

Engineering Part IB P8 – Elective (2)
Engineering for Renewable Energy Systems
Wind Turbines - Vibration and Noise
Dr Michael Sutcliffe

Aims of this course

- Apply Part I vibration theory to wind turbine design
- Understand causes of vibration and critical vibration modes
- Outline principal sources of noise for wind turbines

Selected bibliography

Guidelines for Design of Wind Turbines, DNV/Risoe Publication, ISBN 8755028705

Aerodynamics of Wind Turbines, MOL Hansen, James and James, 2000

Mechanical vibration analysis and computation, DE Newland, Longman, 1989

1 Introduction

- Principals
- Sources of non-steady loading

2 Modelling of Wind Turbines vibrations

- Continuous beam model – application to tower
- Equivalent lumped mass model – application to tower
- Tjaereborg tower example

- Blade vibration – example of multi-degree of freedom model
- Variable mass and stiffness blade model
- Blade modelling of torsional/edgewise vibration – Tjaereborg example

3 Noise – background reading handout

1 Introduction

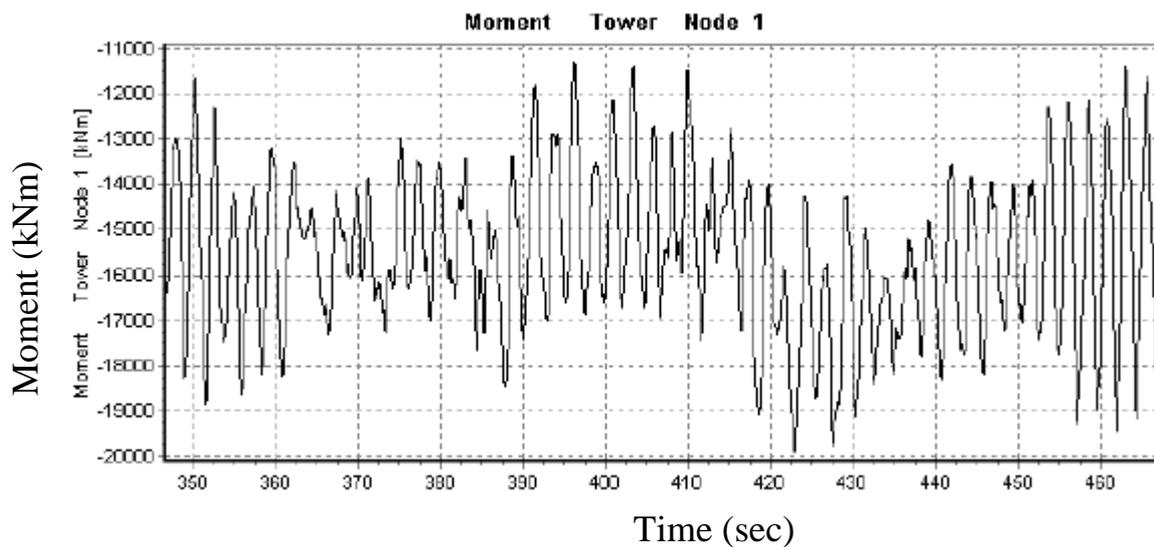
Need to consider vibration of the tower, blades, torsion of the shaft, coupled modes...

Critical frequencies: 1P is the rotation frequency of ω , 3P is the passing frequency (for three blade design). Need also to consider harmonics (2P, 4P...)

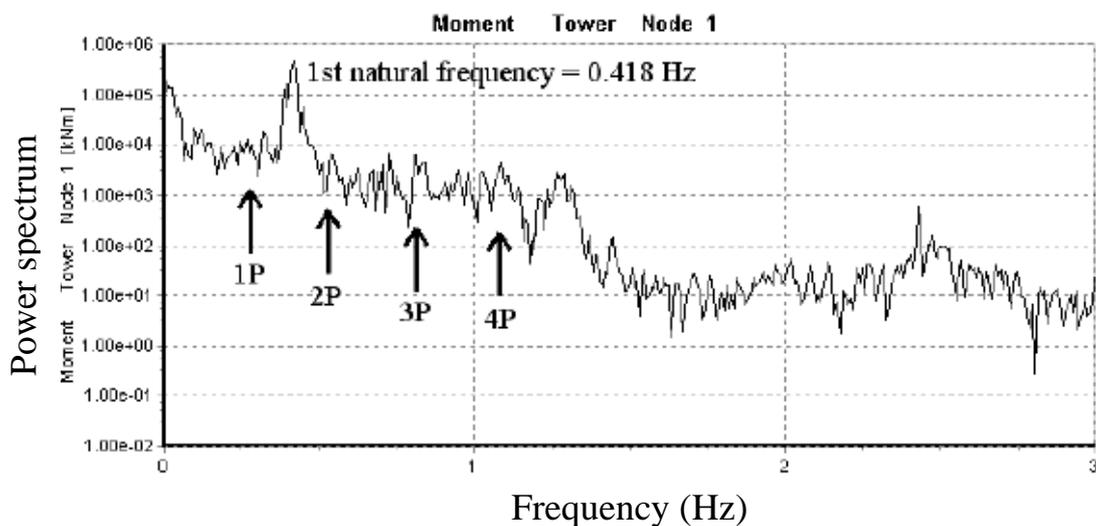
Guidelines (DNV/Risoe)

Blades: as a minimum the two lowest frequencies of the blade in the flapwise and edgewise direction should be considered and the operating frequency should not be within 12% of these frequencies.

For the **tower** the lowest frequency should not be within 10% of the rotor and blade passing frequencies (1P and 3P).



Moment time series at bottom of tower



Power spectrum of moment at bottom of tower [Risoe/DNV]

Drive train: similar considerations apply.

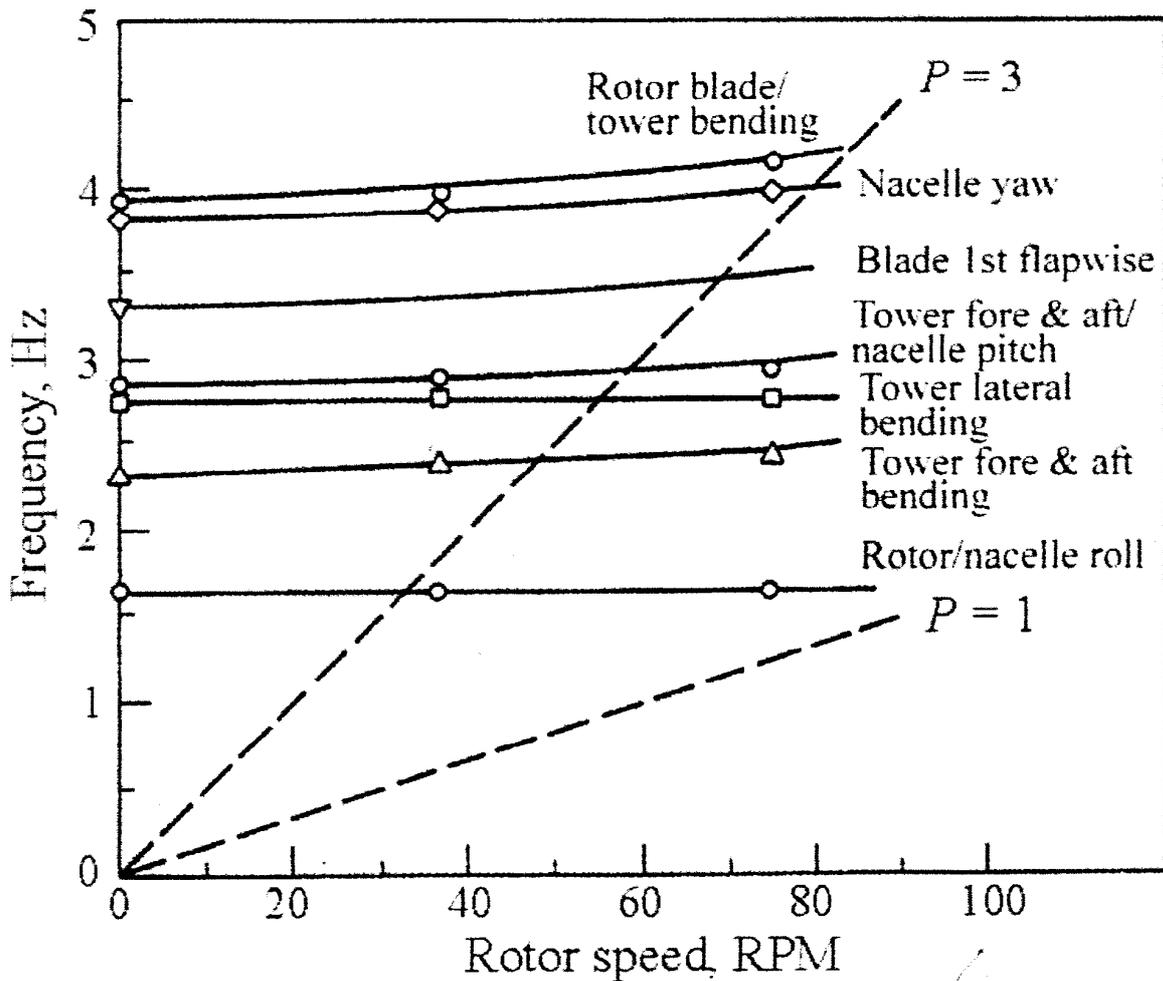
Dampers may be introduced to reduce vibration at resonance.

May need to be able to **pass through** resonant modes during start-up/slow down.

At operating/peak load certainly need to avoid resonances.

Fixed speed operation reduces challenge in vibration avoidance, while variable speed operation (to optimise power production) gives more difficulties.

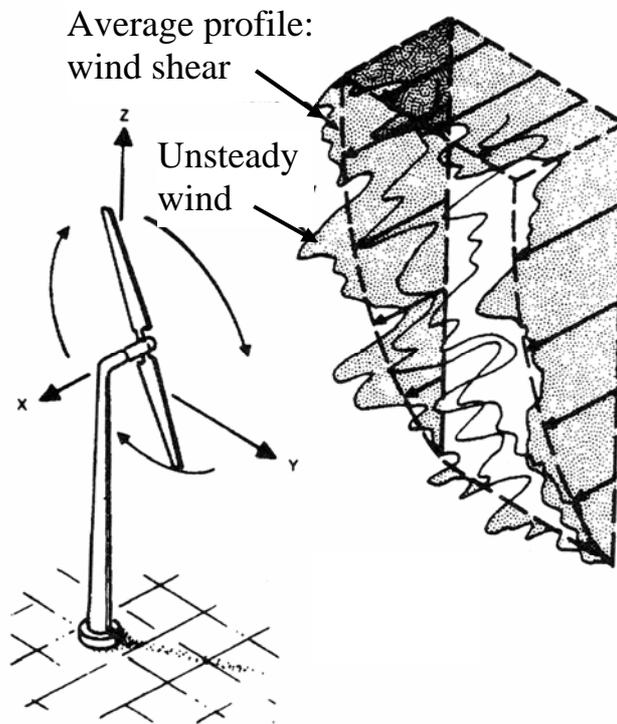
Campbell chart plots the resonant frequencies as a function of rotor speed. Need to avoid resonances at operating frequencies.



[Eggleston and Stoddard, 1987]

1.1 Sources of non-steady loading

Unsteady wind conditions – a broad spectrum of frequencies with some contribution in the critical range down to a few Hertz, e.g. due to (see typical wind variation, Materials handout).



[Walker and Jenkins]

gives 1P loading on blades and 3P loading on tower

Out-of-balance in mass and pitch giving 1P loading on tower

Self-weight gives 1P loading on blade.

Tower shadow effects – 1P loading on blade and 3P loading on tower, plus harmonics. May not be important for upwind configuration.

Aeroelastic and flutter effects.

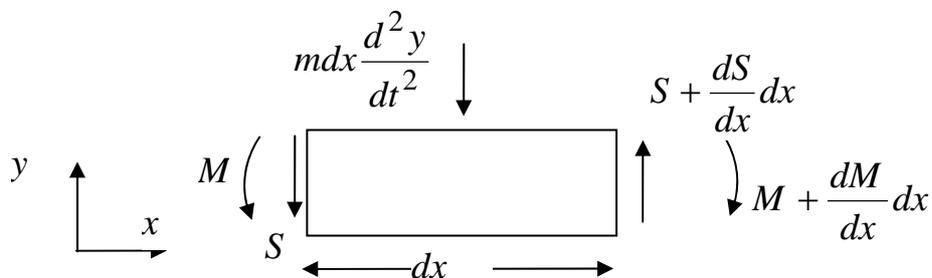
Vortex shedding.

2. Modelling of wind turbine vibrations

- Application of single and multi-degree-of-freedom models to wind turbine.
- Explain by example. Start with simpler tower models and then move on to more complex blade models, though all models applicable to blade and tower.
- Compare continuous with discrete models.
- Need to consider changes in cross-sectional geometry and area giving changing stiffness and mass along structure. Does shape matter?

2.1 Continuous beam model

Uniform cross section, bending stiffness EI , mass per unit length m

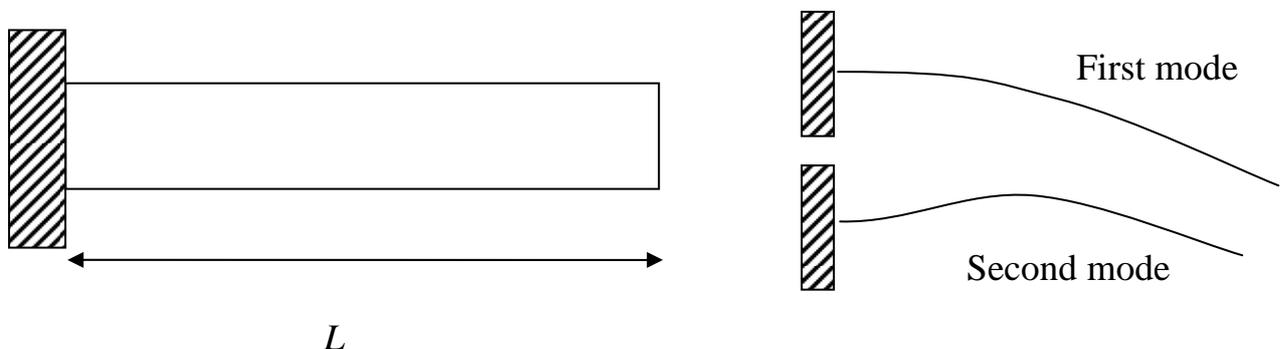


Governing equation for beams (neglecting shear):

$$M = EI \frac{d^2 y}{dx^2}; \quad \frac{dS}{dx} = -m \frac{d^2 y}{dt^2}, \quad \frac{dM}{dx} = S$$

$$\frac{d^2 M}{dx^2} = EI \frac{d^4 y}{dx^4} = -m \frac{d^2 y}{dt^2}$$

Analytical/numerical solutions are available for simple cases.



$$f \begin{cases} \text{First mode} \\ \text{Second mode} \end{cases} = \begin{cases} 3.52 \\ 22.0 \end{cases} \frac{1}{2\pi} \sqrt{\frac{EI}{mL^4}}$$

Application to tower

Consider tower of height L , constant circular cross section, wall thickness t much less than diameter D , density ρ and Young's Modulus E .

$$I = \frac{\pi}{8} D^3 t, \quad m = \pi D t \rho$$

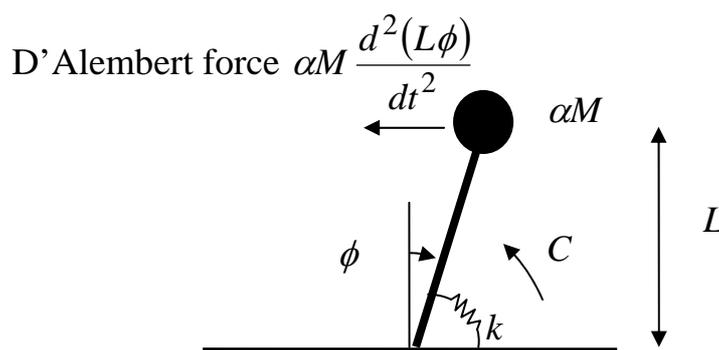
$$\text{For the fundamental mode } f = \frac{3.52}{2\pi} \frac{D}{L^2} \sqrt{\frac{E}{8\rho}}$$

Note how increasing the wall **doesn't** change the vibration frequency. As rotation speed will roughly scale inversely with tower height, we need D to scale with height to retain frequency relative to 1P and 3P.

2.2 Equivalent lumped mass model – application to tower

To model more complicated geometries it is helpful to think of the structure as a series of lumped connected via

For the tower sway the simplest model is to have a lumped mass αM equal to a proportion of the tower mass at the tip of the tower, with a spring element at the base which provides a moment resisting rotation.



Restoring couple C provided by spring = $k\phi$

Taking moments about the base

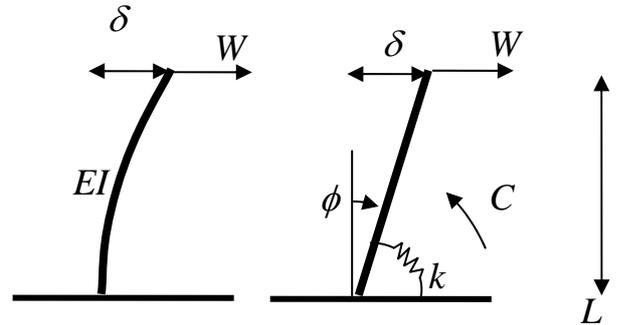
$$k\phi + \alpha M L^2 \frac{d^2\phi}{dt^2} = 0 \text{ to give simple harmonic motion with}$$

To calculate an appropriate k for the tower, consider a beam in bending with end load W

Clamped cantilever beam model: $\delta = \frac{WL^3}{3EI}$

From the spring model:

and



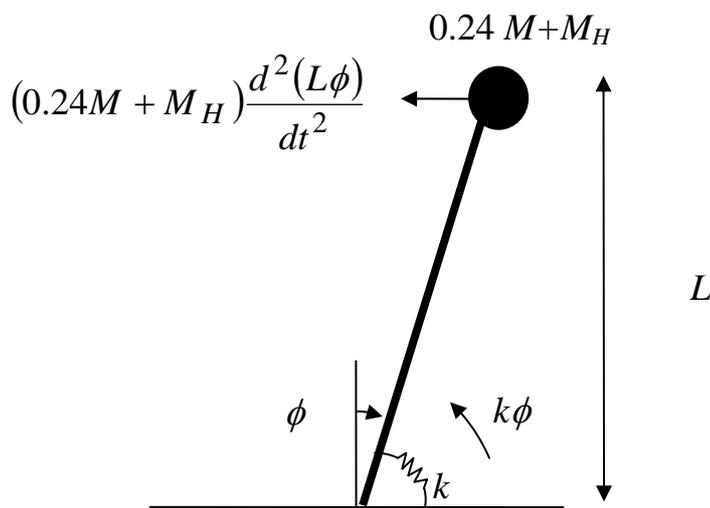
Hence $k = \frac{3EI}{L}$

Hence the model predicts $f = \frac{1}{2\pi} \sqrt{\frac{3EI}{\alpha ML^3}}$

To match the exact solution $f = \frac{3.52}{2\pi} \sqrt{\frac{EI}{mL^4}}$ (noting that $M = mL$) choose $\alpha = 0.24$.

The effective mass is less than the actual mass because the mass is not all at the end.

Consider adding a tower head mass M_H to the top of the tower.



$$k\phi + \left((0.24M + M_H)L^2 \right) \frac{d^2\phi}{dt^2} = 0 \text{ to give } f = \frac{1}{2\pi} \sqrt{\frac{3EI}{L^3(0.24M + M_H)}}$$

Note the very significant effect of tower head mass in reducing the frequency.

2.3 Tjaereborg Tower example:

Turbine

2 MW

Rotor speed constant at 22.36 rpm at rated power (1P=0.37 Hz, 3P = 1.11 Hz)

Three GFRP blades

Dimensions

Blade diameter 61.1 m

Tower height 57m

Hub height 61 m

Masses

Blade mass 9 tonnes

Hub 22.1 tonnes

Tower mass 665 tonnes

Towerhead mass (i.e. blades, nacelle...)
224 tonnes

Total mass 890 tonnes

Tower details

Reinforced concrete,

Wall thickness 0.25 m

Diameter tapering from 7.25m at base to 4.25m near top

Assume Young's modulus of 50 GPa



$$I = \frac{\pi}{8} D^3 t = 18.7 \text{ m}^4 \text{ using an average diameter of 5.75m}$$

$$\text{Exact solution for tower alone: } f = \frac{3.52}{2\pi} \sqrt{\frac{EI}{ML^3}} = \frac{3.52}{2\pi} \sqrt{\frac{50 \times 10^9 \times 18.7}{665 \times 10^3 \times 57^3}} =$$

Approximate solution for structure:

$$f = \frac{1}{2\pi} \sqrt{\frac{3EI}{L^3(0.24M + M_H)}} = \frac{1}{2\pi} \sqrt{\frac{3 \times 50 \times 10^9 \times 18.7}{57^3 \times (0.24 \times 665 + 224) \times 10^3}} =$$

Measured frequency of tower sway is

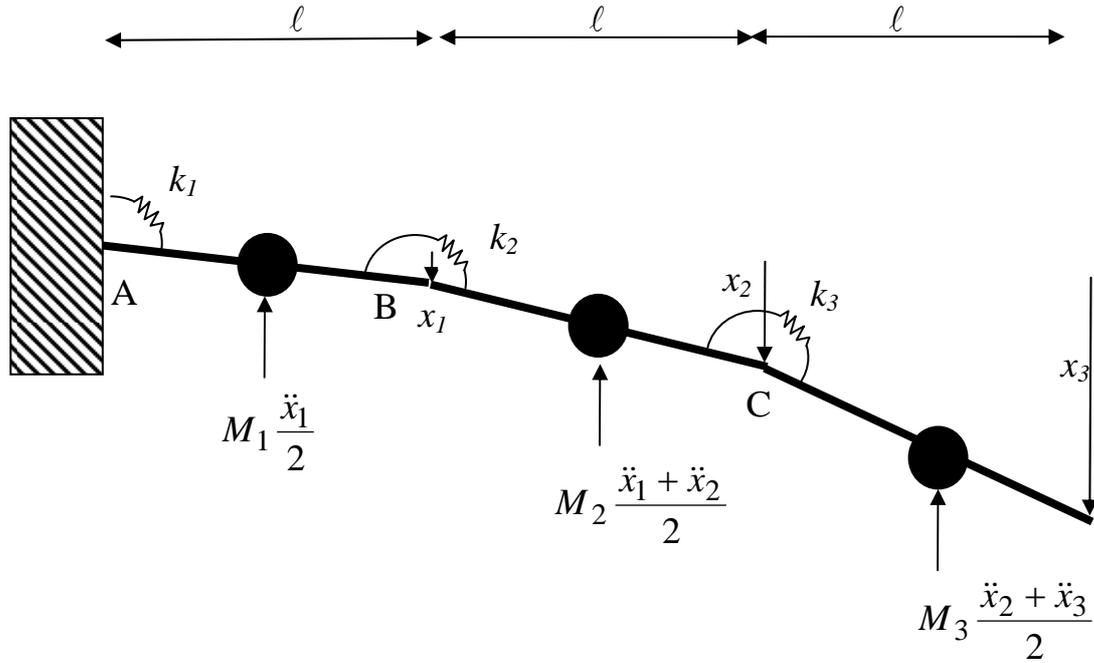
Ball park figure is right, but more accuracy needed to be sure.

Very significant effect of towerhead mass.

These numbers are similar to 1P and 3P, so it is important to get them right!

2.4 Blade vibration – example of multiple degree-of-freedom model

In this section we consider a three degree-of-freedom beam model to model vibrations, using the blade as an example problem. Here the distribution of mass and stiffness along the blade will be critical. This method is also applicable for towers (see examples paper).



$$\text{Moments about C to tip: } k_3 \left(\frac{(x_3 - x_2)}{l} - \frac{(x_2 - x_1)}{l} \right) + \frac{\ell}{2} M_3 \frac{(\ddot{x}_2 + \ddot{x}_3)}{2} = 0$$

$$\text{Moments about B to tip: } \frac{k_2}{l} (x_2 - 2x_1) + \frac{\ell}{2} M_2 \frac{\ddot{x}_1 + \ddot{x}_2}{2} + \frac{3\ell}{2} M_3 \frac{\ddot{x}_3 + \ddot{x}_2}{2} = 0$$

$$\text{Moments about A to tip: } \frac{k_1 x_1}{l} + \frac{\ell}{2} M_1 \frac{\ddot{x}_1}{2} + \frac{3\ell}{2} M_2 \frac{\ddot{x}_1 + \ddot{x}_2}{2} + \frac{5\ell}{2} M_3 \frac{\ddot{x}_3 + \ddot{x}_2}{2} = 0$$

In matrix form:

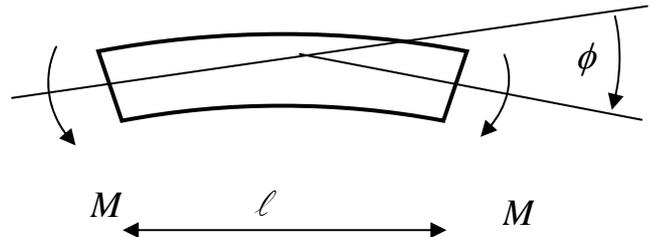
$$\frac{\ell}{4} \begin{pmatrix} 0 & M_3 & M_3 \\ M_2 & M_2 + 3M_3 & 3M_3 \\ M_1 + 3M_2 & 3M_2 + 5M_3 & 5M_3 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} + \frac{1}{l} \begin{pmatrix} k_3 & -2k_3 & k_3 \\ -2k_2 & k_2 & 0 \\ k_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

This equation, of the form $[m][\ddot{x}] + [k][x] = 0$, has harmonic solutions which satisfy the problem

$[m]^{-1}[k][x] = \omega^2[x]$ where the eigenvalues give the square of the resonant frequencies, and the eigenvectors give the

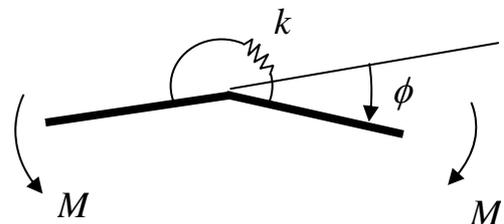
To calculate an appropriate k for the beam (rather than entire beam), consider a beam section in bending with constant moment M

From the beam model: $\frac{M}{EI} = \frac{d^2 y}{dx^2} = \frac{d\phi}{dx} = \frac{\phi}{\ell}$



From the spring model: $M = k\phi$

Hence $k = \frac{EI}{\ell}$



Check on model

Putting $M_1 = M_2 = M_3 = M/3$, $k_1 = 2\frac{EI}{\ell}$, $k_2 = k_3 = \frac{EI}{\ell}$, $L = 3\ell$

gives the lowest two frequencies as:

$$f \begin{cases} \text{First mode} \\ \text{Second mode} \end{cases} = \begin{cases} 3.41 \\ 22.2 \end{cases} \frac{1}{2\pi} \sqrt{\frac{EI}{mL^4}}$$

giving good agreement with the analytical values of the constants of 3.52 and 22.0.

See code below; note that the Matlab command **eig(k,m)** (or Octave command **qz(k,m)**) solves the relevant eigenvalue problem with **k** and **m** the stiffness and mass matrices.

```
L=1/3
mm=[1 1 1]/3;
EI=[1 1 1];
k1=2*EI(1)/L;k2=EI(2)/L;k3=EI(3)/L;
m1=mm(1);m2=mm(2);m3=mm(3);
mmatrix=[0 m3 m3;m2 (m2+3*m3) 3*m3;(m1+3*m2) (3*m2+5*m3) 5*m3]*L/4;
kmatrix=[k3 -2*k3 k3;-2*k2 k2 0 ;k1 0 0]/L;
[v d]=eig(kmatrix,mmatrix)
%octave[aa bb q z v ww d]=qz(kmatrix,mmatrix)%ww are eigenvectors, d are eigenvalues
f=sqrt(d)/(2*pi)
```

Variable mass and stiffness blade model

For the Tjaereborg blade, model the 30 m blade by three masses at distances 5, 15 and 25 m from root, and three springs with effective EI values at 2.5, 10 and 20 m from root. The following very approximate mass and EI distributions are quoted in Hansen, page 100.

Distance from root (m)	mass/unit length (kg/m)	EI (flapwise) (MNm ²)
2.5		1700
5	400	
10		240
15	200	
20		20
25	90	

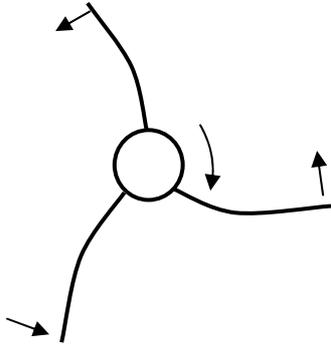
Putting $L = 30$ m, $\ell = 10$ m, $M = m \times \ell$, $k_1 = 2 \frac{EI(2.5m)}{\ell}$, $k_2 = \frac{EI(10m)}{\ell}$, $k_3 = \frac{EI(20m)}{\ell}$ gives a lowest natural frequency of 1.05 Hz, compared with the measured value of

A good estimate – though really a more sophisticated model is needed with the rather dramatic change in properties.

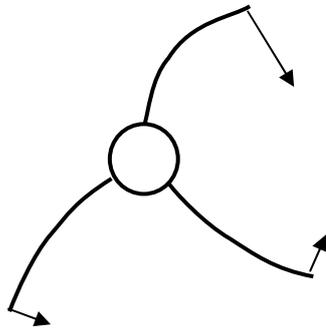
```
%Tjaereborg blade
LL=30
EI=[1700 240 20]*1.e6%EI1 flapwise
mm= [400 200 90]
L=LL/3;
k1=2*EI(1)/L;k2=EI(2)/L;k3=EI(3)/L;
m1=mm(1)*L;m2=mm(2)*L;m3=mm(3)*L;
mmatrix=[0 m3 m3;m2 (m2+3*m3) 3*m3;(m1+3*m2) (3*m2+5*m3) 5*m3]*L/4;
kmatrix=[k3 -2*k3 k3;-2*k2 k2 0 ;k1 0 0]/L;
[v d]=eig(kmatrix,mmatrix)
%octave[aa bb q z v ww d]=qz(kmatrix,mmatrix)%ww are eigenvectors, d are eigenvalues
f=sqrt(d)/(2*pi)
```


Modes

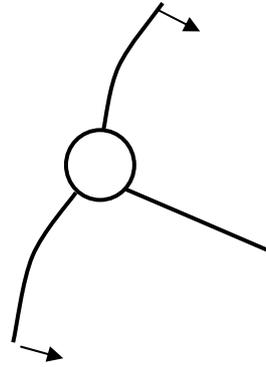
13.4 Hz
Coupled mode

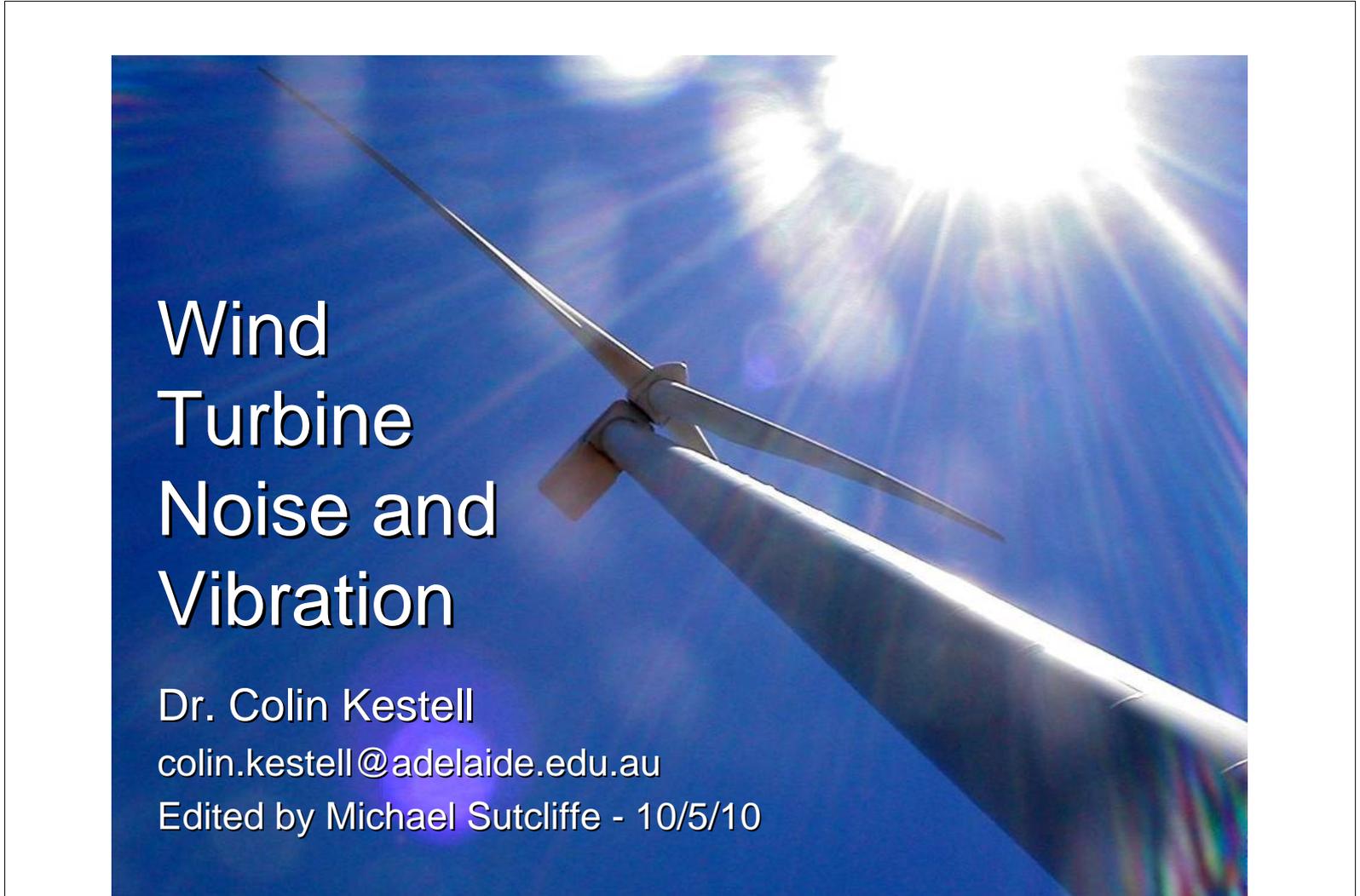


2.3 Hz (A)
Asymmetric blade
mode (hub stationary)



2.3 Hz (2×A+B)
Symmetrical mode
(hub stationary)



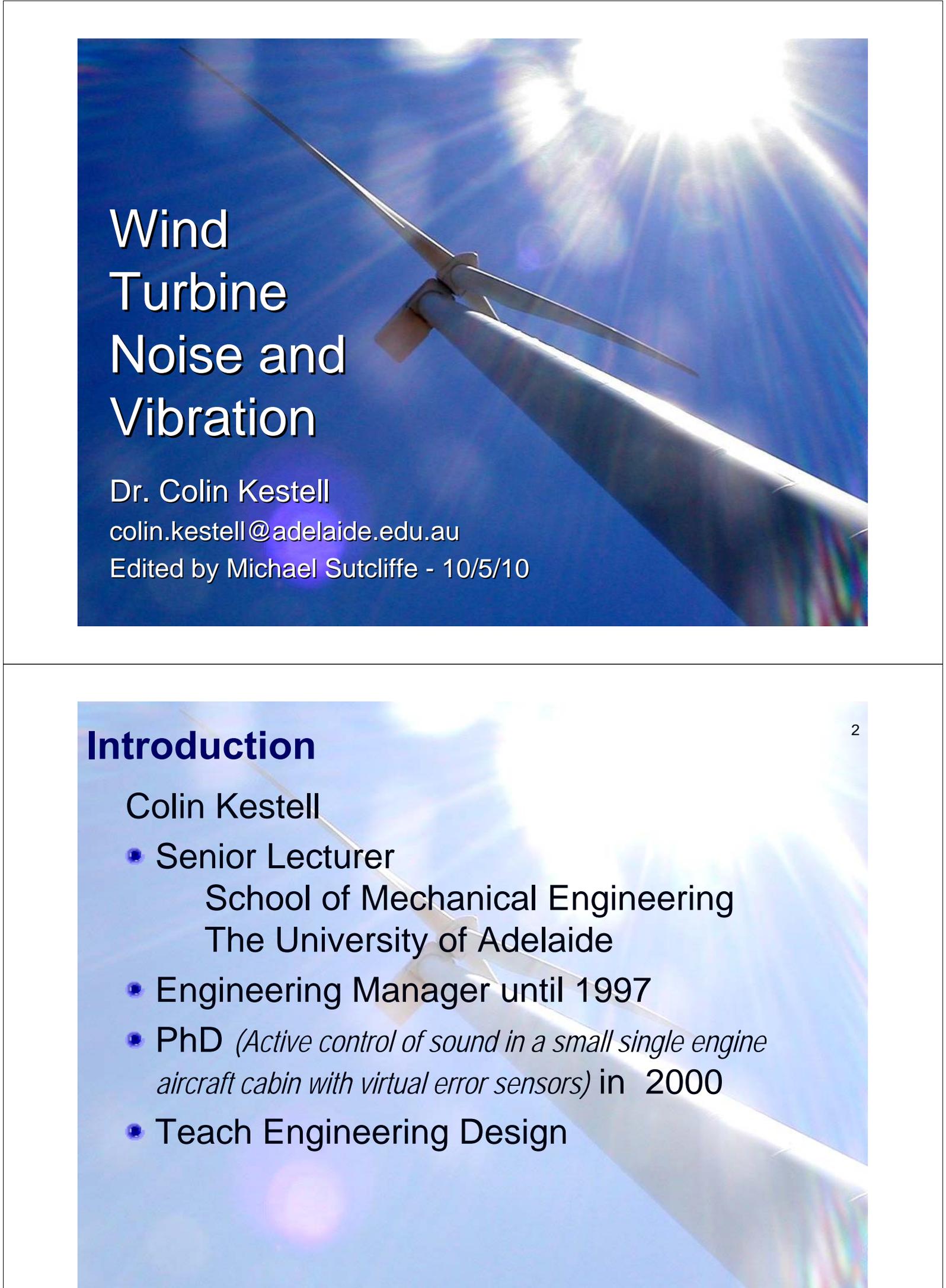


Wind Turbine Noise and Vibration

Dr. Colin Kestell

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Edited by Michael Sutcliffe - 10/5/10



Introduction

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Colin Kestell

- Senior Lecturer
School of Mechanical Engineering
The University of Adelaide
- Engineering Manager until 1997
- PhD (*Active control of sound in a small single engine aircraft cabin with virtual error sensors*) in 2000
- Teach Engineering Design

Sound

$$dB = 10 \log \frac{p^2}{p_{ref}^2}$$

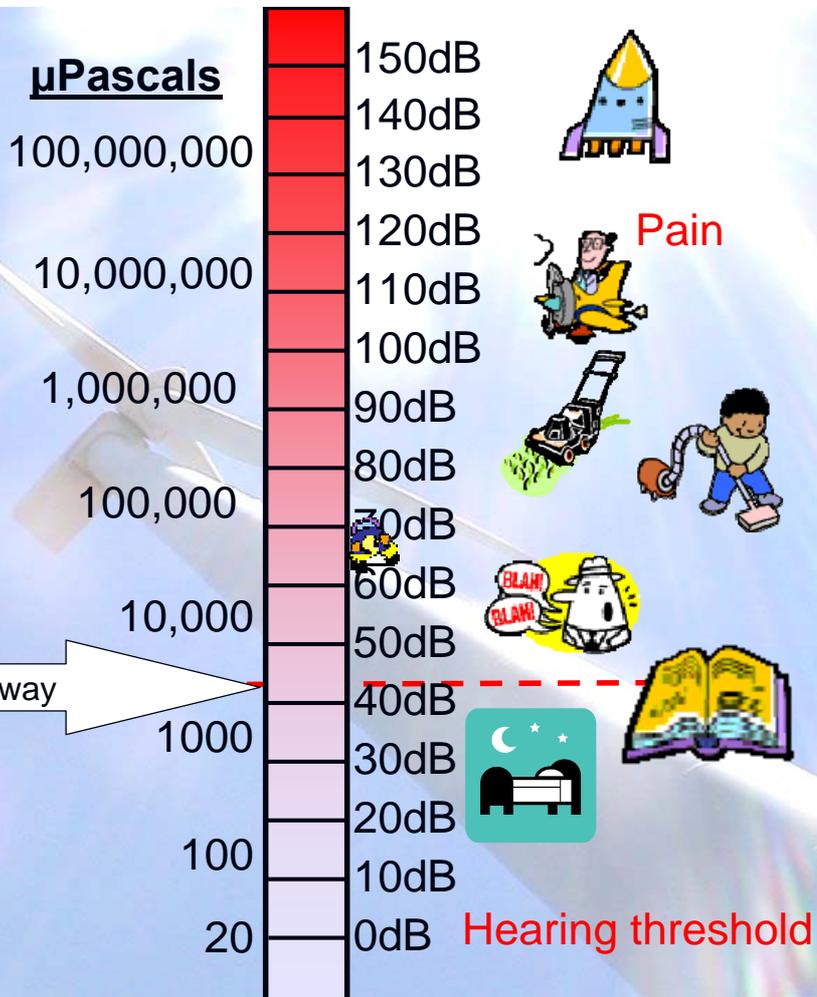
$$= 20 \log \frac{p}{p_{ref}}$$

$p_{ref} = 20 \times 10^{-6}$ Pascals



350m away

Important to state distance from source



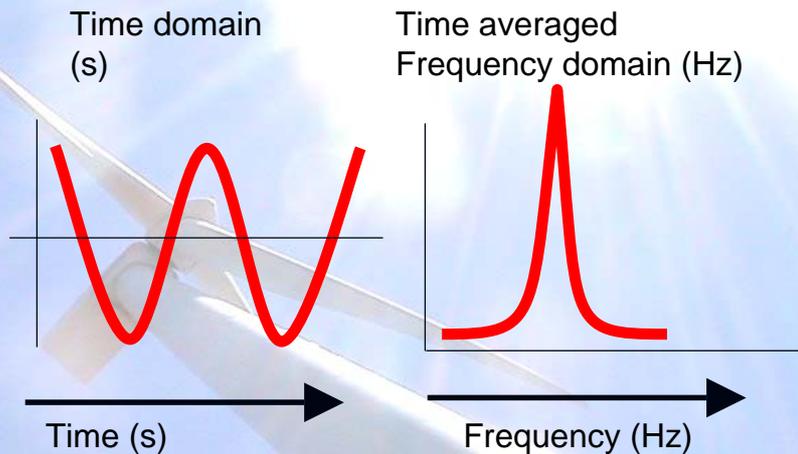
Measuring Sound

Types of noise

- Tonal Noise



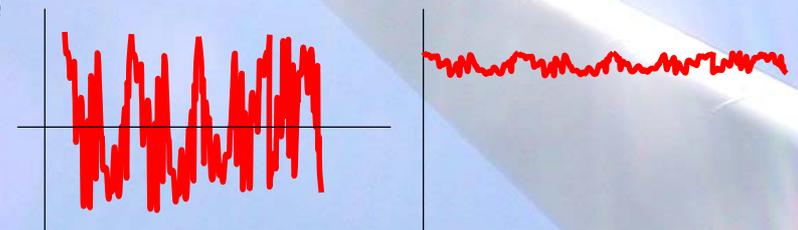
- 440Hz
- 1000Hz
- 10000 Hz



- Random Noise



- White Noise
- Pink Noise



Measuring Sound

Instrumentation - sound



A Sound Level Meter



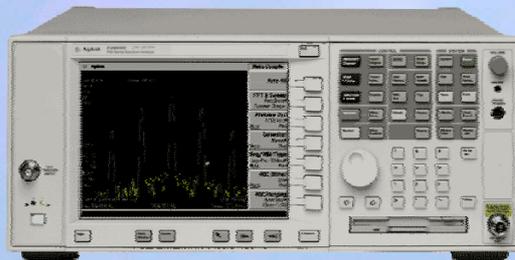
A windsock



A Microphone



Conditioning amplifier



Spectrum Analyser

Measuring Vibration

Vibration

Measured in terms of

- Displacement
- Velocity
- Acceleration – sometimes in terms of 'g' (gravity)
- *dB also used ... or*
- *absolute units on either a logarithmic or linear scale*

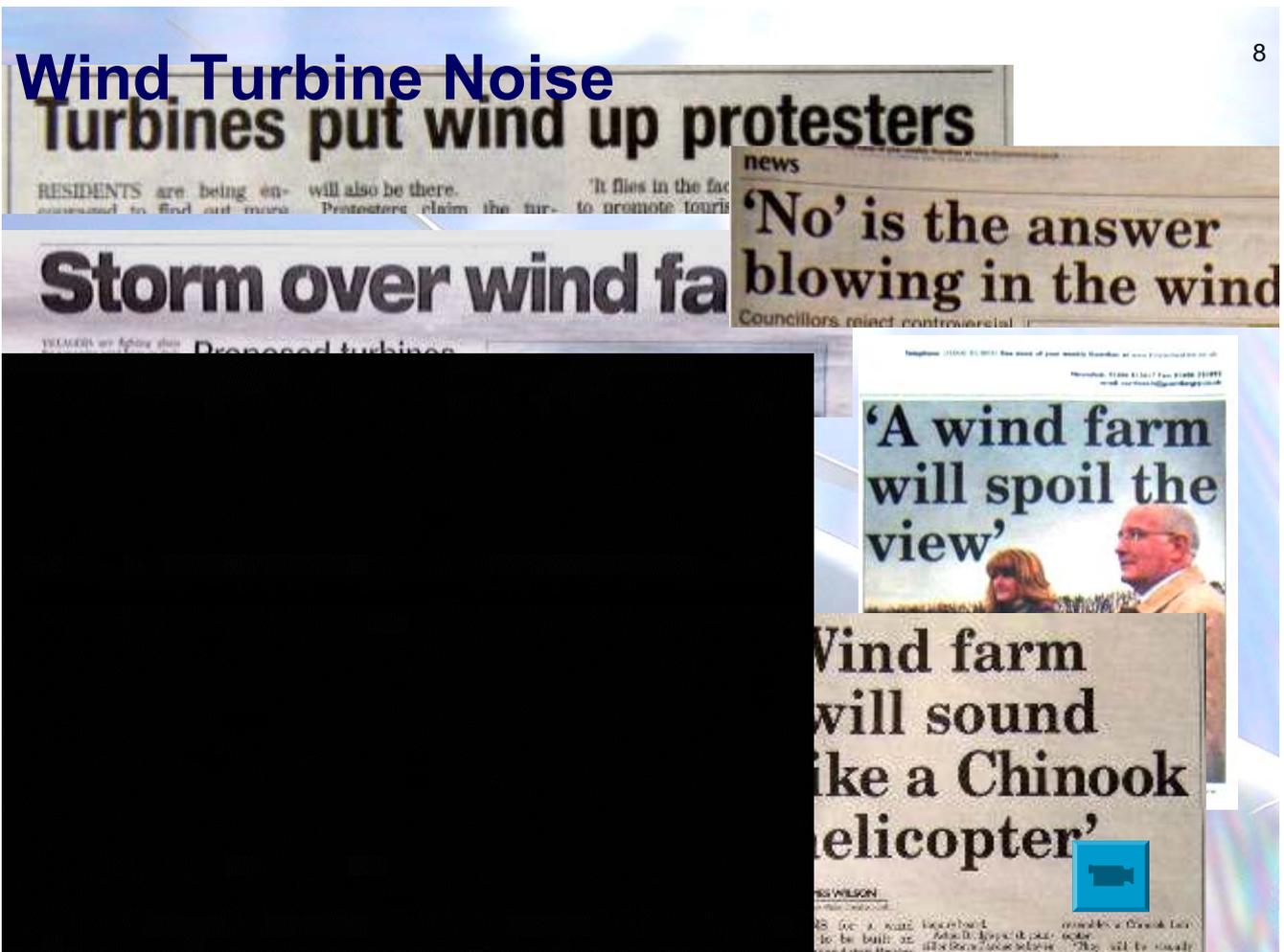
Be careful of units!

*All this and more covered in **Part IIA 3C6 Vibration**
and **Part IIB 4C6 Advanced Linear Vibration***

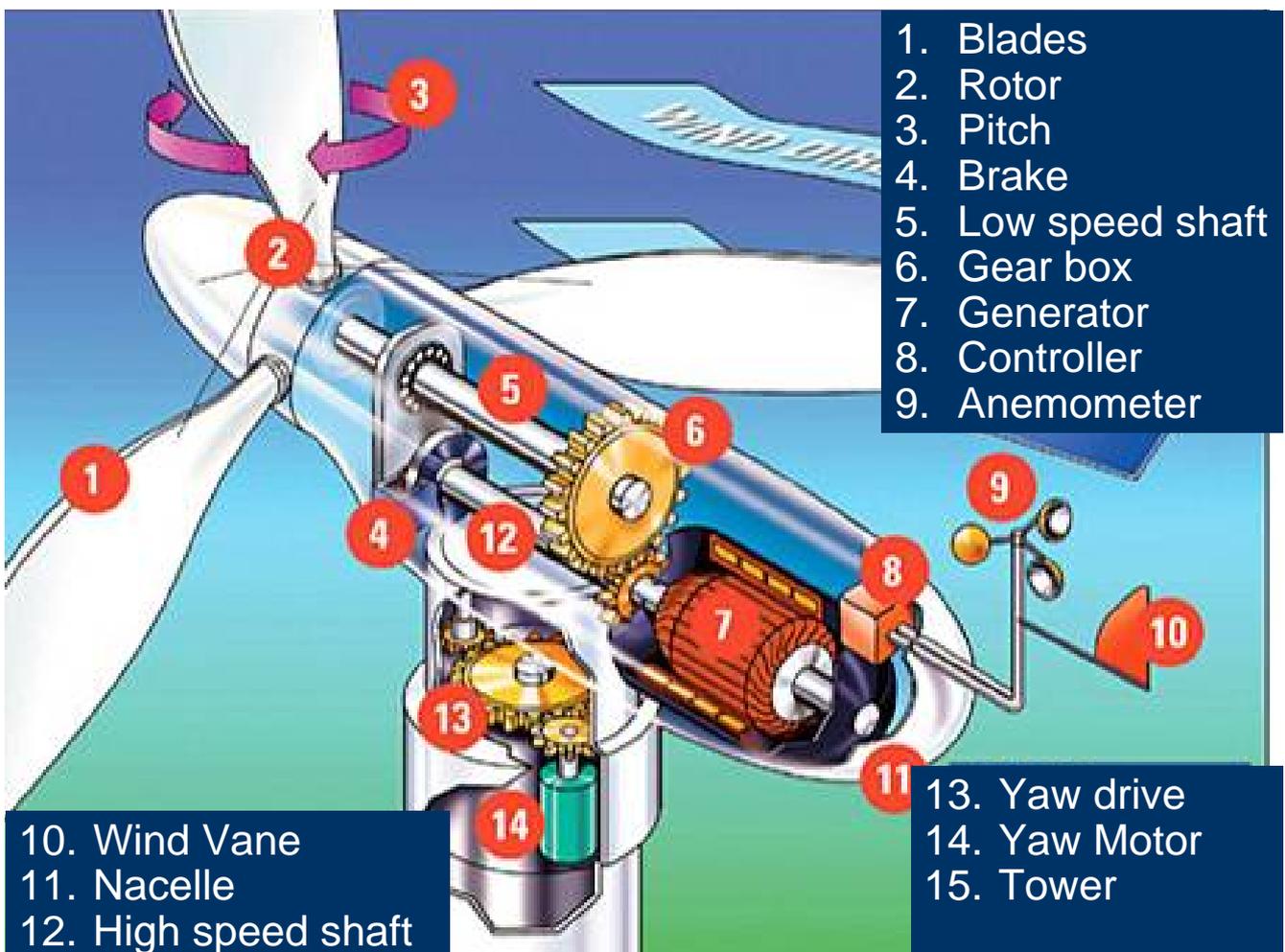
Wind turbines

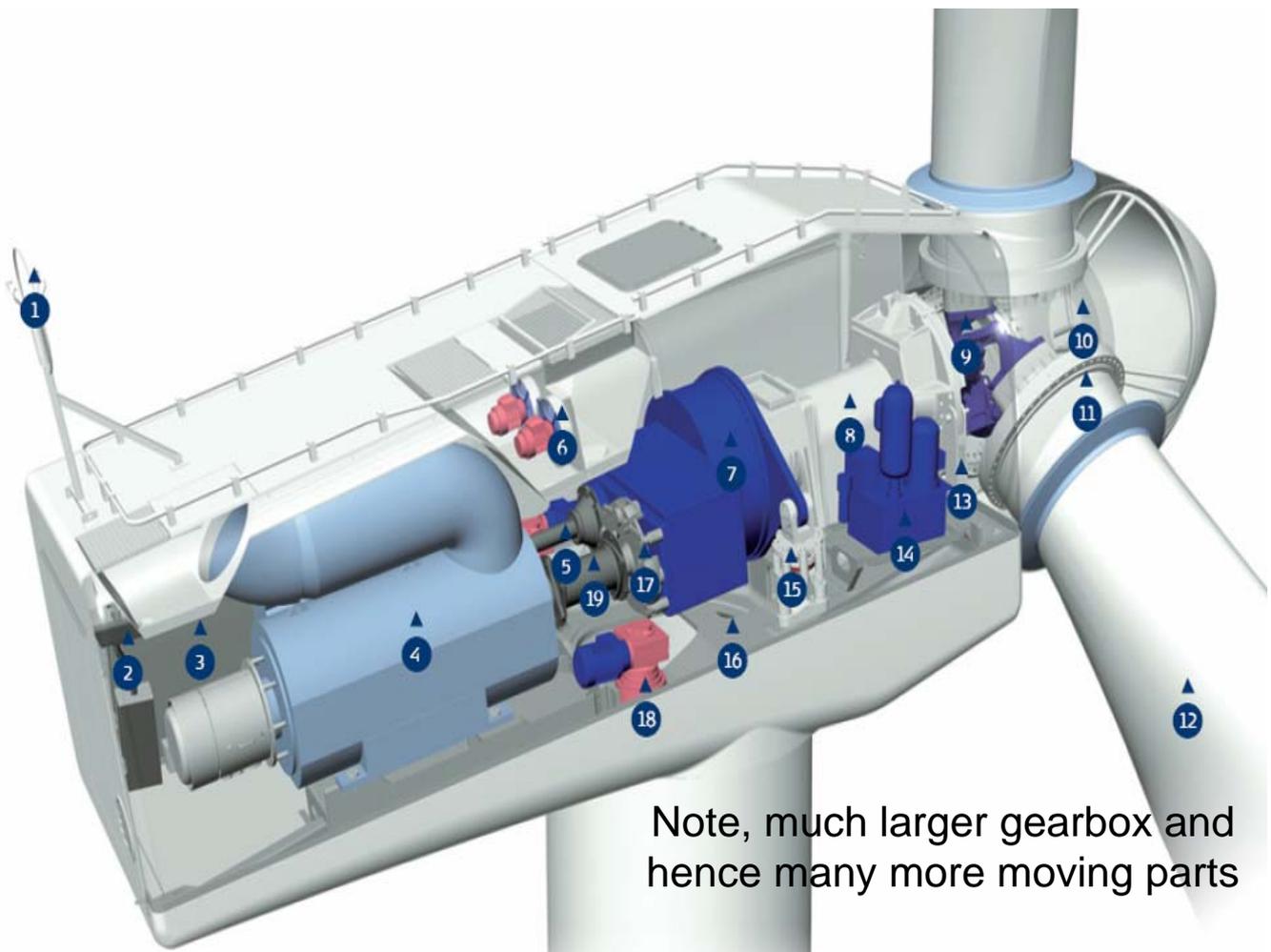


Wind Turbine Noise



What generates the noise in wind turbines ?

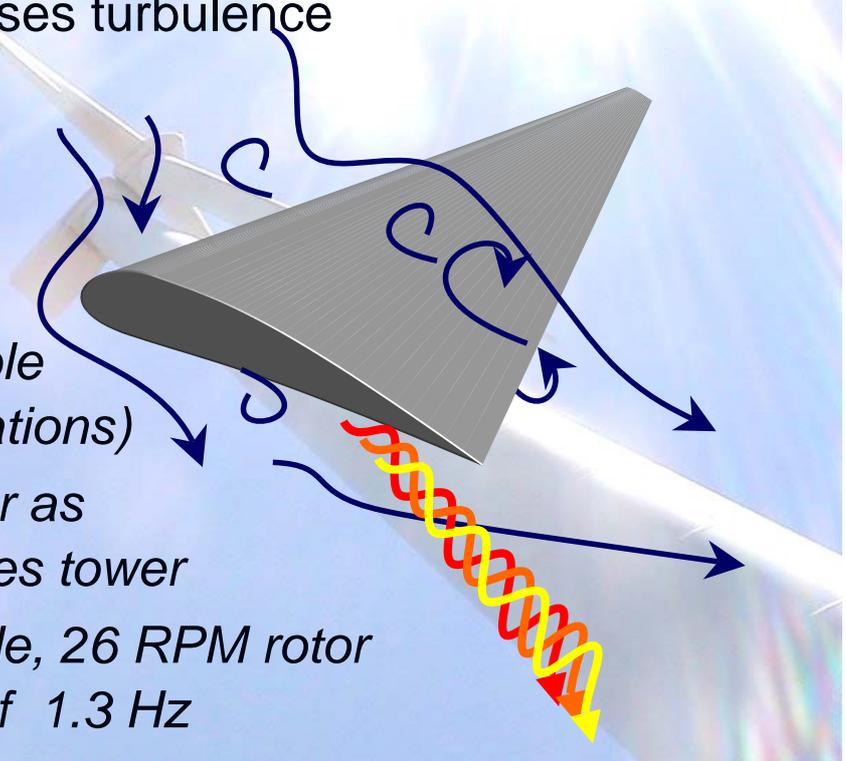




Aerodynamic Noise

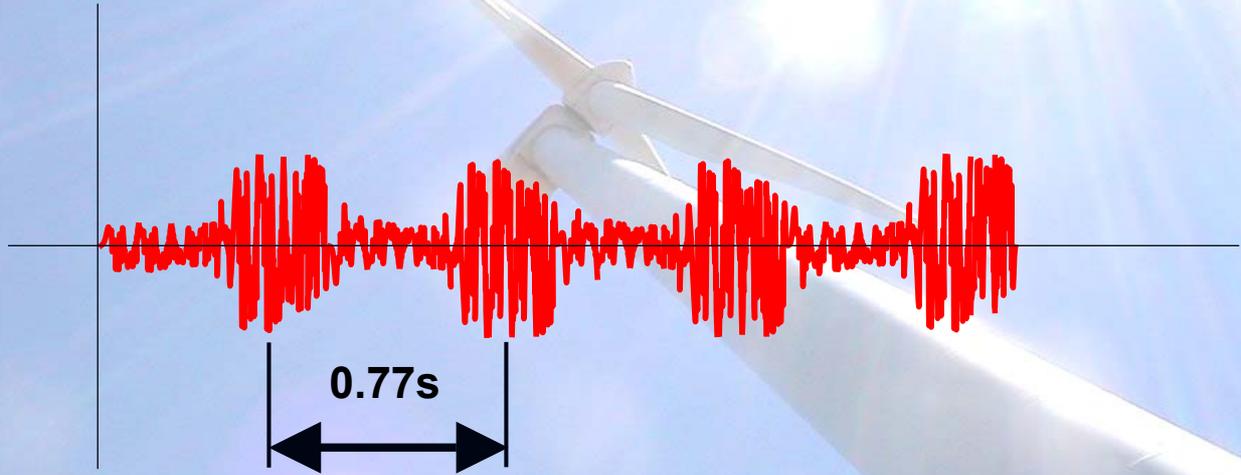
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- Blade passes through turbulent, often gusty flow
- Blade motion causes turbulence
- Wing tip vortices cause turbulence
- Turbulence creates sound (*broadband audible pressure perturbations*)
- *Turbulence higher as each blade passes tower*
- *Consider a 3 blade, 26 RPM rotor will have a BPF of 1.3 Hz*



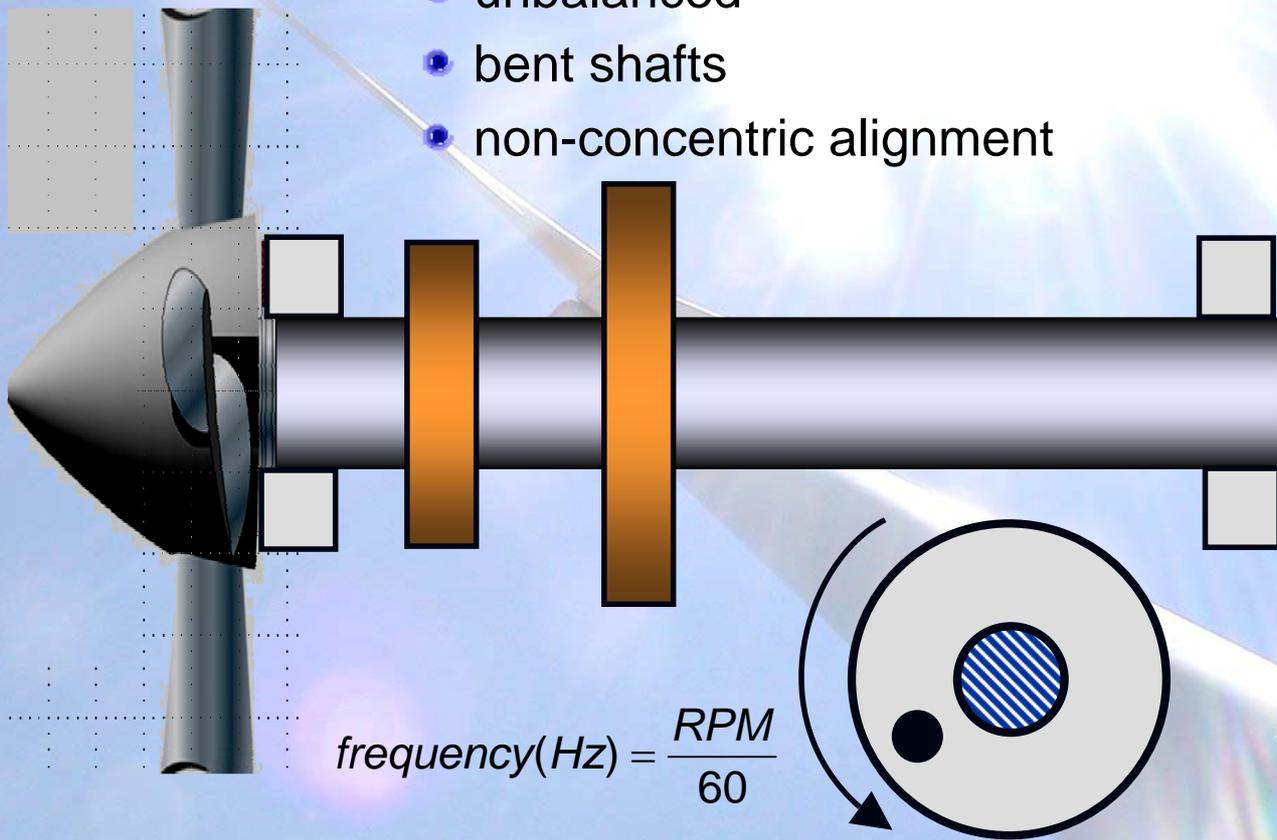
Aerodynamic Noise

- A 3 blade, 26 RPM rotor will have a tower BPF of 1.3 Hz, or a periodic time of 0.77 seconds



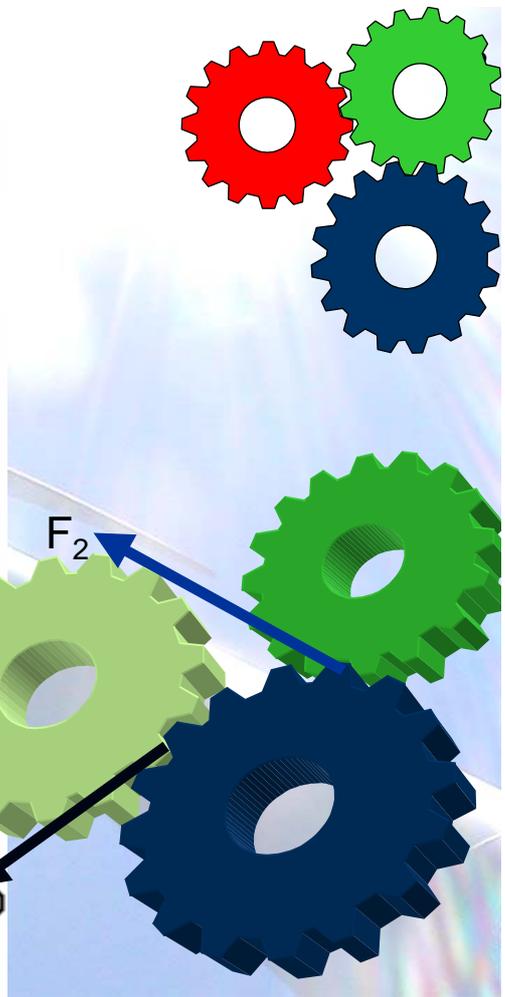
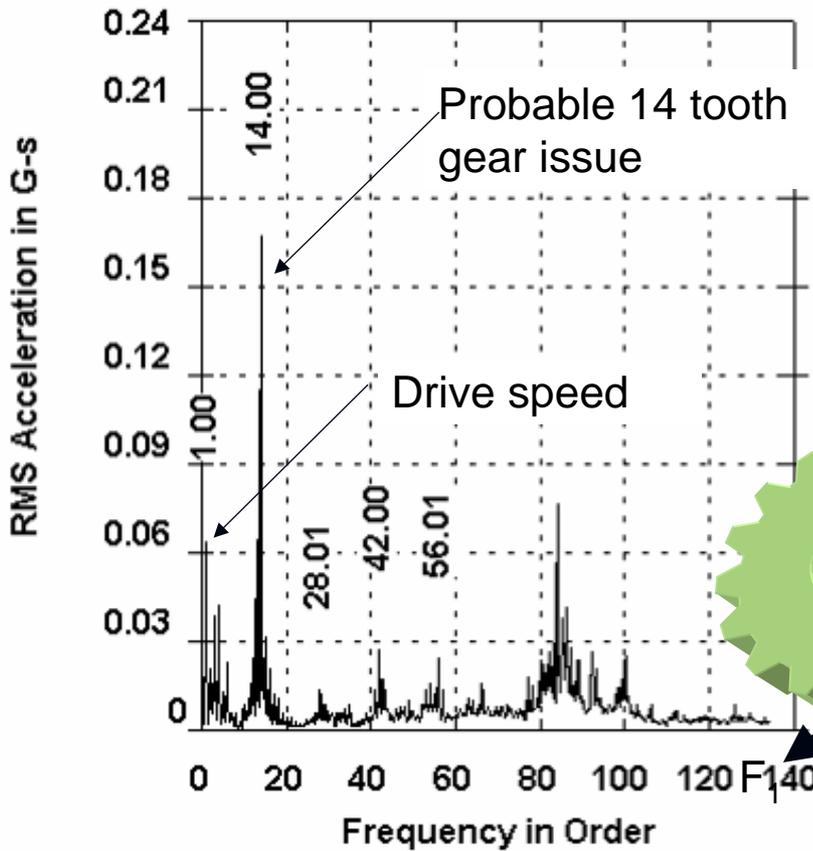
Shaft noise

- unbalanced
- bent shafts
- non-concentric alignment



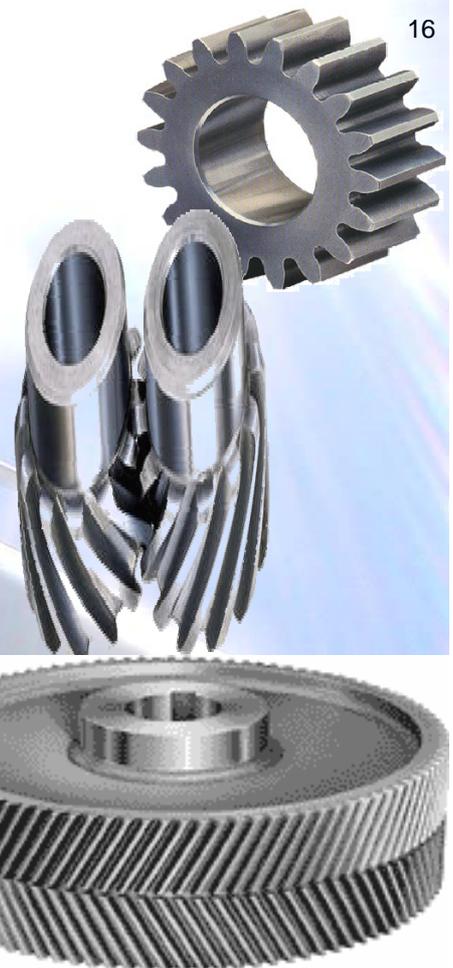
$$\text{frequency(Hz)} = \frac{\text{RPM}}{60}$$

Gear Noise



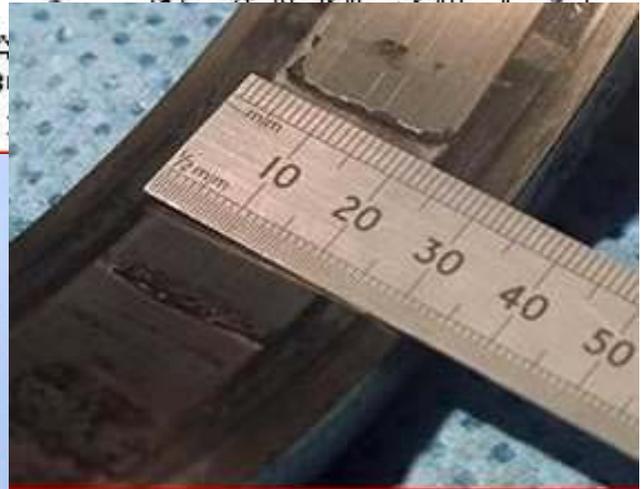
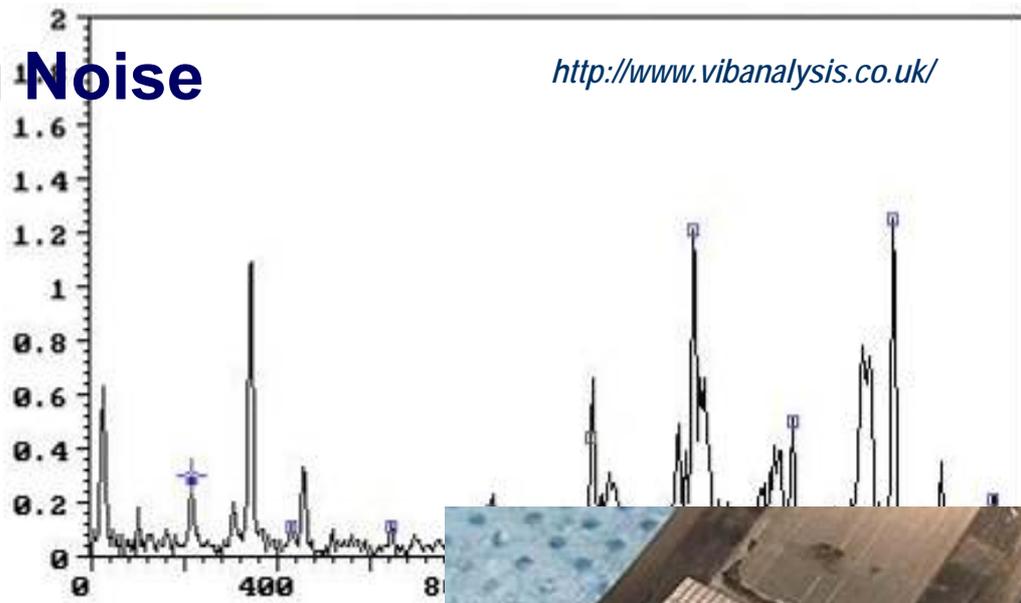
Gear Noise

- Normal spur gears – cost effective
- Helical gears – smoother mesh, more expensive, produce an axial force component as well as a tangential
- Herring bone gears (far more expensive) realign the resultant force



Bearing Noise

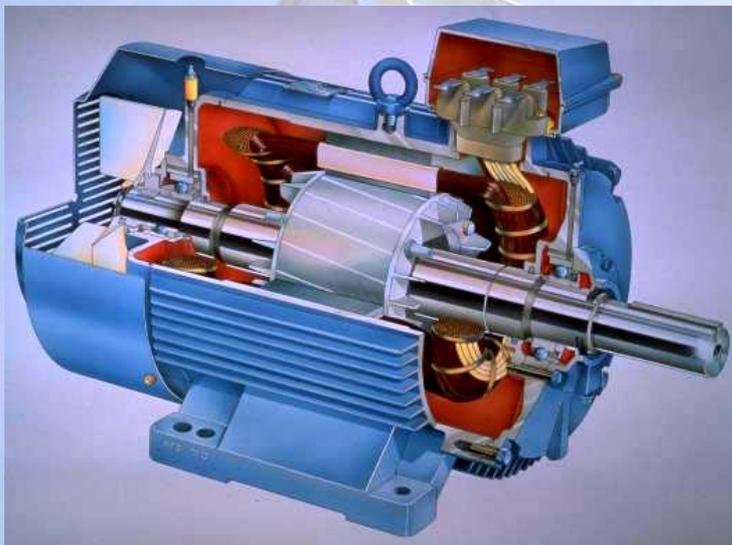
<http://www.vibanalysis.co.uk/>



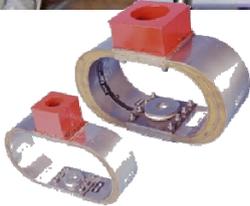
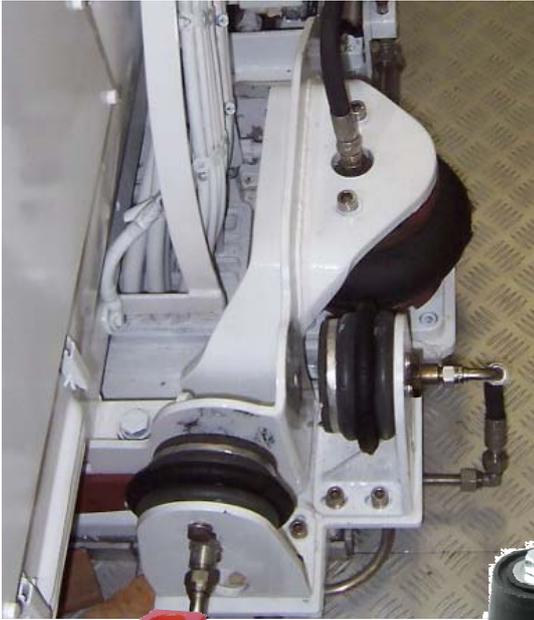
Generator Noise

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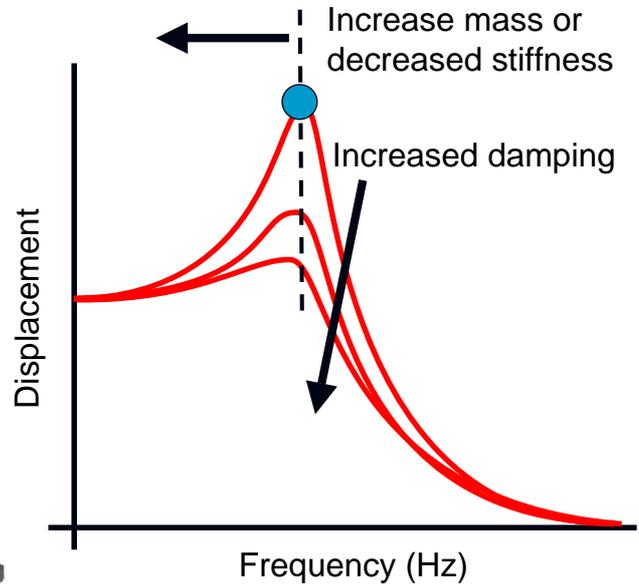
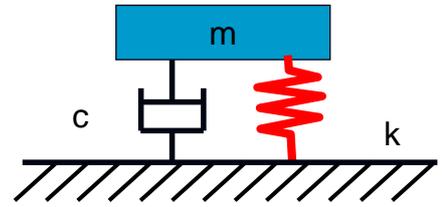
- A typical 3-phase generator will have
- 3 pairs of (6) opposing wound coils
- 4 rotating permanent magnets
- Producing 12 pulses per revolution



Vibration isolation

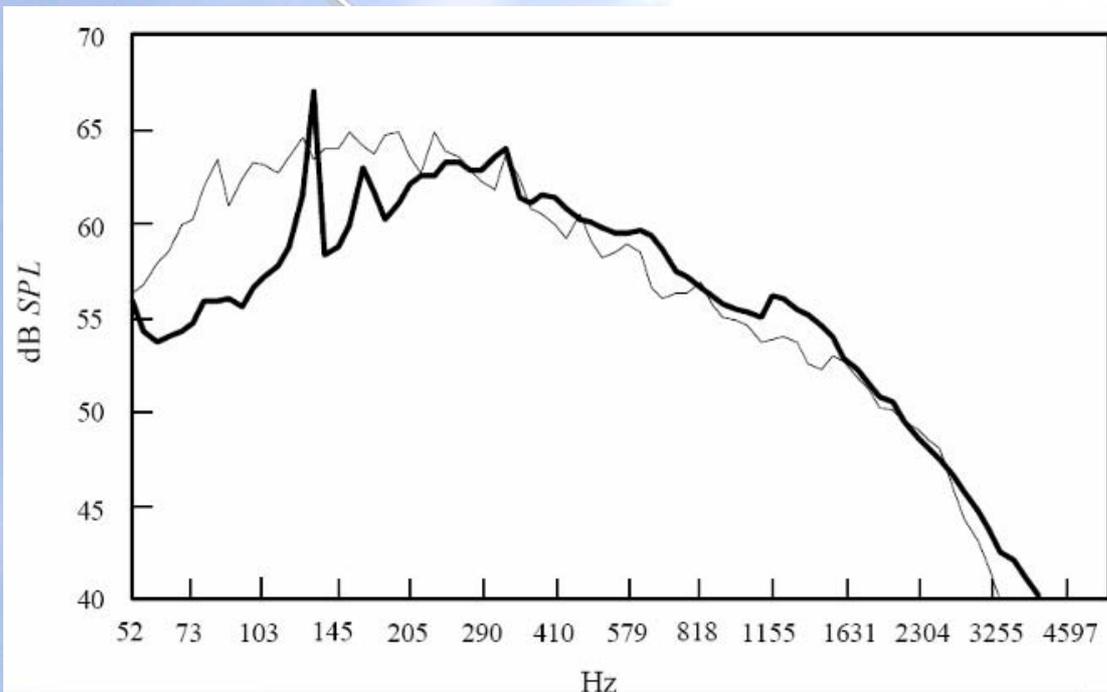


$$\omega = \sqrt{\frac{k}{m}}$$



Examples

- Vesta V52-850 kW, 3 Blade, 26 RPM

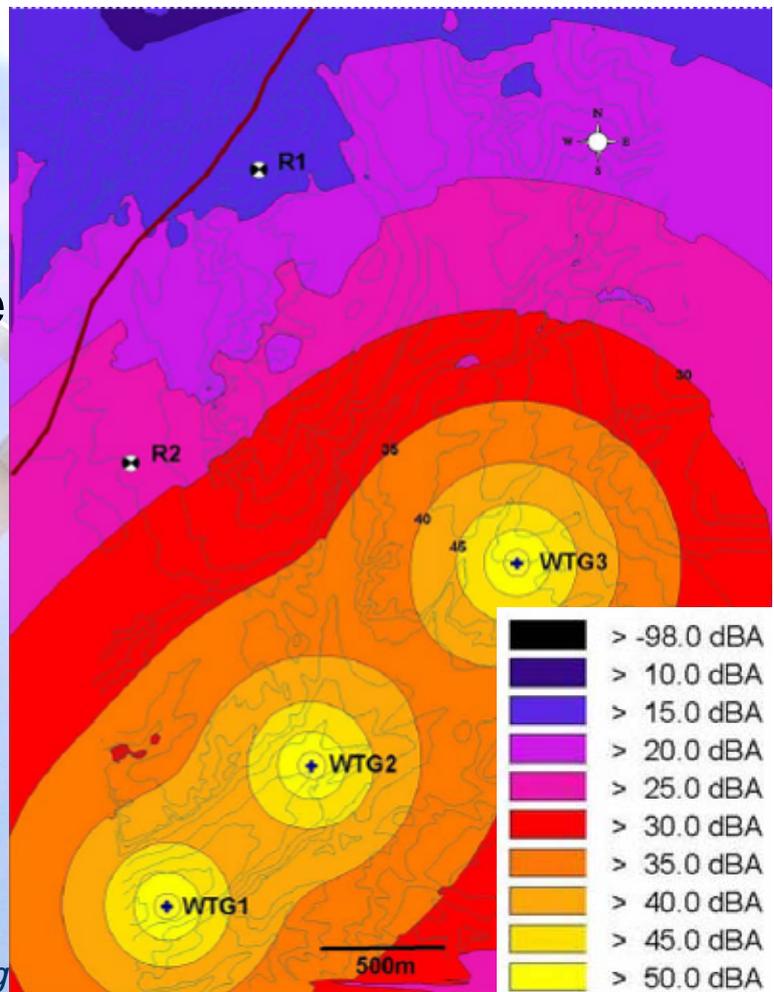


*Psycho-acoustic characters of relevance for annoyance of wind turbine noise.
K. Persson Waye and E. Og Hrstrog M. journal of sound and vibration (2002) 250(1), 65-73*

Examples

- Acoustic model of typical wind turbine sound propagation

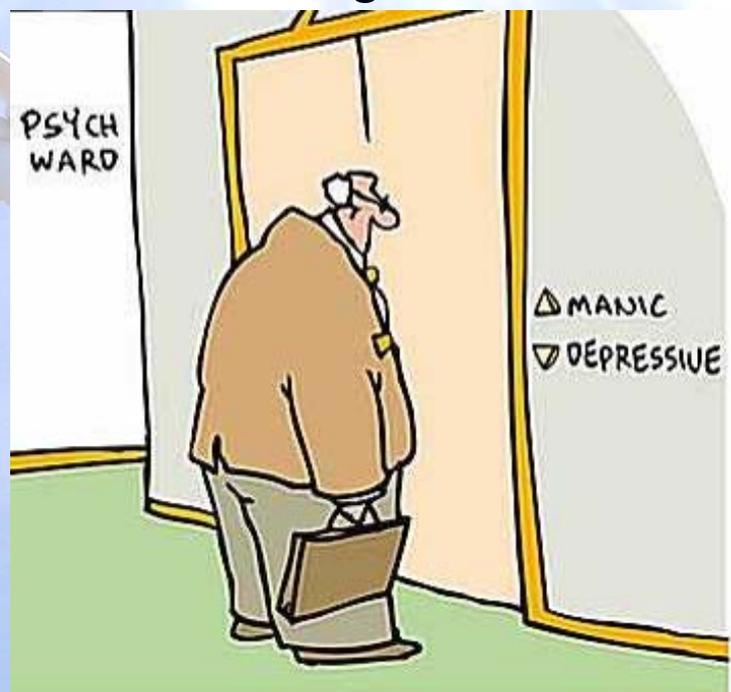
Wind turbines and sound: Review and best practice guidelines. HGC Engineering



Noisy or quiet?

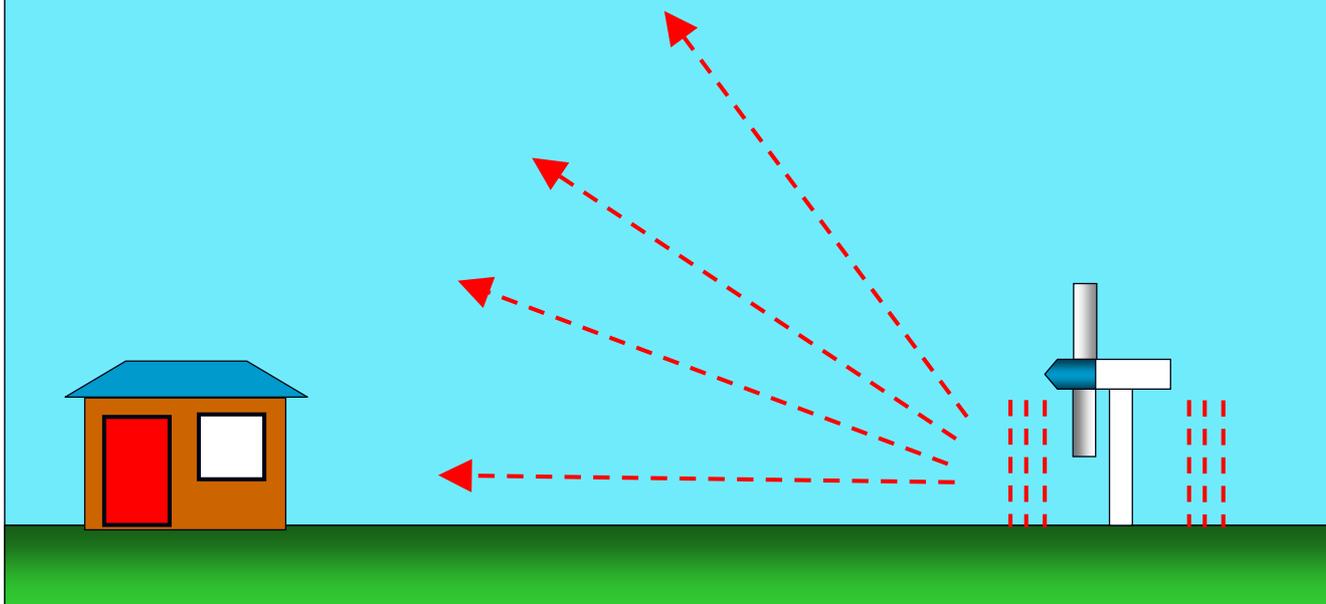
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- Many claim that the noise is worse at night or in the early hours of the morning.
 - Less masking (other noises covering it up)
 - Physiological issues
 - Thermal inversion layers



Noisy or quiet?

- Normal conditions



Noisy or quiet?

- One of a few meteorological effects

