at any Node i, and that the associated actual cost or utility function is a quadratic function in quantity q_i, either cost, $A_iq_i + B_iq_i^2$ ($q_i \geq 0$), or utility, $-A_iq_i - B_iq_i^2$ ($q_i \leq 0$), respectively, where each A_i and B_i is assumed to be positive. Note that a consumer at Node i has a utility that is non-decreasing in the quantity consumed ($-q_i$) in the range $[0, A_i/(2B_i)]$. At the equilibria in all the presented numerical results, a consumer at Node i is dispatched a quantity in this range.

We further assume that the generator (or consumer) bids $a_iq_i + b_iq_i^2$ (or $-a_iq_i - b_iq_i^2$) to the ISO in the form of a pair of coefficients (a_i, b_i) (in both cases). The ISO solves the following problem of minimizing the apparent social cost over all $N + 1$ nodes in the network:

$$\begin{array}{lll} \underset{q_0,\cdots,q_N}{\text{minimize}} & \sum_{i=0}^{N}(a_iq_i + b_iq_i^2) & \\ \text{subject to} & q_0 + q_1 + \cdots + q_N = 0 & : \lambda \\ & -C_{ij} \leq \sum_{k=1}^{N} \phi_{ij,k}q_k \leq C_{ij}, \quad i < j,\ ij \in \mathcal{L}, & : \underline{\mu}_{ij},\ \overline{\mu}_{ij} \\ & q_i \geq 0, \quad i: \text{ generator}, \quad q_i \leq 0, \quad i: \text{ consumer}, & : \nu_i \end{array} \qquad (1)$$

where C_{ij} denotes the transmission limit on link ij, $\phi_{ij,k}$ denotes the distribution factor, that is, the contribution of an injection (or withdrawal) at Node k to the link ij. These distribution factors are determined by the susceptance of the transmission lines and the choice of the swing bus. (Designating the swing bus as Node 0 means that $\phi_{ij,0} = 0$ for all links ij, and explains why $k = 0$ is omitted from the sum indices in the capacity constraints.) Lagrange multipliers are displayed after the corresponding constraints in (1). The optimal solution to (1) is denoted by $q = (q_0(a, b), \cdots, q_N(a, b))$, or simply $q = (q_0, \cdots, q_N)$, a vector-valued function of $(a, b) = (a_0, \cdots, a_N, b_0, \cdots, b_N)$. The above Lagrange multipliers λ, $\underline{\mu}_{ij}$, $\overline{\mu}_{ij}$ and ν_i are taken to be those corresponding to the optimal solution.

The ISO sets p_k, the price of electricity at Node k, equal to

$$p_k = -\lambda \;\; - \sum_{i<j,\, ij\in\mathcal{L}} (-\underline{\mu}_{ij} + \overline{\mu}_{ij})\phi_{ij,k}.$$

That is, at Node k, either a consumer would pay $-p_kq_k$ units to the ISO for consumption of $-q_k$ units, or a producer would be paid p_kq_k by the ISO for generation of q_k units.

[10]In the numerical examples in this paper, we allow for multiple participants (generators or consumers) at a single node. Generators or consumers located at the same node are assumed to have identical cost or utility functions. However, they are treated as independent participants where they play the game.

Notice that in the absence of binding transmission constraints, p_k equals the shadow price ($-\lambda$) of the physical (as opposed to economic) requirement that electricity generated equals electricity consumed. As a result, with no binding transmission constraints, p_k is the standard CSFE price.

It is also true that, if $q_k \neq 0$, then, at equilibrium, p_k equals the marginal cost/value of each player at Node k [9], that is,

$$p_k = a_k + 2b_k q_k. \tag{2}$$

(as can be seen from the first equation of the ISO's Karush-Kuhn-Tucker or KKT conditions in (4) below).

Some remarks on the problem (1) are in order. First, in real electricity markets, the format of bids is typically a step-wise function and more operating constraints are included in the problem, see [2]. We only include the transmission constraint to see its role in competition among market participants. This makes sense when the transmission network is in a stable state of operation because other operating constraints, such as reliability and security issues become inactive. In some markets, such as the Australian National Electricity Market, bids are *not* firm. Instead, generators are allowed to change their quantity offers, however, their price offers are not allowed to change after the closing time (12:00 noon) of a trading day.

2.2 A market participant's bidding problem

Consider the question of how a profit-maximizing player can take advantage of the market clearing procedures described above. That is, what supply/demand function should player i submit to the ISO to achieve the player's own maximal profit?

Recognizing, from Equation (2), that player i's price is $a_i + 2b_i q_i$, then its profit maximization problem is:

$$\begin{array}{ll} \underset{a_i, b_i}{\text{maximize}} & (a_i + 2b_i q_i)q_i - (A_i q_i + B_i q_i^2) \\ \text{subject to} & \underline{A}_i \leq a_i \leq \overline{A}_i \\ & \underline{B}_i \leq b_i \leq \overline{B}_i \\ & q_i \text{ such that } q = (q_0, \ldots, q_i, \ldots, q_N) \text{ solves (1)} \\ & \text{given other participants' bids } (a_{-i}, b_{-i}) \end{array} \tag{3}$$

recalling that $A_i q_i + B_i q_i^2$ ($q_i \geq 0$) is the generator's actual cost, and $-A_i q_i - B_i q_i^2$ ($q_i \leq 0$) is the consumer's actual utility.

The constants $\underline{A}_i, \overline{A}_i$ and $\underline{B}_i, \overline{B}_i$ are lower and upper bounds for a_i and b_i that are based on industry knowledge (available to all participants) and imposed by the ISO (hence the resulting equilibria are CSFE). The constants are assumed to satisfy

$$0 < \underline{A}_i \leq A_i \leq \overline{A}_i \quad \text{and} \quad 0 < \underline{B}_i \leq B_i \leq \overline{B}_i.$$

This problem is a bilevel program, where the lower-level problem is that q must solve the optimal power flow problem of the ISO, (1). Thus we refer to (1) and (3), where the latter refers to the collection of problems for each player i, as a bilevel game. Looking ahead to Sections 3 and 4, note that our main solution procedure starts by replacing the constraint on q in (3) with the KKT conditions of (1), that is, reformulating the bilevel problem as a mathematical program with equilibrium (indeed complementarity) constraints, MPEC, and then replacing all participants' MPECs by stationary conditions for these MPECs. This overall reformulation renders the entire game into the form that has recently come to be called an equilibrium problem with equilibrium constraints, EPEC [41], or, more precisely, an EP with complementarity constraints, EPCC.

While we examined the case where participants bid pairs of coefficients, (a_i, b_i), these generated multiple equilibria (as might be expected, in the absence of network constraints, when a bid is applied to a unique and known demand function [34, 37, 23, 38]; see also the discussion in Subsection 3.4 below). As a result, the examples in the paper further assume all participants know either A_i for all players, or B_i for all players, and the ISO restricts bidding for each firm to its unknown parameter, fixing its known coefficient to the true value. That is, where A_i is known then b_i only is bid, requiring the additional constraint for the game that $a_i = A_i$ (or define $\underline{A}_i = A_i = \overline{A}_i$). Refer to this as the bid b case. The bid a game is correspondingly similar.

In this paper we are interested in whether interaction among the participants leads the market to any *stable*, that is, *equilibrium*, state—see discussion in Subsection 3.1.

2.3 The networks modelled

We examine the following two networks:

A three-node network, Figure 1, which is extensively used [9, 26, 28, 20, 39, 47] to illustrate loop effects, or the difference between transmission networks and ordinary commodity networks.

The distribution factors for the three-node network are $\phi_{01,1} = 2/3, \phi_{01,2} = 1/3, \phi_{02,1} = 1/3, \phi_{02,2} = 2/3, \phi_{12,1} = 1/3, \phi_{12,2} = -1/3$. The distribution factors have an intuitive explanation for this three-node example. For example, power injected at Node 1 can flow directly to Node 0 (the swing bus) via Link 01, or by transiting Link 12 and then Link 02. If we assume all lines have the same physical parameters, so transit via the two links is twice as "hard" as by the single link, then two thirds of the power flows through Link 01 and one third through Link 12 and Link 02.

A five-node network, Figure 2, which is taken from a PJM's tutorial course material [42]

The distribution factors of power injected at the five nodes without Node 0 are:

Link 01: $\phi_{01,1} = 0.6363, \phi_{01,2} = 0.3636, \phi_{01,3} = 0.5454, \phi_{01,4} = 0.4545$.

Link 02: $\phi_{02,1} = 0.3636, \phi_{02,2} = 0.6363, \phi_{02,3} = 0.4545, \phi_{02,4} = 0.5454$.

Link 12: $\phi_{12,1} = 0.2727, \phi_{12,2} = -0.2727, \phi_{12,3} = 0.0909, \phi_{12,4} = -0.0909$.

Link 13: $\phi_{13,1} = 0.0909, \phi_{13,2} = -0.0909, \phi_{13,3} = -0.6364, \phi_{13,4} = -0.3636$.

Link 24: $\phi_{24,1} = -0.0909, \phi_{24,2} = 0.0909, \phi_{24,3} = -0.3636, \phi_{24,4} = -0.6364$.

Link 34: $\phi_{34,1} = 0.0909, \phi_{34,2} = -0.0909, \phi_{34,3} = 0.3637, \phi_{34,4} = -0.3637$.

While we use these two networks throughout, we vary the capacities, C_{ij} on each link (as well as the bounds on bids, $\underline{A}_i, \overline{A}_i, \underline{B}_i, \overline{B}_i$).

We will also vary the number and type of players at each node.

Figure 1: A three-node network

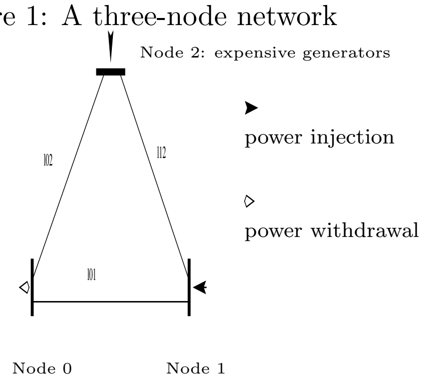
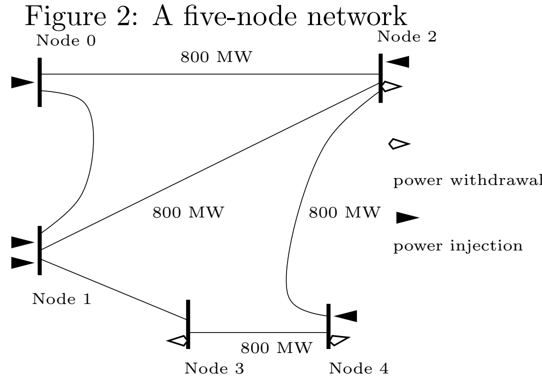


Figure 2: A five-node network



3 Calculation of Nash stationary equilibria

Motivated by questions of existence and computational tractability, in Subsection 3.1 we propose two solution concepts: Nash stationary and local equilibria, which are weak variants of the classical Nash equilibrium concept.

In Subsection 3.2 we formulate the bilevel problem faced by each player as a mathematical program with equilibrium constraints. We explain how the stationary conditions of the latter problems yield a large KKT system, called All-KKT, that represents Nash stationary equilibria. Based on previous work [19], which analyzed a nonlinear programming method applied to MPECs, we anticipated that the PATH package [13, 18] for solving KKT systems would be useful in solving All-KKT and hence in finding Nash stationary points.

Subsection 3.3 provides details on solving the All-KKT system. Preliminary numerical tests on markets over the five-node network with randomly generated cost/utility data indicate, as hoped, that finding Nash stationary points need not be computationally intractable. Checking second-order optimality conditions established that all the stationary points were *local* Nash equilibria.

Subsection 3.4 concludes with a discussion of uniqueness of Nash equilibria. We find that restricting bids to a single parameter, a_i or b_i, is important.

3.1 Solution concepts

If we express the dispatch q from the ISO's problem as a vector-valued function of (a, b) or, since we are thinking of the ith player's strategy, $(a_i, b_i, a_{-i}, b_{-i})$, then the bilevel game, (1) and (3), takes the form of a standard game theory model. Participant i has a profit function which can be written as $\pi_i(a_i, b_i, a_{-i}, b_{-i})$, that it seeks to maximize by choosing its strategies within prescribed bounds: $\underline{A}_i \leq a_i \leq \overline{A}_i$ and $\underline{B}_i \leq b_i \leq \overline{B}_i$. Two difficulties immediately arise. First, the profit function π_i is generally nonconcave in (a_i, b_i), which throws doubt on the existence of pure strategy Nash equilibria. Indeed, lack of existence is well-known in this situation; see, for example, [5, Footnote 8, p. 143], [48] and also [29] where a small example of this type is analyzed. Second, the dispatch function $q(a_i, b_i, a_{-i}, b_{-i})$ is usually nondifferentiable and cannot be readily, if at all, expressed analytically. We therefore propose a solution concept, for the bilevel game, that is weaker than the pure Nash, but is somewhat more amenable to computation.

To relax the typical concavity assumption on profit functions, we apply a local version of the Nash equilibrium concept. A *local Nash equilibrium* is a point which is a Nash equilibrium for the restricted problem that is defined by restricting the strategy space (the Cartesian product of individual participants' strategy spaces) to a sufficiently small neighborhood of this point. An even weaker equilibrium concept stems from computational practice in optimization, where algorithms are commonly designed only to find a stationary point of an optimization problem, that is, a point satisfying a first order optimality condition. Based on this, we call a point a *Nash stationary equilibrium* if the associated strategy of each participant satisfies the stationarity condition for their profit maximization problem. It is not yet clear what such a stationary condition for (3) might be; we develop this next.

3.2 Model development

Since the ISO's problem (1) is a strictly convex quadratic program, a dispatch $q(a,b)$ solves (1) given (a,b) if and only if $q(a,b)$ is stationary for (4), that is, there exist multipliers corresponding to the constraints that satisfy the usual KKT conditions at $q(a,b)$ [4]. Therefore given other participants' bids (a_{-i}, b_{-i}), participant i's problem becomes the following mathematical program with equilibrium constraints:

$$
\begin{array}{ll}
\underset{a_i, b_i, q, \lambda, \underline{\mu}, \overline{\mu}, \nu}{\text{maximize}} & (a_i + 2b_i q_i) q_i - (A_i q_i + B_i q_i^2) \\
\text{subject to} & \underline{A}_i \le a_i \le \overline{A}_i, \quad \underline{B}_i \le b_i \le \overline{B}_i \\
\text{(ISO-KKT)} & \left\{ \begin{array}{l}
\text{where, for each } j = 0, \dots, N \quad \text{and } mn \in \mathcal{L} \text{ with } m < n: \\
a_j + 2b_j q_j + \lambda + \sum_{mn \in \mathcal{L}} \phi_{mn,j} (\overline{\mu}_{mn} - \underline{\mu}_{mn}) - \nu_j = 0 \\
q_0 + q_1 + \cdots + q_N = 0 \\
C_{mn} + \sum_{k=1}^N \phi_{mn,k} q_k \ge 0, \ \underline{\mu}_{mn} \ge 0, \ \underline{\mu}_{mn} (C_{mn} + \sum_{k=1}^N \phi_{mn,k} q_k) = 0 \\
C_{mn} - \sum_{k=1}^N \phi_{mn,k} q_k \ge 0, \ \overline{\mu}_{mn} \ge 0, \ \overline{\mu}_{mn} (C_{mn} - \sum_{k=1}^N \phi_{mn,k} q_k) = 0 \\
q_j \ge 0, \ \nu_j \ge 0, \ q_j \nu_j = 0 \quad \text{if player } j \text{ is a generator} \\
q_j \le 0, \ \nu_j \le 0, \ q_j \nu_j = 0 \quad \text{if player } j \text{ is a consumer}
\end{array} \right.
\end{array}
\tag{4}
$$

where $\underline{\mu}, \overline{\mu}, q, \nu$ denote vectors, the respective components of which are $\underline{\mu}_{mn}, \overline{\mu}_{mn}, q_j, \nu_j$. This problem may also be called a mathematical program with complementarity constraints.

In [28] we formulate the standard first order necessary (that is, KKT) conditions of (4) for each player i. We then form a system consisting of all players' KKT conditions in which the ISO's KKT conditions, that appear as (ISO-KKT) in the constraints of (4), are in common (so the variables $q, \lambda, \underline{\mu}, \overline{\mu}, \nu$ are in common), hence appear only once. We call this system All-KKT. However, each participant's KKT system will include different KKT multipliers for the corresponding common constraints. Note further that a solution of All-KKT is necessarily a Nash stationary equilibrium of the bilevel game; see [29] for discussion of a more general situation under which these concepts are equivalent. For brevity, we omit formulation of

All-KKT here; see [28, Chapter 7] for details.

Formally, we are just using a KKT system (and solver) in the same way that a classical game with constrained strategy sets can be modelled by aggregating the KKT systems of all players. However, the KKT system for (4) is not an "ordinary" KKT system because the problem is not an ordinary nonlinear program. MPECs are atypical in the class of general nonlinear programs in that standard numerical stability conditions on the constraints, termed constraint qualifications, do not hold; see [28] and references therein, including the monograph [35], for a general exploration of MPECs. Nevertheless, in many situations, standard NLP methods are very effective. In particular, the analysis of [19], which explains why the sequential quadratic programming method for nonlinear programs can be efficient for MPECs, is a strong indicator for applying a sequential linear complementarity method, such as the path search method [43] that is embodied in PATH, to All-KKT.

3.3 On computation

PATH is applied directly to All-KKT to find a Nash stationary point of the bilevel game. We write the All-KKT system, and subsequent checking procedures mentioned next, in the GAMS modelling language, and present results compiled from the WHEEL machine at The University of Wisconsin.

The algorithm can fail to find a Nash stationary point. According to our computational experience with the bilevel game (1) and (3), several factors such as the capacity limits on links, the location and number of generators and consumers, their cost/utility functions, bounds on bid variables, as well as starting points for PATH, may have significant effects on the solvability of the game. For example, using the built-in uniformly distributed random number generator of GAMS [7], we produced 30 cost/utility functions for a five-node network—see Table 1. We then imposed a capacity limit 500 MW on link 13 and 800 MW on other links of the five-node network in Figure 2, and required $\underline{A}_i = 0.0, \overline{A}_i = 100, \underline{B}_i = 0.0001$ and $\overline{B}_i = 2$ for all generators. We tried to find an equilibrium for the 90 games generated by allowing bids of a only, b only, and a and b together, under each of these 30 network and bid constraint configurations. Out of these 90 problems, 82 were solved from starting points determined by the default action of the GAMS/solver interface (30, 27, and 25, respectively, for bid a, bid b, and jointly bid a and b).

If a Nash stationary equilibrium is obtained, we check to see if a second-order sufficient

Table 1: Location of generators and consumers and actual cost/utility functions

node	# generators	# consumers	coefficient a	coefficient b
0	1	0	uniform(30.16,30.45)	uniform(0.030,0.035)
1	2	0	uniform(30.20, 30.40)	uniform(0.035, 0.040)
2	1		uniform(33.5, 33.60)	uniform(0.06, 0.085)
		1	uniform(262.15,262.80)	uniform(0.06,0.065)
3		1	uniform(240.15,240.85)	uniform(0.065,0.068)
4	1		uniform(30.30, 30.45)	uniform(0.02, 0.04)
		1	uniform(243.45,243.50)	uniform(0.055,0.058)

GAMS default seed = 3141 is applied here

condition holds, that is, we attempt to verify that the stationary point for each player is indeed a local maximum of its profit/utility problem (4). Given that MPECs have properties that are atypical in general nonlinear programming, checking sufficient conditions is not always straightforward; once again, we refer the reader to [28] for a full description. Nevertheless in all our experiments, by good luck or, in some cases, a favorable MPEC structure, all Nash stationary points were found to be local Nash equilibria.

3.4 Bidding strategies considered

We focus on the two scenarios in which either each participant only bids the linear part a_i of its cost/utility function, with the actual coefficient B_i of the quadratic part assumed to be known to the ISO, or each participant only bids the quadratic coefficient b_i, with the linear coefficient A_i assumed to be known, for the following reasons.

1. For bid a and bid b games, our numerical experience indicates that algorithm convergence is more or less independent of the starting points, that is, when successful, the algorithms we tested arrived at the same Nash stationary equilibrium regardless of the initial states of the game. In contrast, when bidding both a and b, given an initial point (a^0, b^0) the algorithm produces a stationary point (a, b) with a at or near a^0; this multiplicity of Nash stationary equilibria makes it difficult to compare the game outcome(s) with other market situations. Indeed, it is not hard to give an example with a continuum of local Nash equilibria, see [28], which is consistent with the related analysis of [14].

2. Suppose we have a Nash stationary equilibrium point for the bid a or bid b game. It can be seen from the KKT conditions for each participant's problem (4) that if the bounds

$\underline{A}_i \leq a_i \leq \overline{A}_i$ and $\underline{B}_i \leq b_i \leq \overline{B}_i$ are inactive at that point for all participants, then the point also satisfies the stationary conditions for jointly bidding a and b. That is, this point must be a Nash stationary equilibrium for the bid both a and b game.

4 Market efficiency: Numerical examples

In this section, a number of comparisons of local (non-cooperative) Nash equilibrium or Nash stationary points are made to illustrate possible impacts of different market environments and regulatory regimes. In all these comparisons, only generators bid strategically,[11] except in Subsection §4.5, where strategic bidding by generators and consumers together is examined.

If generators and consumers were to bid truthfully, then the ISO would be able to maximize economic efficiency. However, in the examined games, TT is not achieved due to the presence of market power and the ISO's, albeit limited, ignorance. In these circumstances, nodal prices are distorted by market participants' strategic behaviors. While TT is used as a benchmark in the analysis that follows, it is important to recognize that a difference between TT and the reported market outcomes does not indicate market failure if TT cannot be realistically achieved [11, 16]. That is, TT need not be a normative benchmark, but instead only the upper, perhaps unachievable, bound for economic efficiency.

TT is not, however, without relevance. In some cases, most especially where transmission constraints are absent, sufficient competition may approximate TT (in line with [5, 50]), suggesting a normative preference for competition and the easing of transmission constraints.[12] Moreover, because of the repeated nature of the bidding process, and the relative simplicity of electricity generation, engineering models of the generation process are relatively well-understood. This, especially when coupled with legal requirements on information revelation, may greatly narrow the ISO's uncertainty about cost functions. Further, requirements, supported by fines, to bid truthfully, could make TT more plausible.[13]

[11] This would occur if the ISO was well-informed about consumer demand, or if the utility functions used aggregate many consumers, competition among which leads to revelation of their summed demand.

[12] We hope to explore the impact of competition on efficiency more fully in future work.

[13] Referees advise us that, in Latin America, it is common for bids to be required to be based on audited costs and to hold for six months; and in Chile, under certain conditions, generators are required to bid their marginal cost.

4.1 Definitions

Define:

- **congestion rent** = (total) payment from consumers − (total) payment to generators.
- **social welfare** = consumers' surplus + generators' surplus + congestion rent (where, as is conventional, consumers' surplus is the difference between utility and payments by consumers for energy consumption; and producers' surplus is profit).
- **dead weight loss** (DWL) = 100×(social welfare in TT − social welfare with gaming)/social welfare in TT (note the unconventional expression of DWL as a ratio).
- **welfare share** = participant's surplus/social welfare in the same scenario.
- **(welfare) share index** = welfare share in gaming/welfare share in TT (for each participant).
- **consumer average price** = sum of all consumers' payments/total consumption (for each given scenario).
- **output** = total generation = total consumption (due to the loss-less network).
- $\Delta(X)(Y)$ = 100×(X with gaming − X in TT)/X in TT, where X stands for the aggregate surplus or average price or (congestion) rent or output and Y stands for generators (G) or consumers (C).

  For example, Δ(price)(C) indicates the percentage change "Δ" in consumer (C) average price relative to TT.

  Similarly, Δ(price)(G) denotes the percentage change in average price paid to generators relative to TT.

4.2 CSFE versus Cournot

In this subsection, we use the three-node network of Figure 1, which has one consumer node and two generator nodes, to examine three different bidding models: bid a, bid b and Cournot games (which could be referred to as bid q). In this context, Cournot competition is more plausible than it might initially seem. First, the Cournot model can be naturally applied to an electricity network when net consumption only occurs at one node, since, in that circumstance, a single price is paid by all consumers (we do not model price discrimination within any node). Second, while the Cournot assumption, that firms compete by supplying a chosen quantity at the price the market will bear, often lacks explanation, in our case one can assume the ISO simply requires quantity bids. That is, the Cournot game is no less unrealistic than the (C)SFE models presented here.

There are n_1 generators at Node 1, each having a quadratic cost function $A_{1,k}q_{1,k} + B_{1,k}{q_{1,k}}^2, k = 1, \ldots, n_1$, and there are n_2 generators at Node 2, each having a quadratic cost function $A_{2,k}q_{2,k} + B_{2,k}{q_{2,k}}^2, k = 1, \ldots, n_2$. Aggregate consumer demand is given by $q_0 = -(A_0 - p)/(2B_0) \leq 0$, where p denotes the price for electricity and $-q_0$ the quantity of electricity in demand.

In the Cournot case (see also [47]), the market price is set equal to $A_0 + 2B_0q_0^*$, where q_0^* equals equilibrium output. Each generator k at Node l ($l = 1, 2$) has the following profit maximization problem given other generators' injections $q_{j,i}$ (supply by generator i at Node j, $j = 1, 2$):

$$\begin{array}{ll} \underset{q_{l,k}}{\text{maximize}} & (A_0 - 2B_0(\sum_{i=1}^{n_1} q_{1,i} + \sum_{i=1}^{n_2} q_{2,i}))q_{l,k} - (A_{l,k}q_{l.k} + B_{l,k}{q_{l,k}}^2) \\ \text{subject to} & -C_{mn} \leq \sum_{j \neq 0} \phi_{mn,j} \sum_{i=1}^{n_j} q_{j,i} \leq C_{mn} \\ & q_{l,k} \geq 0 \end{array} \tag{5}$$

where $\phi_{mn,j}$ is the distribution factor of Node j to link mn and C_{mn} is the transmission limit of link mn.

In the results presented in Table 2 and Table 3, there are two generators at Node 1, who have the cheaper generation costs, whereas the single generator at Node 2 has expensive costs, and one consumer at Node 0. The transmission limit on the link between the two generators is varied to demonstrate the uncongested and congested cases, while limits on the other two links are fixed sufficiently large so that they are never reached. In the ISO case, we set bounds constraining the generators' bids, but they turned out to be non-binding.

In Table 2 there is no congestion on the network. In the bid a and bid b SFE, the bids from the two generators located at Node 1 with cheaper generation costs are much further above the true cost function than in the case of the third generator at Node 2. As a result, the ISO has to dispatch more generation from the expensive generator compared with TT. Under Cournot competition, prices and firm profits are higher and output, overall efficiency and consumer welfare lower, than in the SFE (see Table 2). That is, the outcomes of the bid a and bid b games lie between TT and the Cournot outcome, as is consistent with standard

Table 2: Cournot competition vs supply function bidding in a uncongested network

	player	A_i	B_i	dispatch	price	profit	welfare share
TT	0 1 (−)	90.00	0.50	-78.50	11.50	3080.88	0.9815
	1 1 (+)	10.00	0.02	37.58	11.50	28.24	0.0090
	1 2 (+)	10.00	0.02	37.58	11.50	28.24	0.0090
	2 1 (+)	10.50	0.15	3.34	11.50	1.68	0.0005

	player	bid	dispatch	Δ(dispatch)	price	Δ(price)	profit	Δ(profit)
bid a	0 1(−)	N/A	-77.38	**-1.42**	12.62	**9.72**	2993.78	**-2.83**
	1 1(+)	11.21	35.37	**-5.87**	12.62	**9.72**	67.68	**139.65**
	1 2(+)	11.21	35.37	**-5.87**	12.62	**9.72**	67.68	**139.65**
	2 1(+)	10.63	6.64	**98.45**	12.62	**9.72**	7.47	**345.32**
	Δ(price)(C): **9.72**			Δ(price)(G): **9.72**			DWL: **0.08**	
bid b	0 1(−)	N/A	-75.90	**-3.31**	14.11	**22.62**	2880.02	**-6.52**
	1 1(+)	0.0623	32.92	**-12.39**	14.11	**22.62**	113.47	**301.80**
	1 2(+)	0.0623	32.92	**-12.39**	14.11	**22.62**	113.47	**301.80**
	2 1(+)	0.1793	10.05	**200.60**	14.11	**22.62**	21.08	**1157.11**
	Δ(price)(C): **22.62**			Δ(price)(G): **22.62**			DWL: **0.35**	
Cournot	0 1(−)	N/A	-58.23	**-25.82**	31.77	**176.19**	1695.32	**-44.97**
	1 1(+)	N/A	20.93	**-44.29**	31.77	**176.19**	446.98	**1482.77**
	1 2(+)	N/A	20.93	**-44.29**	31.77	**176.19**	446.98	**1482.77**
	2 1(+)	N/A	16.36	**389.36**	31.77	**176.19**	307.88	**18259.93**
	Δ(price)(C): **176.19**			Δ(price)(G): **176.19**			DWL: **7.71**	

For this and similar tables following: under *player*, the node is denoted by the first digit and the player at the node by the second. Consumers/producers are respectively indicated by −/+.

(non-bilevel) SFE problems [34, 23], see also [5, footnote 8].

However, when the network is congested, this need not be true. Cournot competition may be more efficient and result in greater consumer welfare. In Table 3, the expensive generator at Node 2 bids much further above its true cost function than do the two cheaper generators. Seeing such a high bid, the ISO decreases the dispatch of the expensive generator, and because of the transmission limit of link 12 and the need for adequate counterflow, this concomitantly forces the ISO to lower the dispatches from the two cheaper generators.

This illustrates a general principle: a transmission constraint may give an otherwise disadvantaged competitor superior market power [5]. Consequently, transmission constraints can be a major obstacle to increasing competition and producing efficient market results.

A second example, illustrated in Table 4, shows that increasing competition can have markedly different effects on the resulting SFE and Cournot equilibria. Assume there are

Table 3: Cournot competition vs supply function bidding in a congested network

Congested case (5MW limit on link 12)								
	player	A_i	B_i	dispatch	price	profit	welfare share	
TT	0 1(−)	90.00	0.50	-74.81	15.19	2798.63	0.9275	
	1 1 (+)	10.00	0.02	22.45	10.90	10.08	0.0033	
	1 2 (+)	10.00	0.02	22.45	10.90	10.08	0.0033	
	2 1 (+)	10.50	0.15	29.91	19.47	134.17	0.0445	
	player	bid	dispatch	Δ(dispatch)	price	Δ(price)	profit	Δ(profit)
bid a	0 1(−)	N/A	-45.84	**-38.73**	44.16	**190.82**	1050.59	**-62.46**
	1 1(+)	10.60	15.21	**-32.26**	11.21	**2.87**	13.79	**36.81**
	1 2(+)	10.60	15.21	**-32.26**	11.21	**2.87**	13.79	**36.81**
	2 1(+)	72.49	15.42	**-48.44**	77.11	**296.01**	991.44	**638.96**
	Δ(price)(C): **190.82**			Δ(price)(G): **133.00**			DWL: **15.03**	
bid b	0 1(−)	N/A	-43.06	**-42.45**	46.94	**209.14**	926.92	**-66.88**
	1 1(+)	0.3035	14.51	**-35.36**	18.81	**72.60**	123.65	**1126.28**
	1 2(+)	0.3035	14.51	**-35.36**	18.81	**72.60**	123.65	**1126.28**
	2 1(+)	2.3017	14.03	**-53.09**	75.08	**285.57**	876.39	**553.20**
	Δ(price)(C): **209.14**			Δ(price)(G): **159.27**			DWL: **18.05**	
Cournot	0 1(−)	N/A	-56.82	**-24.05**	33.18	**118.49**	1614.33	**-42.32**
	1 1(+)	N/A	17.96	**-20.03**	33.18	**204.44**	409.73	**3963.45**
	1 2(+)	N/A	17.96	**-20.03**	33.18	**204.44**	409.73	**3963.45**
	2 1(+)	N/A	20.91	**-30.08**	33.18	**70.39**	408.64	**204.57**
	Δ(price)(C): **118.49**			Δ(price)(G): **118.49**			DWL: **5.79**	

eight generators at Node 1 with the same cheaper cost function $10q + 0.02q^2$ and that other conditions are the same as in the congested case in Table 3. In the bid a and bid b cases, the introduced competition only marginally increases efficiency, and has an ambiguous effect on consumer welfare (it falls when firms bid a and rises when firms bid b). In contrast, in the Cournot market, the introduced competition leads to a substantial increase in both overall efficiency and consumer welfare, a result that seems to arise because, without the capacity to bid their supply functions, generators at the downstream node of a congested link have less market power.

In conclusion, this subsection shows that different market regimes (Cournot vs supply function bidding of different types) can yield important differences in market outcomes, most notably, (1) a simple regime where generators bid quantities only to an ISO can be more efficient than when supply function bids are required, even when, in the latter case, the ISO knows everything except the value of a single parameter; and (2) the difference between bidding

Table 4: Cournot competition produces better results with a congested network

Congested case (5MW limit on link 12)								
	node	bid	dispatch	Δ(dispatch)	price	Δ(price)	profit	Δ(profit)
TT	0	N/A	-75.13	N/A	14.87	N/A	2822.09	N/A
	1	N/A	5.63	N/A	10.23	N/A	0.63	N/A
	2	N/A	30.06	N/A	19.52	N/A	135.58	N/A
	(price)(C): 14.87			(price)(G): 13.94			DWL: N/A	
	(surplus)(C): 2822.09			(profit)(G): 140.65			(rent): 69.70	
bid a	0	N/A	-46.14	**-38.58**	43.86	**194.89**	1064.63	**-62.28**
	1	10.02	3.82	**-32.16**	10.17	**-0.50**	0.38	**-40.84**
	2	72.87	15.57	**-48.20**	77.54	**297.24**	1007.53	**643.15**
	Δ(price)(C): **194.89**			Δ(price)(G): **135.99**			DWL: **14.91**	
	Δ(surplus)(C): **-62.28**			Δ(profit)(G): **618.46**			Δ(rent): **624.81**	
bid b	0	N/A	-46.14	**-38.58**	43.86	**194.91**	1064.45	**-62.28**
	1	0.0233	3.82	**-32.16**	10.18	**-0.46**	0.39	**-38.66**
	2	2.1529	15.57	**-48.21**	77.54	**297.26**	1007.48	**643.11**
	Δ(price)(C): **194.91**			Δ(price)(G): **136.01**			DWL: **14.91**	
	Δ(surplus)(C): **-62.28**			Δ(profit)(G): **618.50**			Δ(rent): **624.82**	
Cournot)	0	N/A	-74.20	**-1.23**	15.80	**6.23**	2752.96	**-2.45**
	1	N/A	5.58	**-1.03**	15.80	**54.50**	31.70	**4895.75**
	2	N/A	29.60	**-1.54**	15.80	**-19.06**	25.40	**-81.27**
	Δ(price)(C): **6.23**			Δ(price)(G): **13.29**			DWL: **0.015**	
	Δ(surplus)(C): **-2.45**			Δ(profit)(G): **98.38**			Δ(rent): **-100.00**	

one of two parameters in our very simple SFE can raise or lower consumer welfare.

The impact of the bidding regime on market equilibria has important practical implications. For example, in Australia, generators must submit their offers (ten pairs of prices and quantities by 12:00 noon for each of 48 half hours of the next day starting from 4:00 am). The ISO predispatches the generators according to forecast loads for each of the 48 half hours. The generators are then allowed to change their quantity, but not price, offers by moving their quantities up or down along the offered price stack, up to 5-minutes before the real-time dispatch. It would seem likely that a model of this market could only be convincing if it included the described rebidding.

4.3 Markets with ISOs versus vertically integrated markets

In this subsection, we compare the equilibrium outcomes of five different market structures with TT. The first two are the bid a and bid b CSFE for independent generators; the third is the outcome when a vertically integrated monopolist controls all generation facilities and the transmission network itself; and the fourth and fifth are the bid a and bid b CSFE when generation is monopolized. A comparison of the vertically integrated monopolist with the generation-only monopolist is of interest because it is speculated in [5, p.156] that vertical integration might increase efficiency where congestion rents are claimed by generators through the exercise of market power. Our results show this need not be so.

The vertically integrated monopolist's problem is

$$\begin{array}{ll} \underset{q_0,\cdots,q_N}{\text{maximize}} & \sum\limits_{i:\text{consumer}} (A_i + 2B_i q_i)(-q_i) - \sum\limits_{i:\text{generation}} (A_i q_i + B_i q_i^2) \\ \text{subject to} & q_0 + q_1 + \cdots + q_N = 0 \\ & -C_{ij} \le \sum\limits_{k=1}^{N} \phi_{ij,k} q_k \le C_{ij}, \quad i < j,\ ij \in \mathcal{L} \\ & q_i \ge 0, \quad i : \text{generation}, \quad q_i \le 0, \quad i : \text{consumer} \end{array} \tag{6}$$

which is a strictly convex quadratic program. Therefore, any stationary point is the unique optimal solution to this problem and *vice versa*.

The generation-only monopolist controls all generating units and bids supply functions for all generating units to an ISO who, just as in the other SFE cases, seeks to maximize economic efficiency taking the submitted bids as true. That is, the generation-only monopolist's problem is

$$\begin{array}{ll} \underset{a_i,b_i,i:\text{generation}}{\text{maximize}} & \sum\limits_{i:\text{generation}} (a_i + 2b_i q_i) q_i - (A_i q_i + B_i {q_i}^2) \\ \text{subject to} & a_{li} \le a_i \le a_{ui}, \quad b_{li} \le b_i \le b_{ui} \\ & q = (q_0, \ldots, q_i, \ldots, q_N) \text{ solves (1) with given } (a, b) \end{array} \tag{7}$$

where $a_j = A_j, b_j = B_j$ for each consumer j.

The results reported in Table 5 are for the three-node network, without and with congestion, where Node 0 has one consumer and one generator, Node 1 has two generators and one consumer, and Node 2 has only one generator.

It can be seen from Table 5 that, unsurprisingly, there are large efficiency and consumer surplus losses under both forms of monopoly, as compared with uncoordinated bidding by independent generators, which only differs somewhat from TT. This confirms the rationale for restructuring of the electricity industry to the extent that it promotes competition in generation. What is more interesting is that in these examples vertical integration tends to increase efficiency losses, and does not reduce them.

These observations are reinforced by examples from a five-node network with parameters as detailed in Table 6. The equilibria are given in Table 7. In all cases, the vertically integrated

Table 5: Monopolistic situations vs supply function bidding

cost/utility functions and locations of players						
node	# generators	cost data		# consumers	utility data	
0	1	$(12.00, 0.022)$		1	$(195.00, 0.50)$	
1	2	$(10.00, 0.02)$		1	$(200.00, 0.55)$	
2	1	$(15.00, 0.05)$		0	N/A	
	Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
Uncongested case						
TT	845.25	347.51	31633.52	15.35	0	N/A
bid a	**63.28**	**-0.88**	**- 1.75**	**10.40**	**0**	**0.05**
bid b	**91.90**	**- 1.29**	**- 2.56**	**15.28**	**0**	**0.10**
VI mon	**1847.92**	**- 49.37**	**- 74.36**	**585.34**	**N/A**	**24.33**
gen mon (bid a)	**1847.56**	**−49.37**	**−74.35**	**585.23**	**0**	**24.33**
gen mon (bid b)	**1845.45**	**−49.30**	**−74.29**	**584.50**	**0**	**24.32**
Congested case with a capacity limit 15 MW on link 12						
TT	756.86	347.49	31633.76	15.35	56.14	N/A
bid a	**155.55**	**- 1.87**	**- 3.69**	**22.12**	**-40.83**	**0.04**
bid b	**668.84**	**- 8.86**	**- 16.92**	**104.93**	**61.92**	**0.78**
VI mon	**2073.38**	**- 49.43**	**- 74.43**	**586.20**	**N/A**	**24.37**
gen mon (bid a)	**2073.16**	**-49.43**	**- 74.42**	**586.09**	**-100.00**	**24.36**
gen mon (bid b)	**2072.34**	**-49.34**	**- 74.33**	**585.07**	**-100.00**	**24.30**

monopoly is considerably worse than the generation-only monopoly, which in turn is quite a bit worse than non-cooperative bidding by separate generators, which is only somewhat worse than TT.

In summary, the games reported in Tables 5–7 show that (1) vertical integration can worsen, rather than improve economic efficiency, and (2) a good correlation between efficiency losses and the weakness of competition.

4.4 Effects of uncertainty on generators' market power

In this subsection, we examine the impact of demand and transmission constraint uncertainty on bidding behavior of generators. In electricity markets, both demand and transmission limits on links (in the latter instance, for example, due to the temperature of the transmission lines) may fluctuate over the course of a day. Periodic variation of demand is used in Green and Newbery [23], by modifying Klemperer and Meyer [34] who consider uncertainty about demand, to study deregulated British electricity markets. When generators make bids that

Table 6: cost/utility functions and locations of players for Table 7

cost/utility functions and location of players				
node	# generators	cost data	# consumers	utility data
0	1	(12.00, 0.010)	0	N/A
1	2	(10.00, 0.025)	0	N/A
2	1	(14.00, 0.005)	1	(100.00, 0.200)
3	0	N/A	2	(80.00, 0.100)
4	1	(20.00, 0.050)	1	(270.00, 0.500)

must cover variation in demand over time, or equivalently, different possible realizations of uncertain demand, the range of possible equilibria outcomes tends to narrow, and firm market power tends to be reduced (in the sense of the capacity to mark-up prices).

We use the five-node network described in Table 6, but only focus on the CSFE when independent generators bid a only or bid b only, and TT. Table 8 compares CSFE when transmission constraints on Link 01 can take one of three values. Under certainty, generators can place a different bid for each of the three cases. The two rows labelled 'bid separately' give the average outcomes of the three CSFE for each of the 'bid a' and 'bid b' cases. For example, the consumer average price of bidding a only in the bid separately situation is defined as the ratio of the sum of consumers' payment over all periods to the sum of total consumption over all periods.

With uncertainty, the generators' bids are fixed for three dispatching periods of equal length regardless of the specific transmission limits on Link 01 (see rows labelled 'bid once for three periods') and the average outcome is reported.[14]

Table 9 compares CSFE when bids specific to two different demand periods can be made, or when one bid must be made for both demand periods. The comparison is made twice for two different transmission constraints. Simple averages are again reported.

In both cases, the impact of uncertainty is to increase, or at least not reduce, market efficiency, and to reduce profits, so presumably market power. However, the effect is small, as is consistent with Mount [36] who used data from PJM in 1999 to show that generators with

[14]The certainty case can be thought of as the expected outcome when three different constraints can occur with equal probability, but the generators know in advance as to which constraint they face. In the uncertain case, the expected outcome is reported when generators must choose one bid in advance of knowing the realized constraint.

Table 7: Monopolistic situations vs supply function bidding for the five-node network

aggregate value and their changes in different operating situations						
	Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
Uncongested case with flows: 155.05 MW on link 02, 494.07 MW on link 13						
TT	3273.15	1073.22	59075.86	18.28	0	N/A
bid *a*	**65.82**	**-2.68**	**-3.82**	**11.67**	**0**	**0.17**
bid *b*	**85.75**	**-3.52**	**-5.00**	**15.33**	**0**	**0.24**
VI mon	**880.07**	**-48.28**	**-73.74**	**290.89**	**0**	**23.67**
gen mon (bid a)	**629.30**	**-48.28**	**-52.90**	**209.98**	**0**	**17.08**
gen mon (bid b)	**628.87**	**-48.26**	**-52.88**	**209.86**	**0**	**17.09**
Congested case (280MW limit on link 13, 800 MW on others)						
TT	2075.81	834.54	44707.11	32.38	11199.69	N/A
bid *a*	**80.36**	**-3.78**	**-4.24**	**6.69**	**0.62**	**0.27**
bid *b*	**111.09**	**-4.57**	**-5.26**	**8.45**	**-1.61**	**0.39**
VI mon	**1445.39**	**-33.49**	**-65.30**	**120.71**	**N/A**	**17.92**
gen mon (bid a)	**1049.96**	**-33.49**	**-37.76**	**75.03**	**-100.00**	**10.84**
gen mon (bid b)	**1049.28**	**-33.46**	**-37.73**	**74.96**	**-100.00**	**10.84**
Congested case (80MW limit on link 02, 800 MW on others)						
TT	3177.06	1067.49	58531.50	18.79	435.97	N/A
bid *a*	**182.12**	**-9.76**	**-13.44**	**41.27**	**34.95**	**3.10**
bid *b*	**319.07**	**-17.07**	**-22.45**	**71.69**	**-45.64**	**5.15**
VI mon	**909.08**	**-48.09**	**-73.57**	**280.82**	**N/A**	**23.52**
gen mon (bid a)	**650.19**	**-48.13**	**-52.55**	**202.10**	**-100.00**	**16.96**
gen mon (bid b)	**649.78**	**-48.10**	**-52.53**	**201.99**	**-100.00**	**16.96**

significant market shares (20% of the expected load) bid up to 80% higher than their true cost in a varying demand environment (see also [6]).

4.5 Bidding by consumers—countervailing power?

This subsection provides some examples of strategic bidding behavior by consumers as well as generators. It has been suggested that strategic bidding from the consumer side, perhaps through retailers, can relieve market power on the generation side of the market [5, 33, 39]. Moreover, in an experimental game, Weiss [50] concludes that "... Active demand-side bidding significantly lowers market power for a given number of sellers and therefore promises to be an excellent market power mitigation strategy". In contrast, our examples (Table 10) demonstrate that strategic bidding by consumers can bring markedly worse CSFE as compared with the case where consumers bid their actual demand functions (for example, if they had no market

Table 8: Under three different network conditions (the five-node network)

cost/utility functions and location of players as in Table 6							
Three periods with different transmission limits 160MW, 200MW, 500MW on Link 01							
		Δ(profit) (G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
bid separately	bid a	**96.94**	**-4.50**	**-6.14**	**17.70**	**19.52**	**0.24**
	bid b	**179.78**	**-6.28**	**-8.74**	**17.61**	**-60.86**	**0.65**
bid once for three periods	bid a	**88.36**	**-3.67**	**-5.14**	**14.94**	**3.97**	**0.17**
	bid b	**128.80**	**-6.21**	**-8.48**	**14.83**	**31.02**	**0.46**

Table 9: Under two different demand periods (locations of generators and their cost functions and the five-node network configuration as in Table 6 and Table 7)

The transmission limits on Link 01 is 200MW (congested)							
		Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
bid separately	bid a	**112.81**	**-6.13**	**-9.62**	**25.35**	**25.09**	**0.47**
	bid b	**207.77**	**-8.87**	**-14.33**	**25.15**	**-31.95**	**1.19**
bid once for two periods	bid a	**100.51**	**-5.56**	**-8.48**	**22.42**	**20.20**	**0.43**
	bid b	**207.66**	**-8.85**	**-14.33**	**22.12**	**-31.90**	**1.19**
The transmission limits on Link 01 is 800MW (uncongested)							
bid separately	bid a	**56.49**	**-2.06**	**-3.43**	**11.59**	**0.0**	**0.09**
	bid b	**77.14**	**-2.86**	**-4.71**	**11.60**	**0.0**	**0.14**
bid once for two periods	bid a	**54.50**	**-2.06**	**-3.30**	**11.19**	**0.0**	**0.08**
	bid b	**76.82**	**-2.82**	**-4.69**	**11.15**	**0.0**	**0.14**

Utility functions for Period 1: $(100, 0.20)$ at Node 2, $(80, 0.10)$ at Node 3, $(270, 0.5)$ at Node 4 .

Utility functions for Period 2: $(150, 0.15)$ at Node 2, $(120, 0.10)$ at Node 3, $(270, 0.45)$ at Node 4.

power).

In Table 10, the *gen* rows describe games when only generators bid strategically and consumers reveal their true utility functions to the ISO. In contrast, in the *gen+con* rows both generators and consumers present bids to the ISO. With the exception of the bid b case for the uncongested three-node network (the fourth and fifth rows), consumer bidding lowers economic efficiency, often sharply. Output also falls in all cases. Consumer welfare rises, but in most cases, not by a great deal. This is not surprising given standard models of bilateral monopoly,[15] and this casts substantial doubt on whether countervailing market power should

[15]Another standard result is that a Nash equilibrium may not exist for bilateral monopoly, a result demonstrated in the context of electricity in [40, p. 12].

Table 10: Only generators bidding vs both generators and consumers bidding

	whoplay	Δ(profit)(G)	Δ(output)	Δ(surplus)(C)	Δ(price)(C)	Δ(rent)	DWL
Uncongested case (players' data and the three-node network as in Table 5)							
bid a	gen	**63.28**	**-0.88**	**-1.75**	**10.40**	**0**	**0.05**
	gen+con	**60.05**	**-1.96**	**-1.69**	**10.00**	**0**	**0.09**
bid b	gen	**91.90**	**-1.29**	**-2.56**	**15.28**	**0**	**0.10**
	gen+con	**60.05**	**-1.96**	**-1.69**	**10.00**	**0**	**0.09**
Congested case (15MW limit on Link 12) (players' data and the three-node network as in Table 5)							
bid a	gen	**155.55**	**-1.86**	**-3.69**	**22.12**	**-40.83**	**0.04**
	gen+con	**147.48**	**-3.47**	**-3.59**	**21.29**	**-42.35**	**0.13**
bid b	gen	**668.84**	**-8.86**	**-16.92**	**104.93**	**61.92**	**0.78**
	gen+con	**561.69**	**-17.51**	**-16.59**	**96.84**	**-14.76**	**3.10**
Uncongested case (players' data and the five-node network as in Table 6 and Table 7)							
bid a	gen	**65.82**	**-2.68**	**-3.82**	**11.67**	**0**	**0.17**
	gen+con	**61.19**	**-4.17**	**-3.64**	**11.03**	**0**	**0.23**
bid b	gen	**85.75**	**-3.52**	**-5.00**	**15.33**	**0**	**0.24**
	gen+con	**78.14**	**-5.79**	**-4.73**	**14.34**	**0**	**0.38**
Congested case (280MW limit on Link 13) (players' data and the five-node network as in Table 6 and Table 7)							
bid a	gen	**80.36**	**-3.78**	**-4.24**	**6.69**	**0.62**	**0.27**
	gen+con	**44.91**	**-8.88**	**6.54**	**-14.88**	**-40.62**	**1.20**
bid b	gen	**111.09**	**-4.57**	**-5.26**	**8.45**	**-1.61**	**0.40**
	gen+con	**76.90**	**-12.27**	**9.88**	**-23.24**	**-64.86**	**2.16**

be relied on to reduce market power. In addition, congestion rent is significantly reduced when consumers bid strategically. This would hide the need for transmission expansion and send poor signals to network investors.

In short, in terms of total output and both short run and long run social welfare, countervailing power from consumers may be far from a panacea in mitigating market power.

5 Conclusion

Dramatic reforms in markets for electricity generation have been undertaken in a number of countries around the world, typically creating mechanisms where generators bid to an ISO which then sets nodal prices. Such reforms came with high hopes for efficient market outcomes at least in the presence of many generators [22, p. 338, 1st column], and see also [5, p. 140]. This paper provides a number examples of this kind of market mechanism for two simple networks where the ISO is well-informed. It shows that even in these simple cases, in the presence of market power, loop flows and transmission constraints, CSFE are not particularly efficient, intuitive or predictable. Indeed, numerous results from simpler models of electricity generation, and standard intuitions, are not supported by the illustrated CSFE. This casts some doubt on whether the undertaken market reforms can be well-understood, let alone improved, without situation-specific modelling of any given electricity market.

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