ENGINEERING TRIPOS PART IB

PAPER 8 - ELECTIVE (2)

Mechanical Engineering for Renewable Energy Systems

Wind Turbines

<u>Aerodynamic fundamentals</u>

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Fundamental fluid mechanics limits to energy generating potential (Betz Limit), including the influence of size and height. Estimates of wind loading. Capacity factor.

These notes contain gaps to be filled in

The derivation of Betz limit can be found in various books , *e.g.*

1. Aerodynamics of Wind Turbines, Hansen M.O.L., 2000
2. Renewable Energy, Sorensen B., 3ed 2004

and online at various websites, links can be followed from www.hughhunt.com

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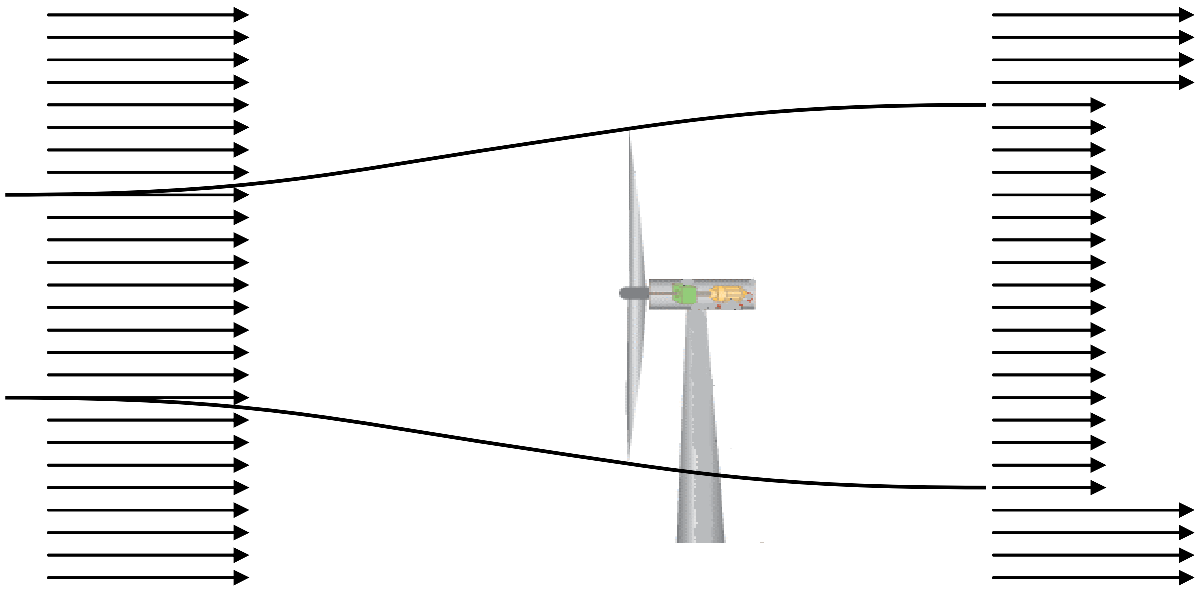
1. Flow around a wind turbine – general considerations



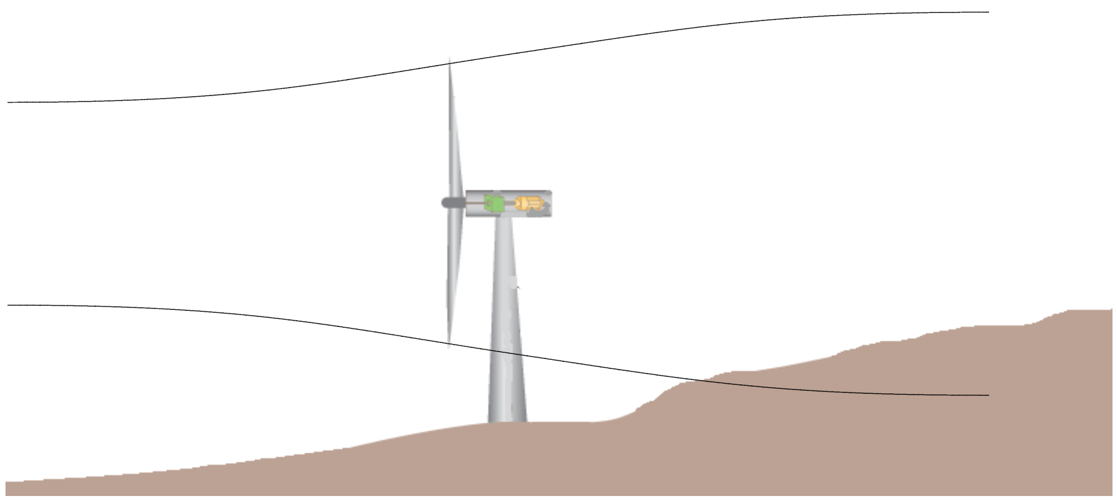
Is this flow realistic?

- air speed increases with height
- adjacent air streams at different speeds?
- swirl and turbulence
- tip vortices
- variable wind speed
- variable wind direction

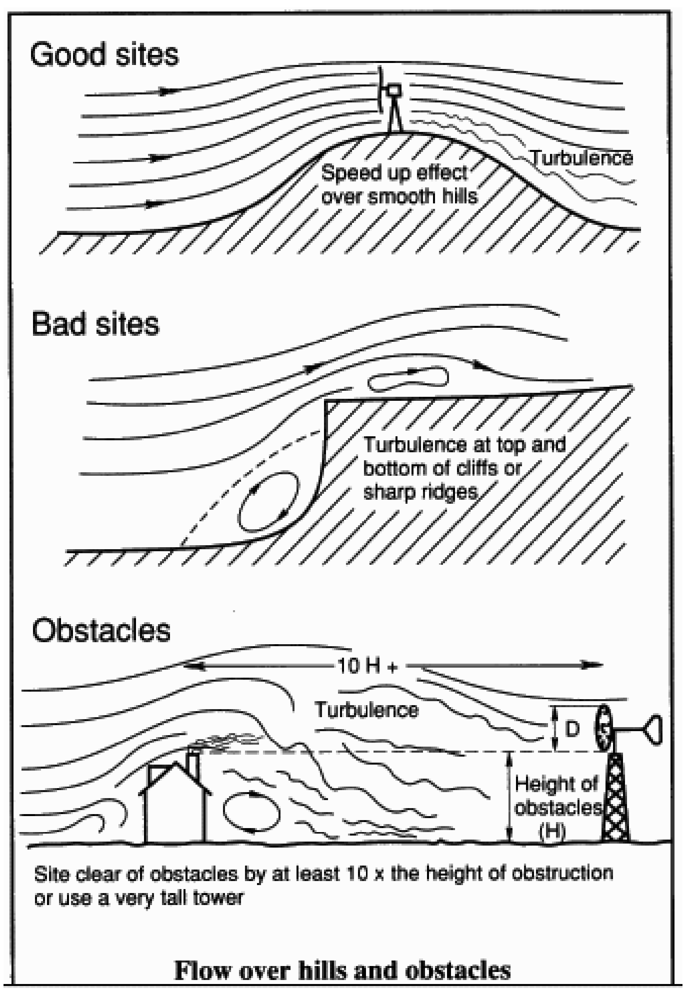
Assumptions:

- laminar incompressible inviscid steady flow

Realistic flow:

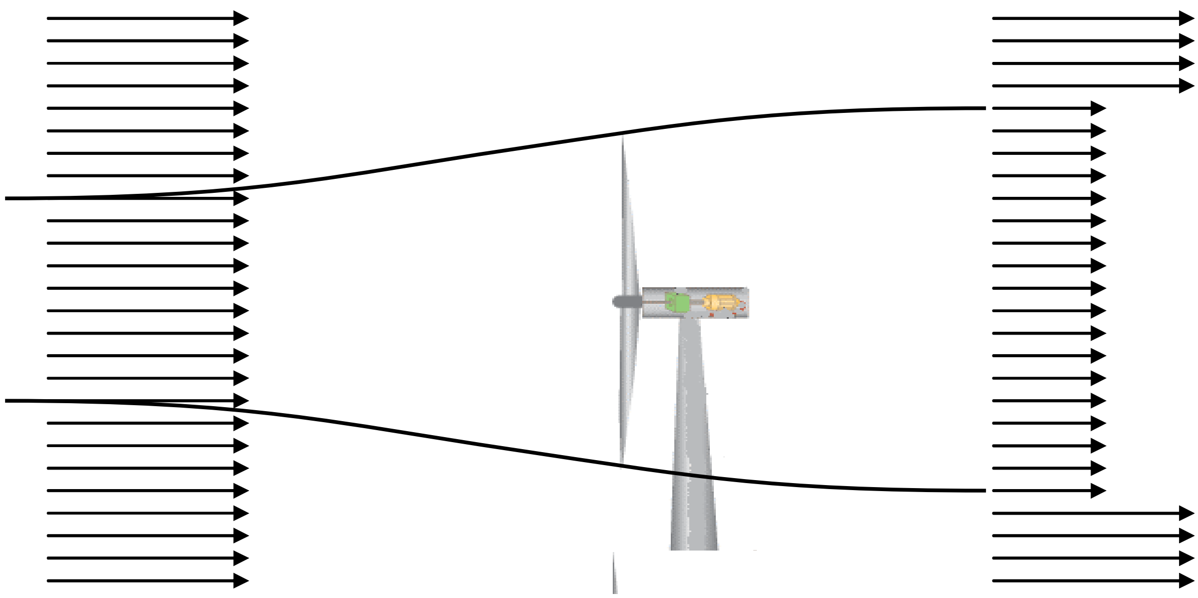


Siting isues



Flow over hills and obstacles

2. Upper limit for efficiency – the Betz Limit



V_o	Free-stream air speed		
$A = \pi R^2$	Swept area of rotor		
p_o	Atmospheric pressure		
p	Pressure just before the rotor		
Δp	Pressure drop across rotor		
u	Velocity at the rotor plane		
u_1	Velocity in the wake		
ρ	Density of air		
T	Thrust on the rotor	$T = \Delta p\, A$	(1)
$\dot{m}$	Mass flow through rotor	$\dot{m} = \rho A u$	(2)
P	Power delivered to the rotor		
a	axial induction factor	$u = (1 - a)\, V_o$	(3)

Apply the Bernoulli equation from far upstream to just before the rotor:

(4)

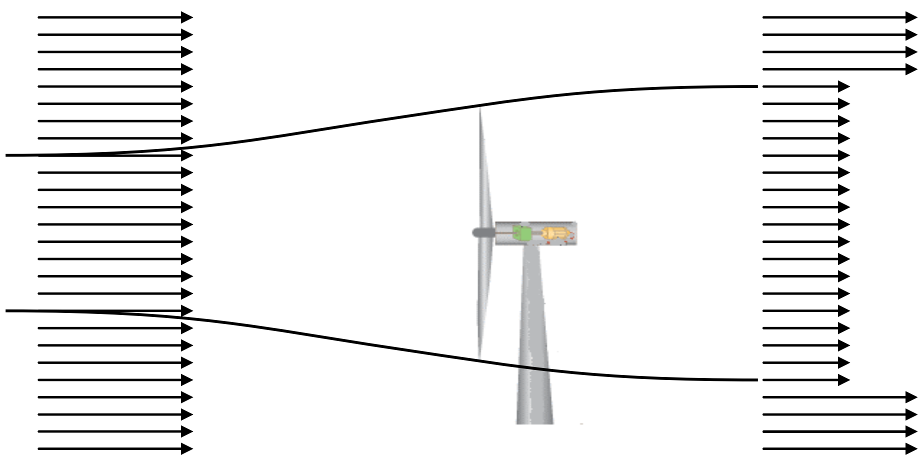
Apply the Bernoulli equation from just after the rotor to far downstream:

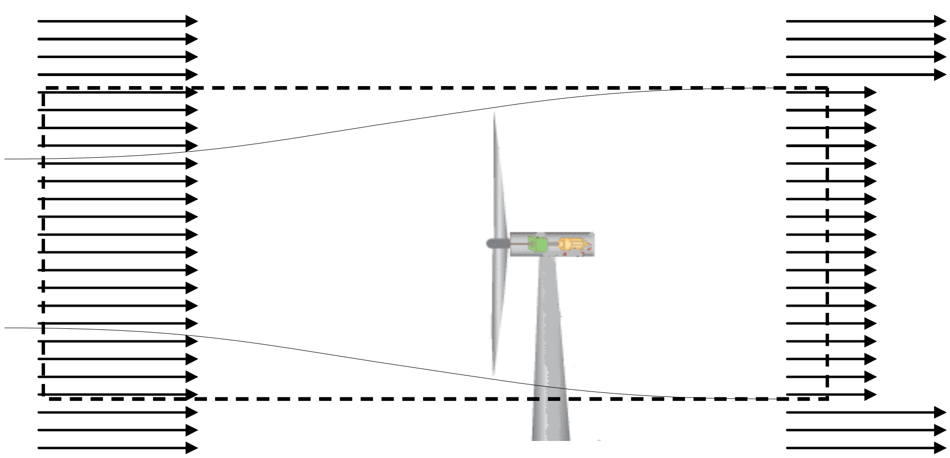
(5)

Combine (4) and (5) to get

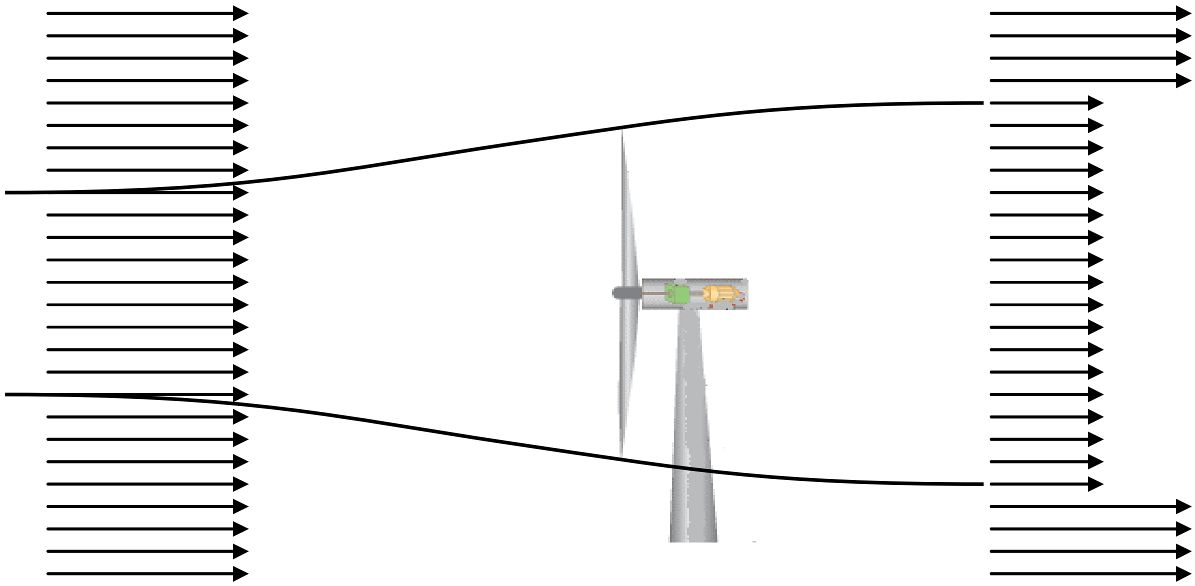
$$\Delta p = \tfrac{1}{2}\rho\,(V_o^{\,2} - u_1^{\,2}) \qquad (6)$$

Axial Momentum considerations: two choices for the control volume





Axial momentum flux



Net momentum flow = thrust on control volume

$$T = \dot{m}(V_o - u_1) \qquad (7)$$

(7a)

Now use the thrust equation from (1) $\quad T = \Delta p\, A$

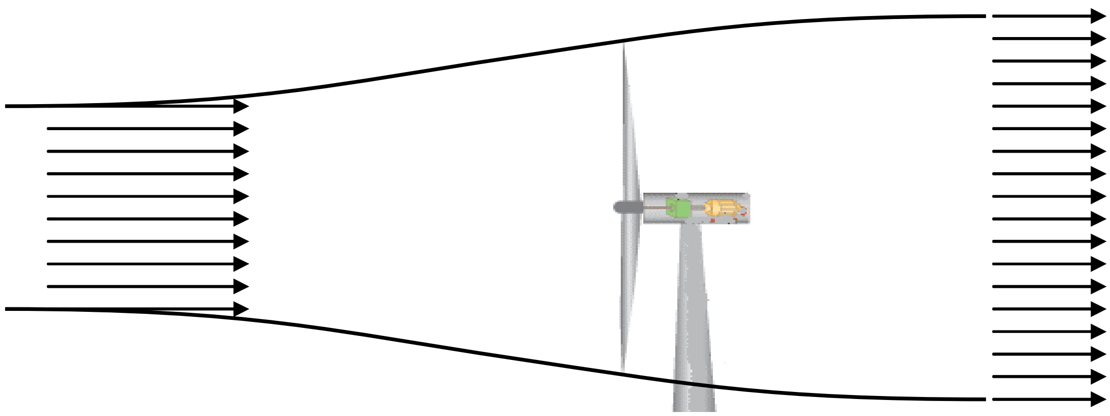
and the Δp equation from (6) $\quad \Delta p = \frac{1}{2}\rho\,(V_o^2 - u_1^2)$

it then follows that $\quad u = \frac{1}{2}(V_o + u_1) \qquad (8)$

Now use the axial induction factor a as defined in (3)

to give

$$u_1 = (1 - 2a)V_o \quad (9)$$



Shaft power

The thrust T acts on fluid moving at a speed u so the power P that is delivered to the turbine is

$$P = T\,u \quad (10)$$

and use T from (7a) to give

$$P = \rho A u^2 (V_o - u_1) \quad (11)$$

Note that this can also be obtained from the change of kinetic energy of the fluid:

Now use u from (3) and u_1 from (9) to give

$$P = 2\rho V_o^3 a(1 - a)^2 A \tag{12}$$

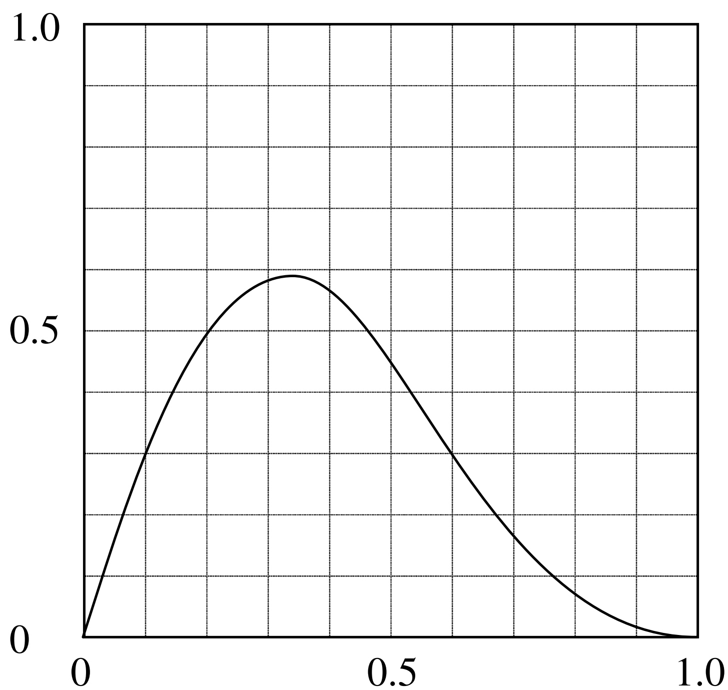
How does this compare with the power *available* to a cross section equal to the swept area A ? This is given by the rate of flow of kinetic energy

$$P_{\text{available}} = \tfrac{1}{2}\dot{m}\, V_o^2 \quad = \quad \tfrac{1}{2}\rho V_o^3 \tag{13}$$

The delivered power is normalized by the available power to give the *Power Coefficient*

$$C_P = \frac{P}{\frac{1}{2}\rho V_o^3} \tag{14}$$

and from (12) $\qquad C_P = 4a(1 - a)^2 \qquad (15)$



Maximum power is obtained when C_P is maximum

The Betz limit is equal to $C_{P,\,Max} = 16/27 = 0.59$

This occurs for $a = 1/3$ (16)

3. Thrust on the turbine

The thrust on the turbine from (3), (10) and (12) is

$$T = 2\rho V_o^2 a(1 - a)\ \mathrm{A} \qquad (17)$$

This has a maximum value of $\frac{1}{2}\rho V_o^2 \mathrm{A}$ at $a = \frac{1}{2}$

For a turbine operating at the Betz limit, $a = 1/3$, the turbine thrust is

(18)

4. <u>Capacity Factor</u>

A wind turbine's output will fall short of its ideal expectations for three reasons:

(a) the peak power in high winds is limited

(b) the wind doesn't blow strongly all the time

(c) the wind turbine has scheduled down-time for maintenance

The *Capacity Factor* is the ratio of the total energy generated in one year by the wind turbine to the maximum possible for the installation if it were to run at full output continuously. Here is an example for a 200kW offshore installation:

Wind speed	Time spent	Power	Energy
maintenance	0.02	0	
5	0.08	10	
10	0.35	80	
15	0.40	270	
20	0.15	640	

We know that if the turbine ran continuously it would produce 200kW steadily, so the Capacity Factor is = 69.4%

5 Summary

Betz limit = 59%

Induction factor = 1/3 at the Betz limit

Thrust on turbine $\leq \frac{1}{2}\rho V_o^2 A$

Capacity Factor is a measure of the effectiveness of an installation

Notes: