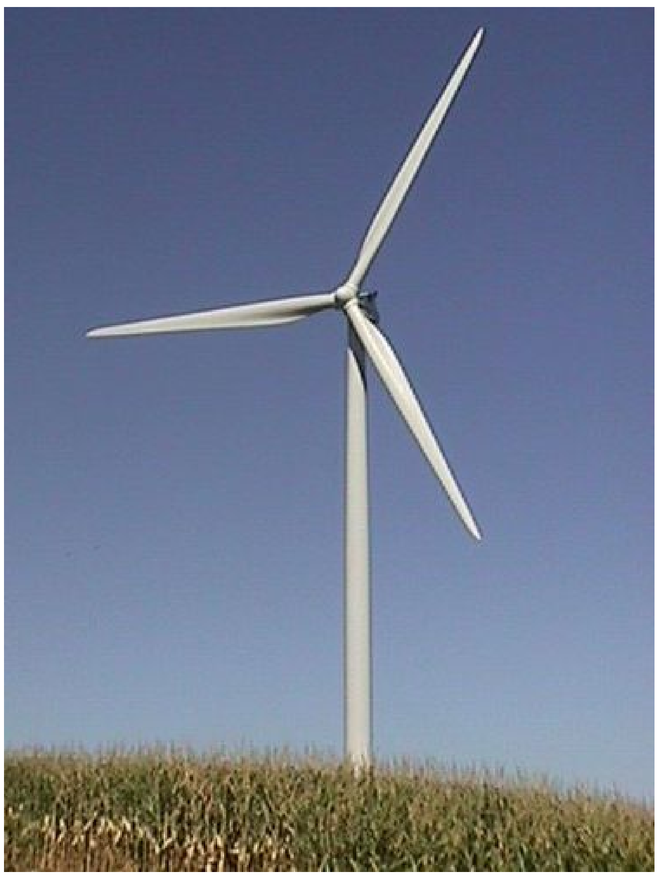
ENGINEERING TRIPOS PART IB

PAPER 8 – ELECTIVE (2)

# Mechanical Engineering for Renewable Energy Systems

**Dr. Digby Symons**

# Wind Turbine Blade Design



Student Handout

**CONTENTS**



More detailed coverage of the material in this handout can be found in various books,

e.g. Aerodynamics of Wind Turbines, Hansen M.O.L. 2000

# 1 INTRODUCTION

### *1.1.1 Aim*

Preliminary design of a wind turbine

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- 
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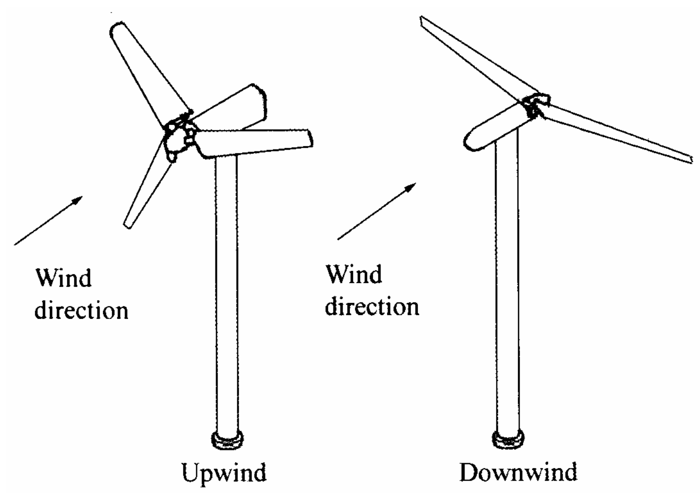
### *1.1.2 Wind turbine type*

Horizontal axis wind turbine (HAWT) with 3 blade upwind rotor – the “Danish concept”:




### *1.1.3 Load cases*

We will consider two load cases:

1) Normal operation – continuous loading
   - Aerodynamic, centrifugal and self-weight loading
   - 
   - 
2) Extreme wind loading – storm loading with rotor stopped
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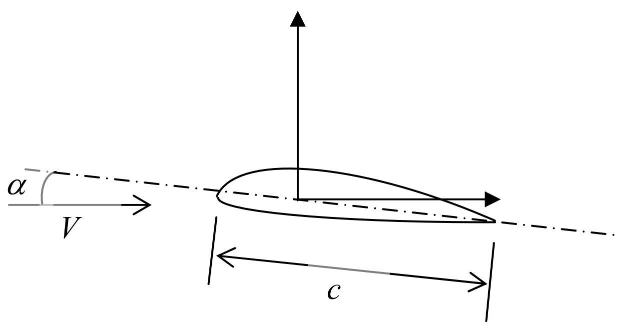
# 2 WIND TURBINE BLADE AERODYNAMICS

## 2.1 Aerofoil Aerodynamics

### *2.1.1 Lift, drag and angle of attack*

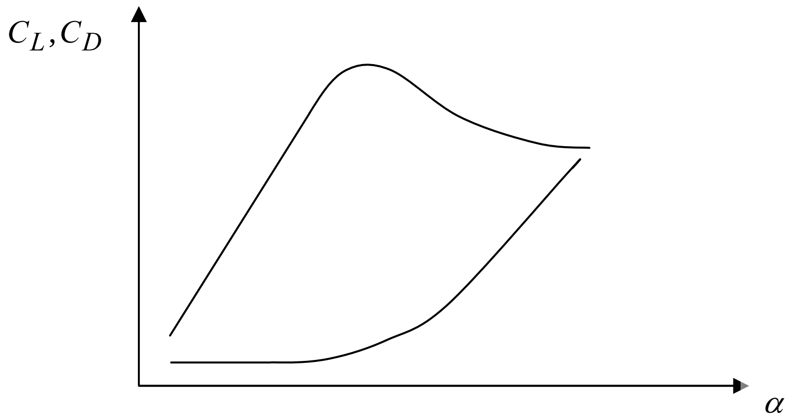



### *2.1.2 Lift and drag coefficients*

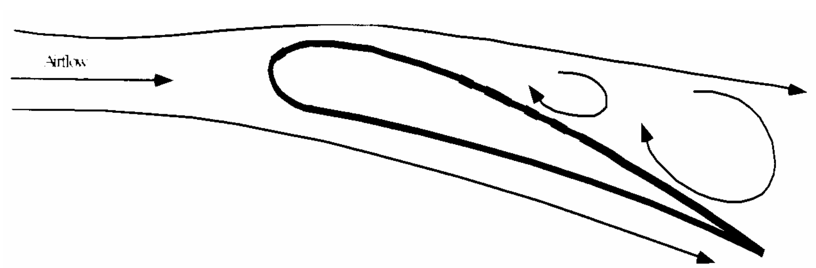
Define non-dimensional lift and drag coefficients

*2.1.3 Variation of lift and drag coefficients with angle of attack*

How does lift and drag vary with angle of attack $\alpha$ ?



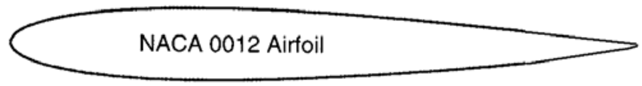

Stall:



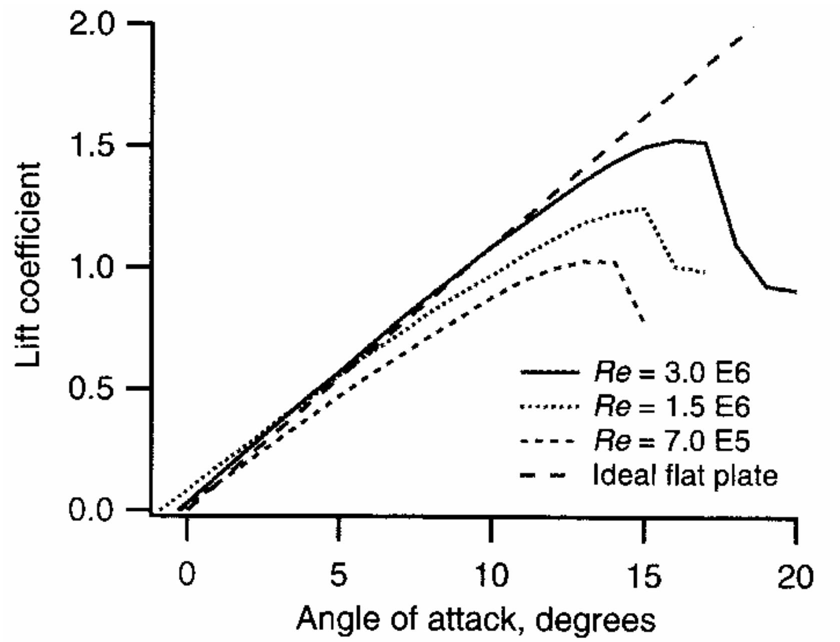

*2.1.4 Application of 2D theory to wind turbines*

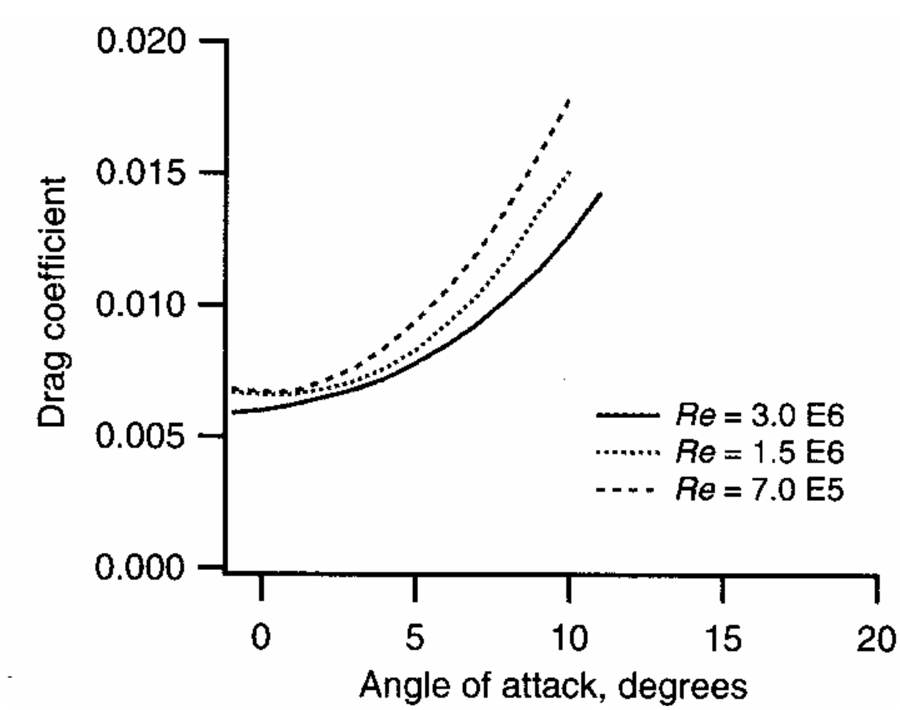
- Tip leakage means flow is not purely two dimensional
- Wind turbine blades are spinning with an angular velocity $\omega$
- The angle of attack depends on the relative wind velocity direction.

*2.1.5 Example aerofoil shape used in wind turbines*



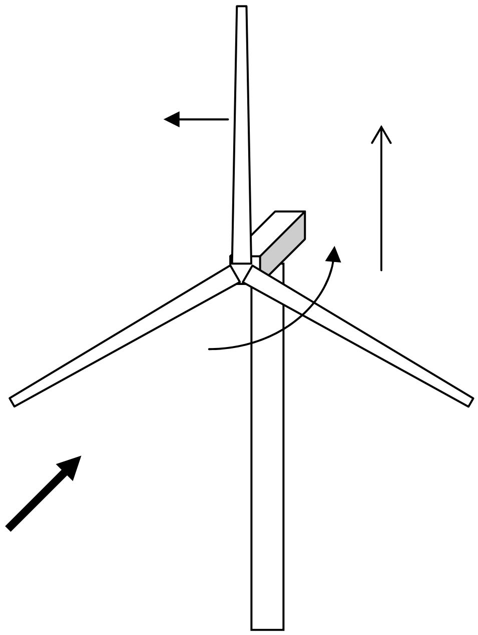

Lift and drag coefficients for the NACA 0012 symmetric aerofoil (Miley, 1982)

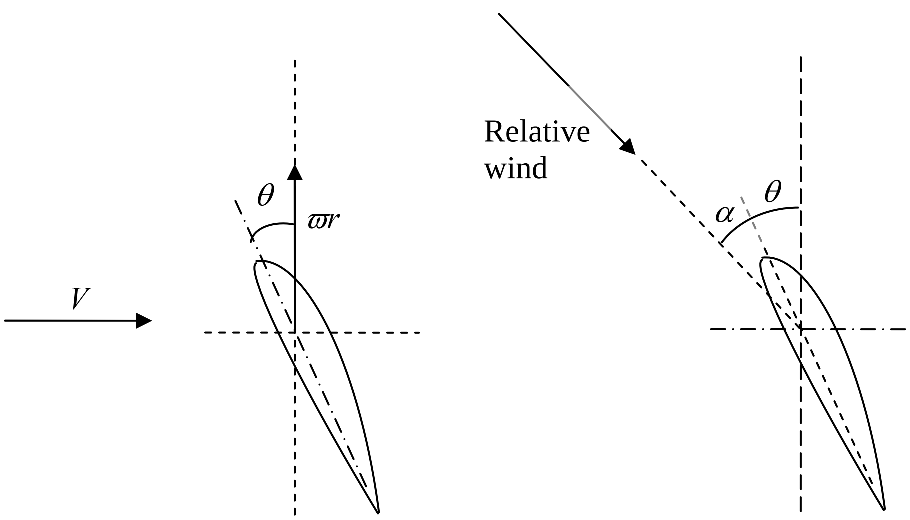


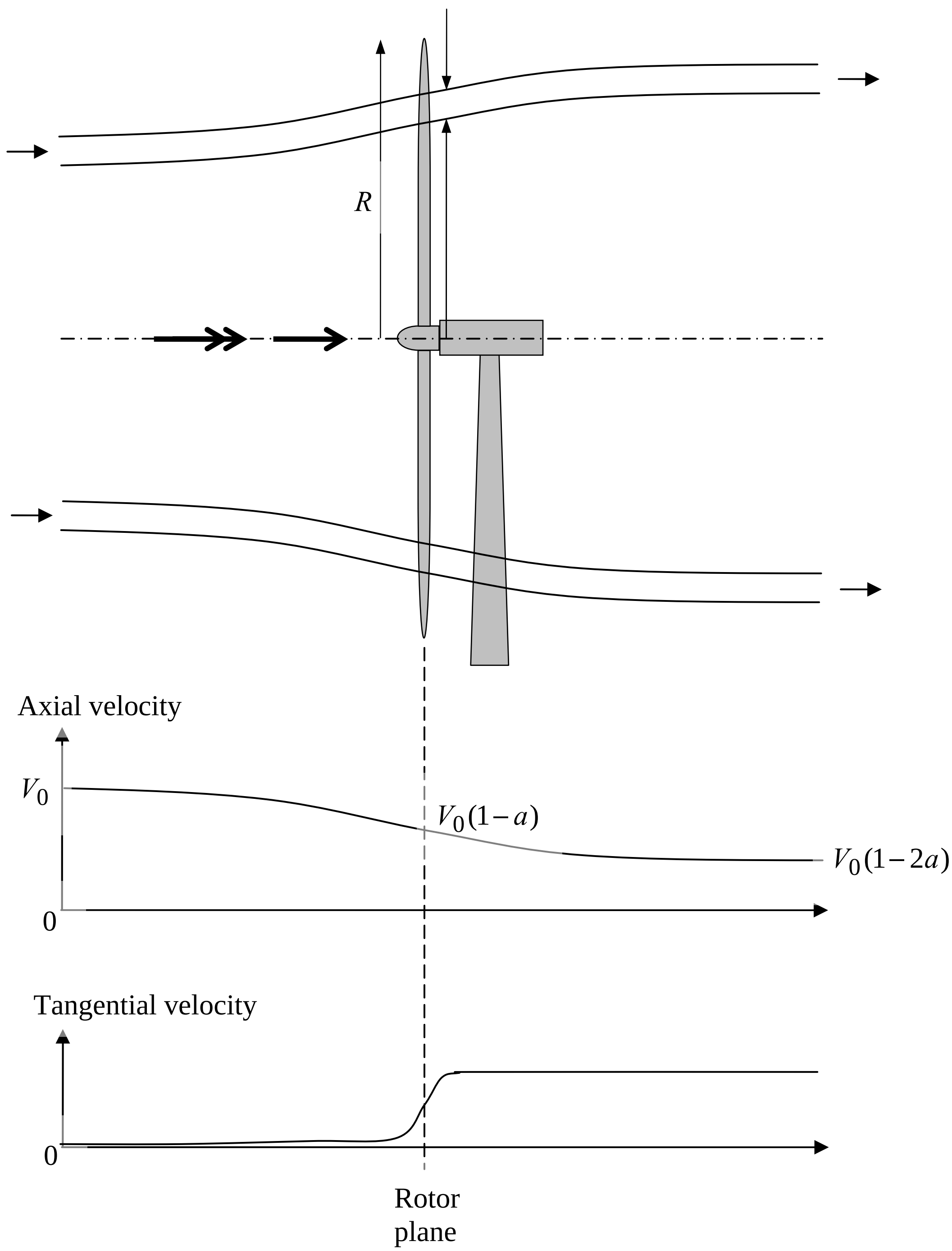
## 2.2 Wind Turbine Blade Kinematics

### *2.2.1 Blade rotation*





*2.2.2 Wake rotation*

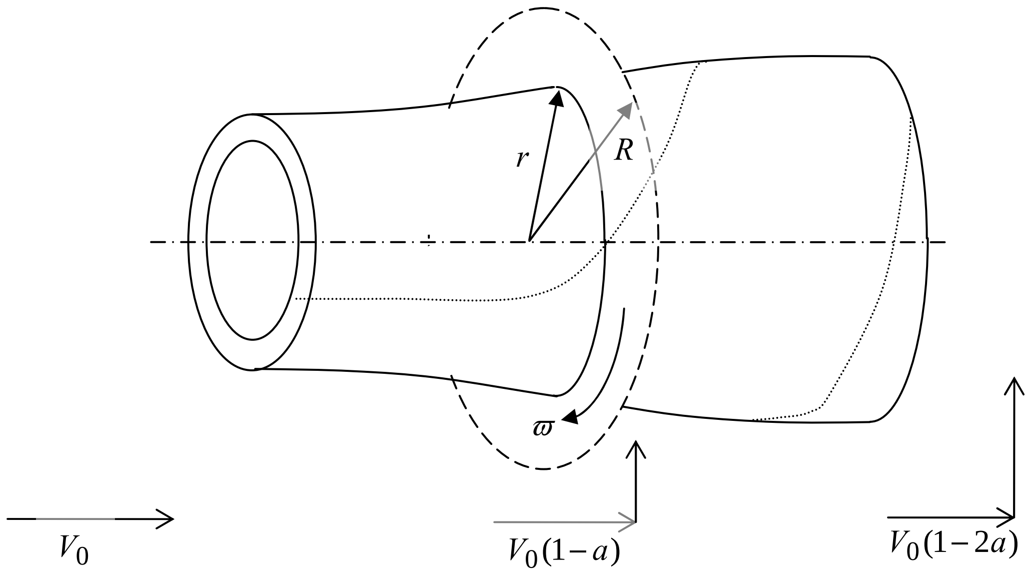



$a$ = axial induction factor

$a'$ = angular induction factor

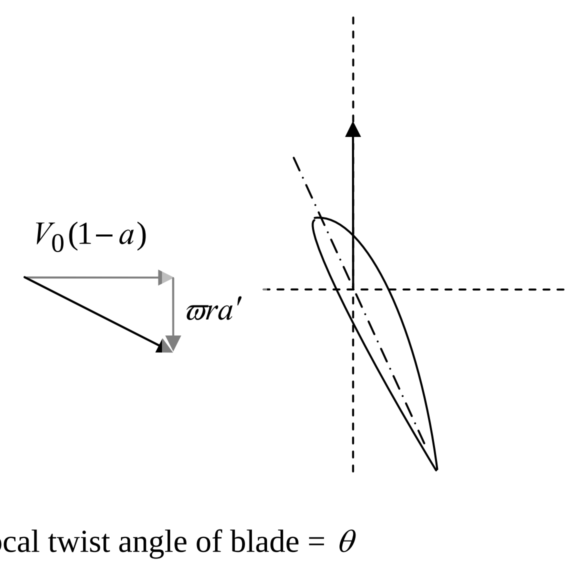
### *2.2.3 Annular control volume*

Wake rotates in the opposite sense to the blade rotation $\omega$



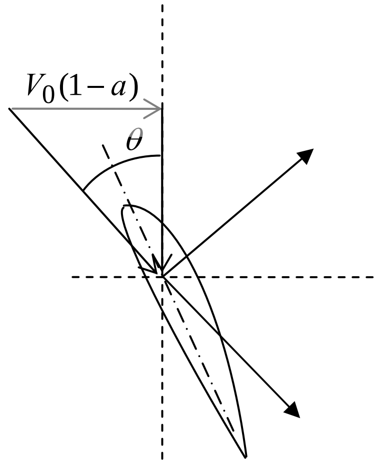

### *2.2.4 Wind and blade velocities*

Induced wind velocities seen by blade + blade motion




Local twist angle of blade = $\theta$

*2.2.5 Blade relative motion and lift and drag forces*




Local angle of attack = $\alpha$

Relative wind speed $V_{rel}$ has direction $\phi = \alpha + \theta$

where

$V_{rel} \sin\phi = V_0(1-a)$

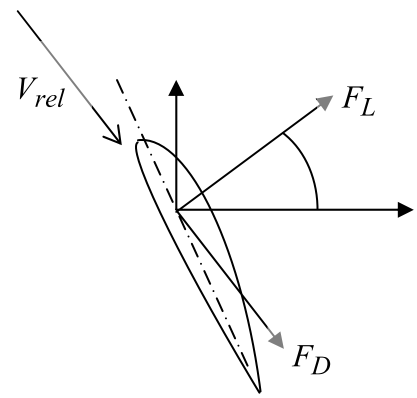
and

$V_{rel} \cos\phi = \omega r(1+a')$

$F_L$ and $F_D$ are aligned to the direction of $V_{rel}$

Obtain $C_L$ and $C_D$ for $\alpha = \phi - \theta$ from table or graph for aerofoil used

*2.2.6 Resolve forces into normal and tangential directions*




We can resolve lift and drag forces into forces normal and tangential to the rotor plane:

We can normalize these forces to obtain force coefficients:

Hence:

$C_N = C_L \cos\phi + C_D \sin\phi$

$C_T = C_L \sin\phi - C_D \cos\phi$

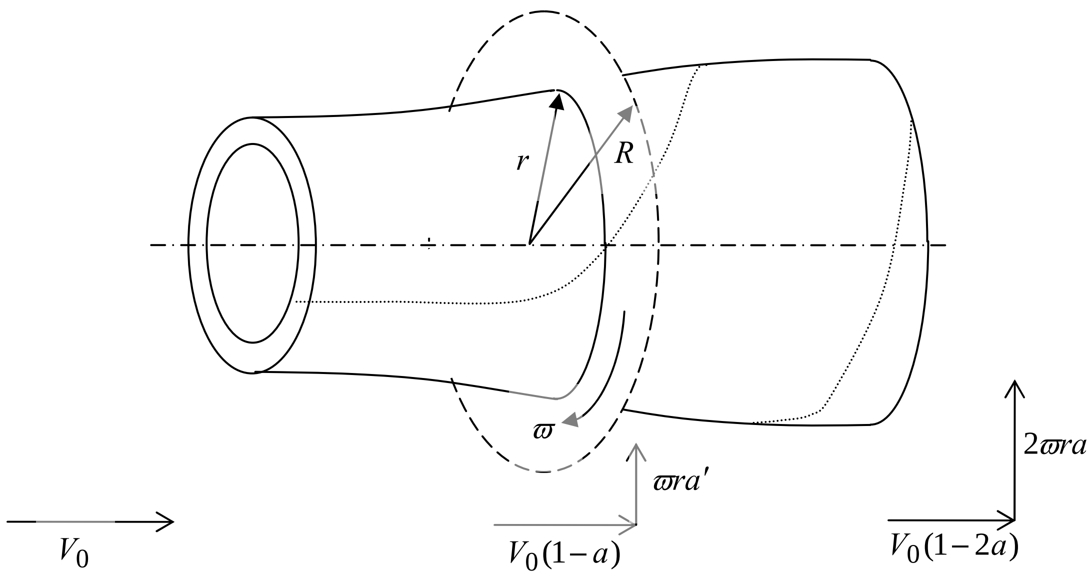
# 3 BLADE ELEMENT MOMENTUM THEORY

Split the blade up along its length into elements.

Use momentum theory to equate the momentum changes in the air flowing through the turbine with the forces acting upon the blades.

Pressure distribution along curved streamlines enclosing the wake does not give an axial force component. (For proof see one-dimensional momentum theory, e.g. Hansen)

## 3.1 Momentum changes




Thrust from the rotor plane on the annular control volume is $\delta N$

$$\delta N = \dot{m}(V_0 - u_1) = 2\pi r \rho u (V_0 - u_1)\delta r$$

Torque from rotor plane on this control volume is $\delta T$

$$\delta T = \dot{m} r u_\theta = 2\dot{m}\varpi r^2 a'$$

## 3.2 Blade forces

Now equate the momentum changes in the flow to the forces on the blades:

### *3.2.1 Normal forces*

$$\delta N =$$

$$4\pi r \rho V_0^2 a(1-a)\delta r =$$

$$= \frac{1}{2} B\rho \frac{V_0^2 (1-a)^2}{\sin^2 \phi} cC_N \delta r$$

Therefore: $$4\pi r a = \frac{1}{2} B \frac{(1-a)}{\sin^2 \phi} cC_N$$

Define the rotor solidity:

Hence:

### *3.2.2 Tangential forces*

$$\delta T =$$

$$4\pi r^3 \rho V_0 (1-a)\omega a' \delta r =$$

$$= \frac{1}{2} rB\rho \frac{V_0 (1-a) r\omega (1+a')}{\sin\phi \cos\phi} cC_T \delta r$$

Therefore: $$4\pi r a' = \frac{1}{2} B \frac{(1+a')}{\sin\phi \cos\phi} cC_T$$

Use the rotor solidity $\sigma$ :

### 3.3 Induction factors

These equations can be rearranged to give the axial and angular induction factors as a function of the flow angle.

Axial induction factor:

Angular induction factor:

However, recall that the flow angle $\phi$ is given by: $\tan\phi = \dfrac{(1-a)V_0}{(1+a')\omega r}$

Because the flow angle $\phi$ depends on the induction factors $a$ and $a'$ these equations must be solved iteratively.

### 3.4 Iterative procedure

Choose blade aerofoil section.

Define blade twist angle $\theta$ and chord length $c$ as a function of radius $r$.

Define operating wind speed $V_0$ and rotor angular velocity $\omega$.

For a particular annular control volume of radius $r$:

1. Make initial choice for $a$ and $a'$, typically $a = a' = 0$.
2. Calculate the flow angle $\phi$.
3. Calculate the local angle of attack $\alpha = \phi - \theta$.
4. Find $C_L$ and $C_D$ for $\alpha$ from table or graph for the aerofoil used.
5. Calculate $C_N$ and $C_T$.
6. Calculate $a$ and $a'$.
7. If $a$ and $a'$ have changed by more than a certain tolerance return to step 2.
8. Calculate the local forces on the blades.

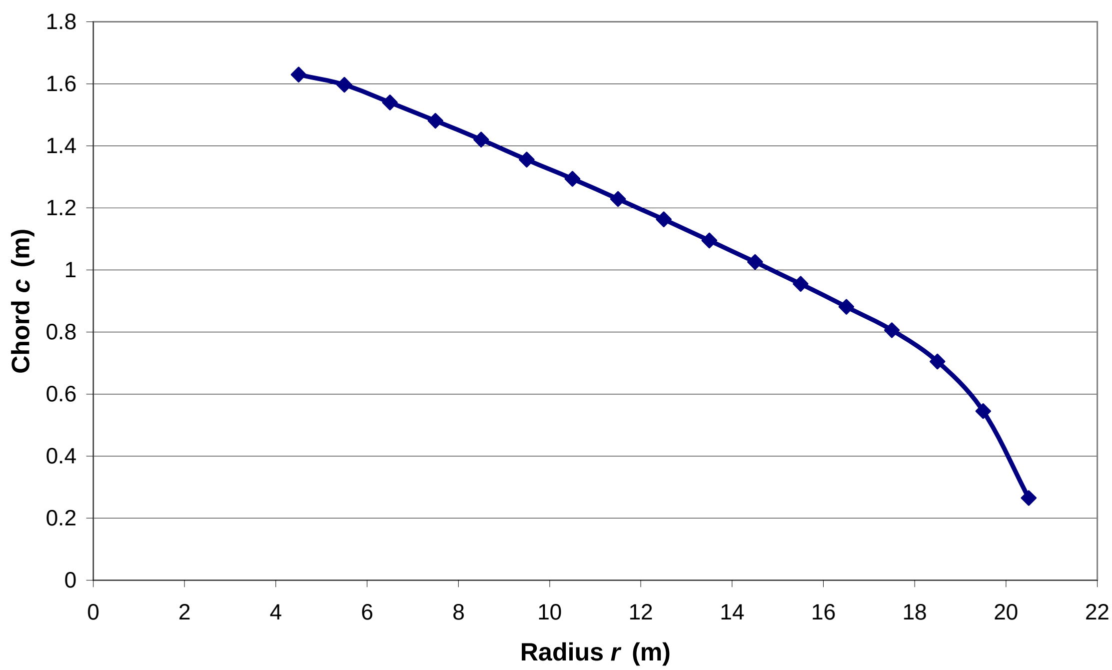
### 3.4.1 Example wind turbine

Blade element theory has been applied to an example 42 m diameter wind turbine with the parameters below. Each element has a radial thickness $\delta r$ = 1m.

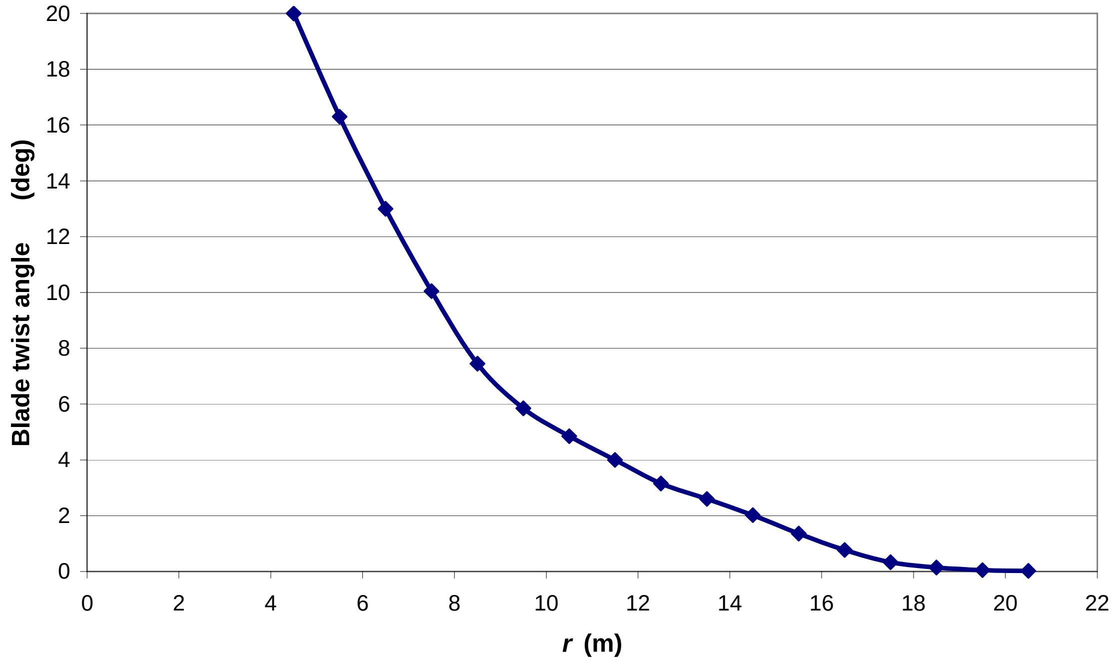
| | | |
|---|---|---|
| Incident wind speed | $V_0$ | 8 m/s |
| Angular velocity | $\varpi$ | 30 rpm |
| Blade tip radius | $R$ | 21 m |
| Tip speed ratio | $\lambda = \omega R / V_0$ | |
| Number of blades | $B$ | 3 |
| Air density | $\rho$ | 1.225 kg/m$^3$ |

Blade shape (chord $c$ and twist $\theta$) are based on the Nordtank NTK 500/41 wind turbine (see Hansen, page 62).

Chord $c$

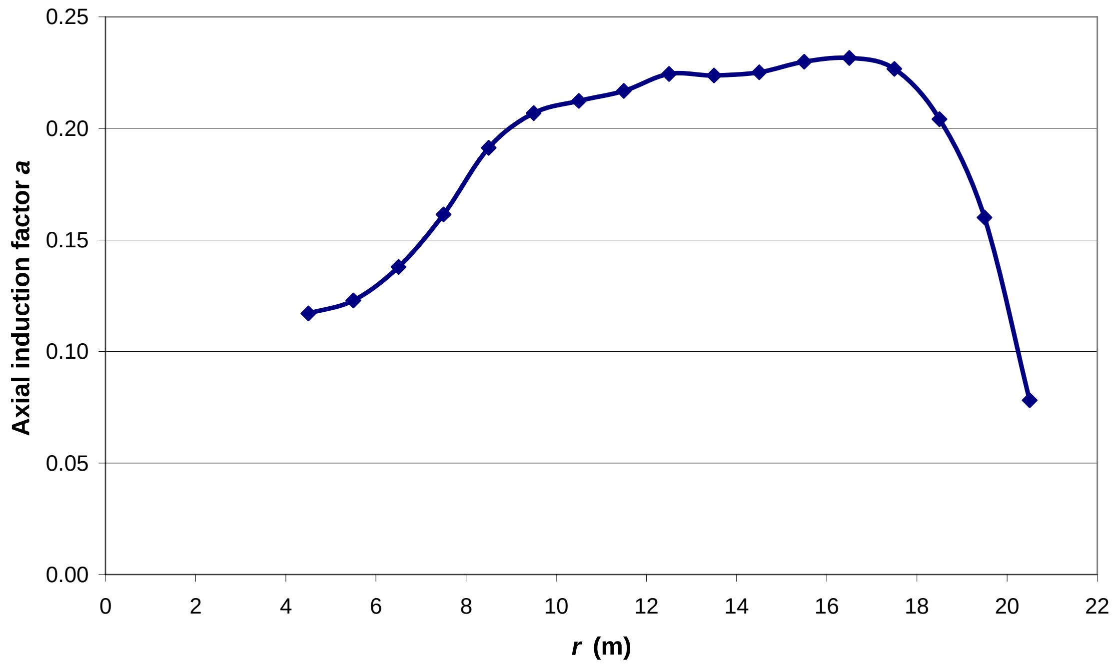


Blade twist angle $\theta$

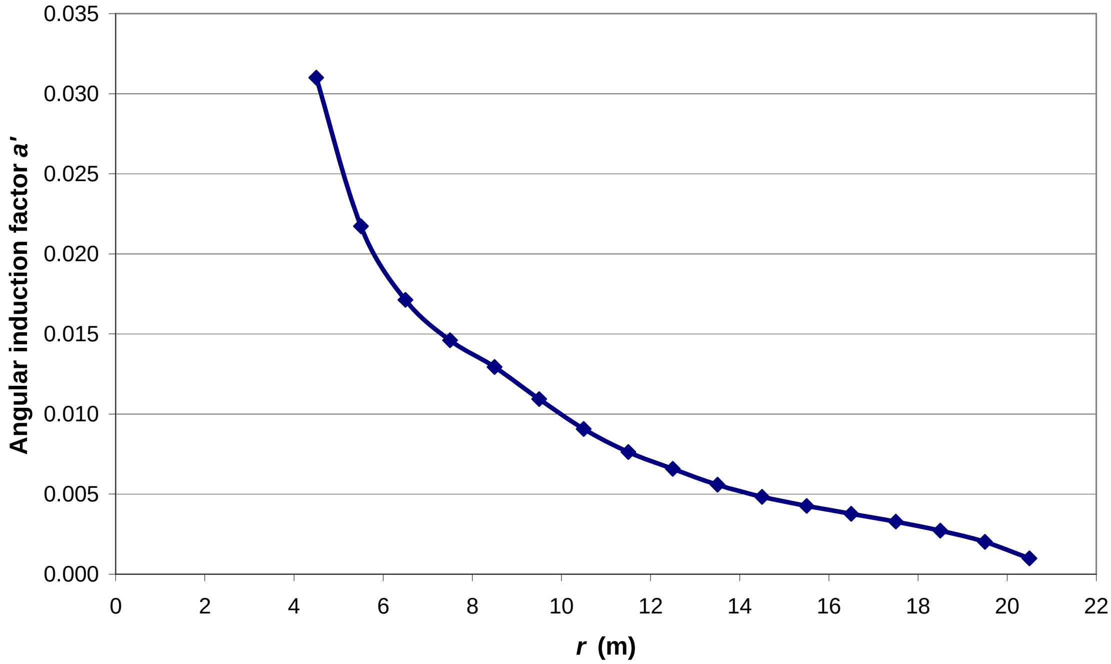



### *3.4.2 Results of BEM analysis*

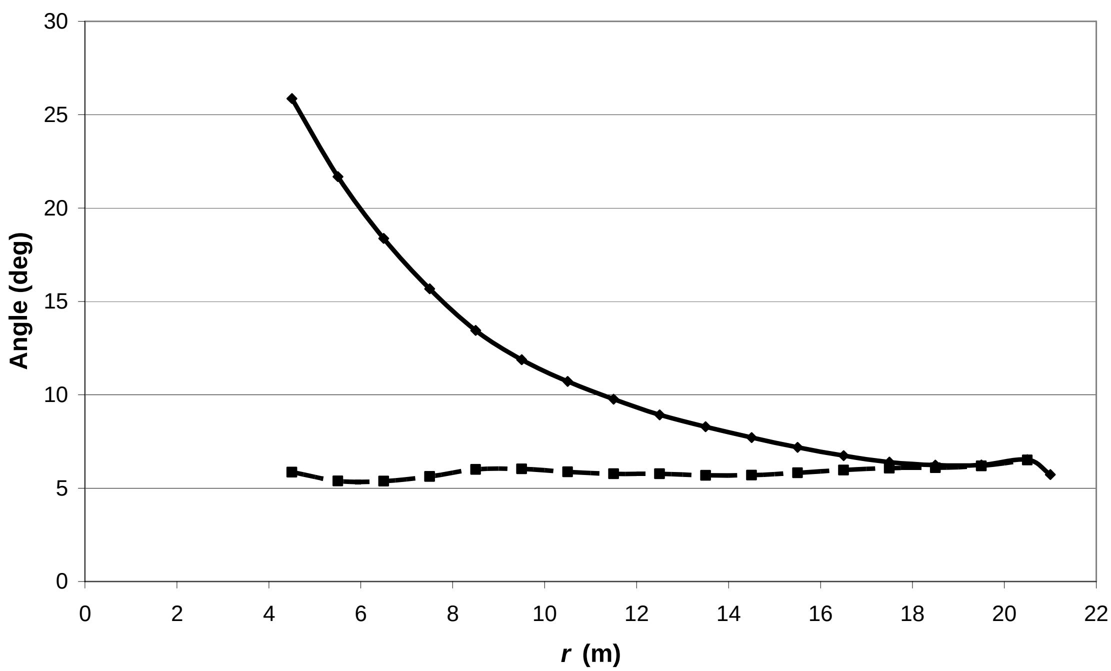
Axial induction factor $a$



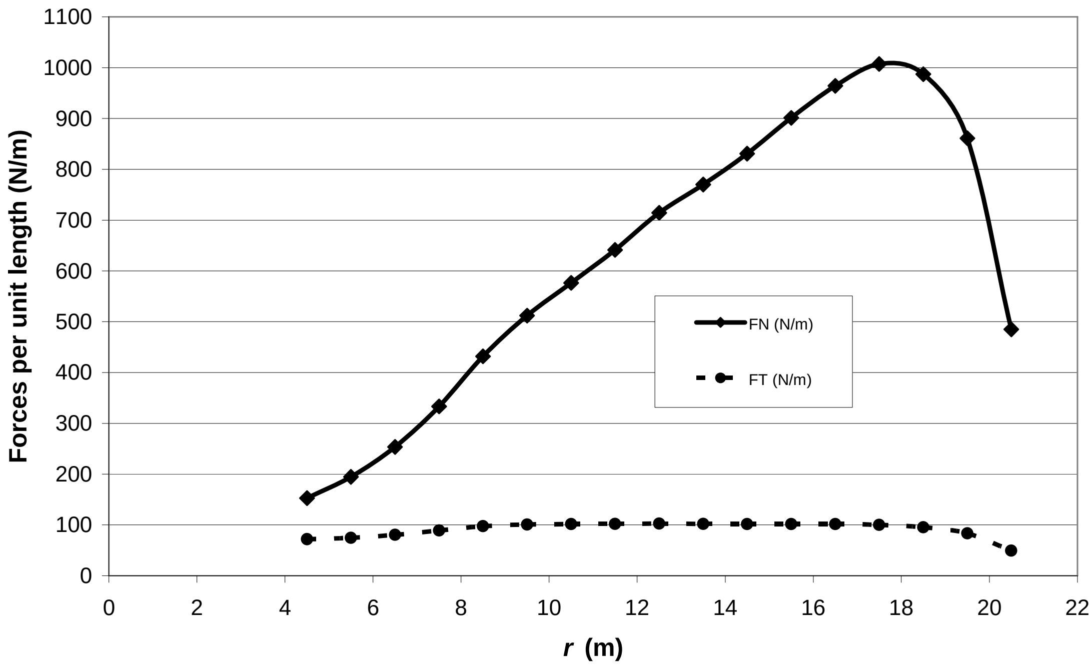
Angular induction factor $a'$




Flow angle $\phi$ and local angle of attack $\alpha = \phi - \theta$



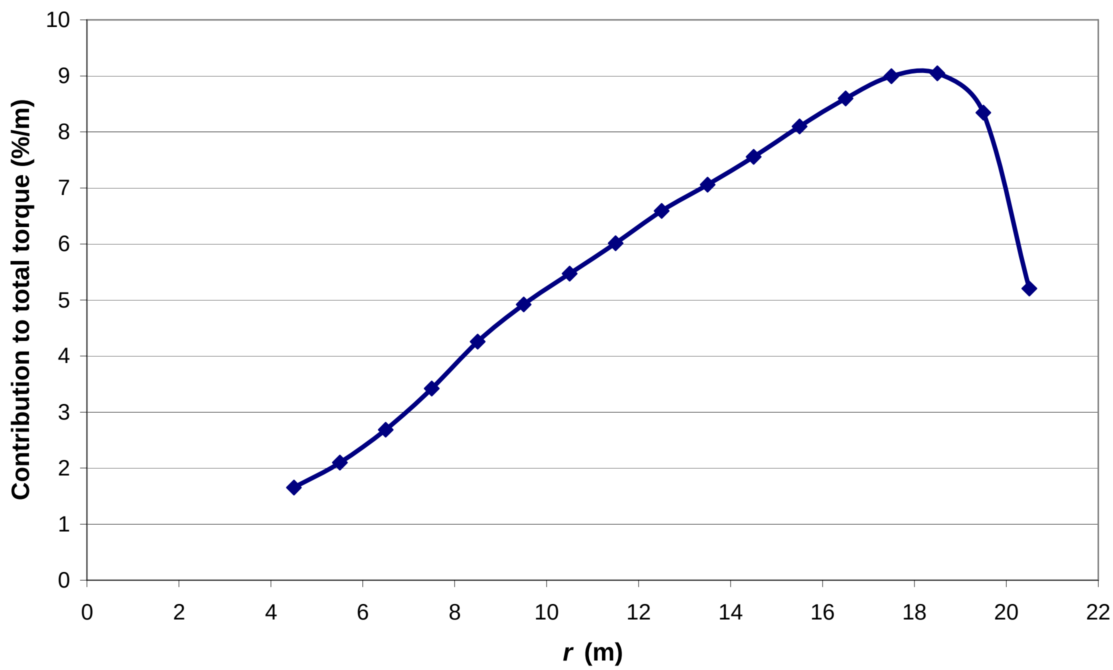
Normal $F_N$ and tangential $F_T$ forces on blade




Total power (3 blades)

Coefficient of performance

Contribution of blade elements to total torque (and therefore power)

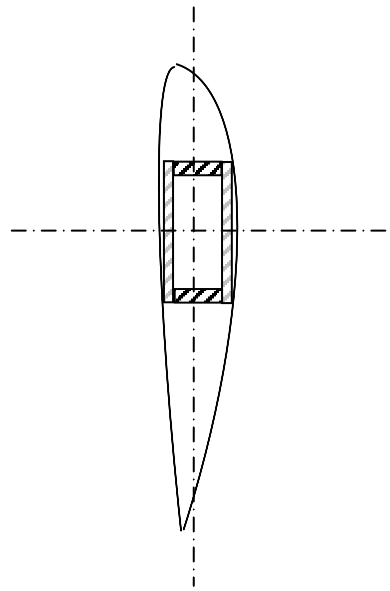


# 4 BLADE LOADING

## 4.1 Aerodynamic Loading

Once values of $a$ and $a'$ have converged the blade loads can be calculated:

$$F_N = \frac{1}{2}\rho \frac{V_0^2(1-a)^2}{\sin^2\phi} cC_N$$



$$F_T = \frac{1}{2}\rho \frac{V_0(1-a)\omega r(1+a')}{\sin\phi\cos\phi} cC_T$$

### *4.1.1 Stresses at blade root*

The normal force $F_N$ causes a "flapwise" bending moment at the root of the blade.

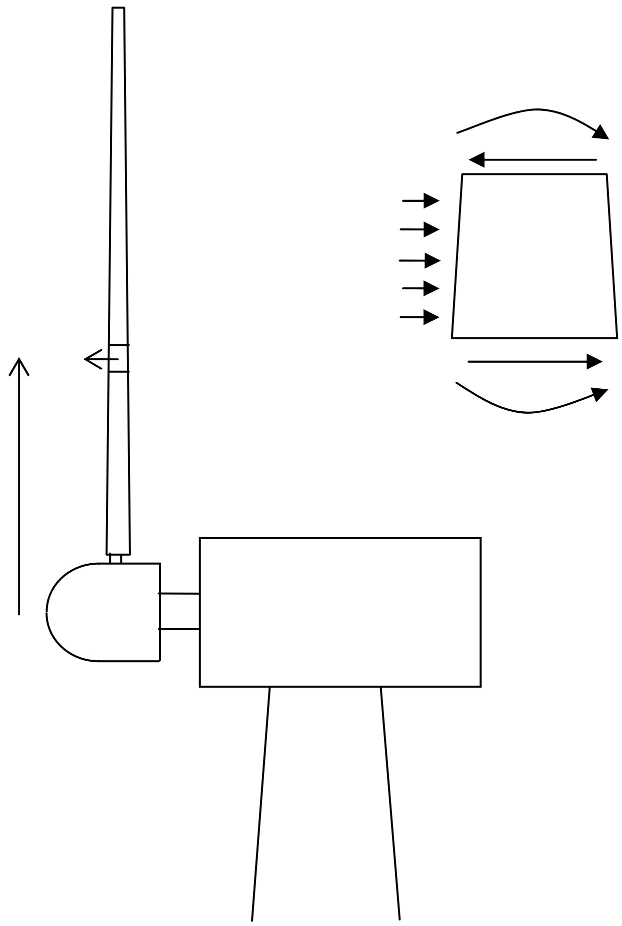
$$M_N = \int_{r_{\min}}^{R} F_N (r - r_{\min}) dr$$

The tangential force $F_T$ causes a tangential bending moment at the root of the blade.

$$M_T = \int_{r_{\min}}^{R} F_T (r - r_{\min}) dr$$

For convenience we will neglect the relatively small twist of the blade cross section and assume that these bending moments are aligned with the principal axes of the blade structural cross section. The maximum tensile stress due to aerodynamic loading is therefore given by:

*4.1.2 Deflection of blade tip*



$$F_N = \frac{dS}{dr}$$

$$S = \frac{dM}{dr}$$

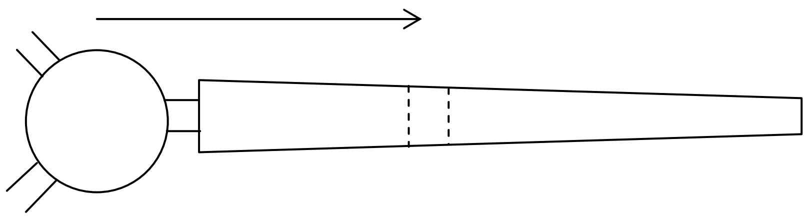
$$-\frac{d^2v}{dr^2} \approx \kappa = \frac{M}{[EI](r)}$$

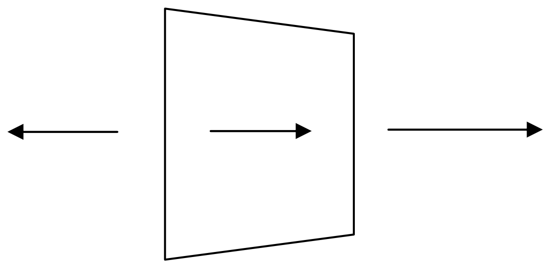
Simplified approach:

- Split blade into elements.
- Assume that for each element the loading $F_N$ and flexural rigidity $EI$ are constant.
- Find the shear force and bending moment transferred between each element.
- Use data book deflection coefficients for each element.
- Find the cumulative rotations along the blade.
- Find the cumulative deflections along the blade.

## 4.2 Centrifugal Loading

The large mass of a wind turbine blade and the relatively high angular velocities can give rise to significant centrifugal stresses in the blade.



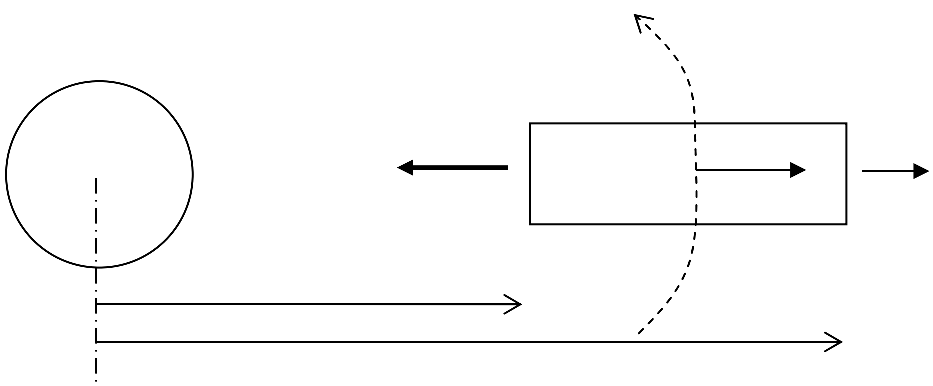


Consider equilibrium of element of blade:

$$\frac{dF_c}{dr} = -m(r)\omega^2 r \qquad \sigma_c = \frac{F_c(r)}{A(r)}$$

Simplified method:

- Split blade up into elements.
- Assume each element has a constant cross-section

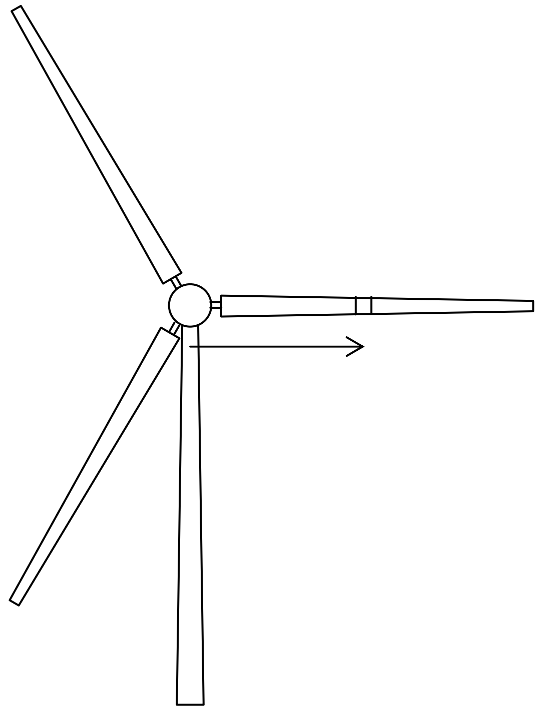


$$F_{c,n} = F_{c,n+1} + m(r_{n+1} - r_n)\frac{1}{2}(r_{n+1} + r_n)\omega^2$$

$$= F_{c,n+1} + \frac{1}{2}m\left(r_{n+1}^2 - r_n^2\right)\omega^2$$

### 4.3 Self Weight loading

The bending moment at the blade root due to self weight loading can dominate the stresses at the blade root. Because the turbine is rotating the bending moment is a cyclic load with a frequency of $f = \omega / 2\pi$ . The maximum self-weight bending moment occurs when a blade is horizontal.



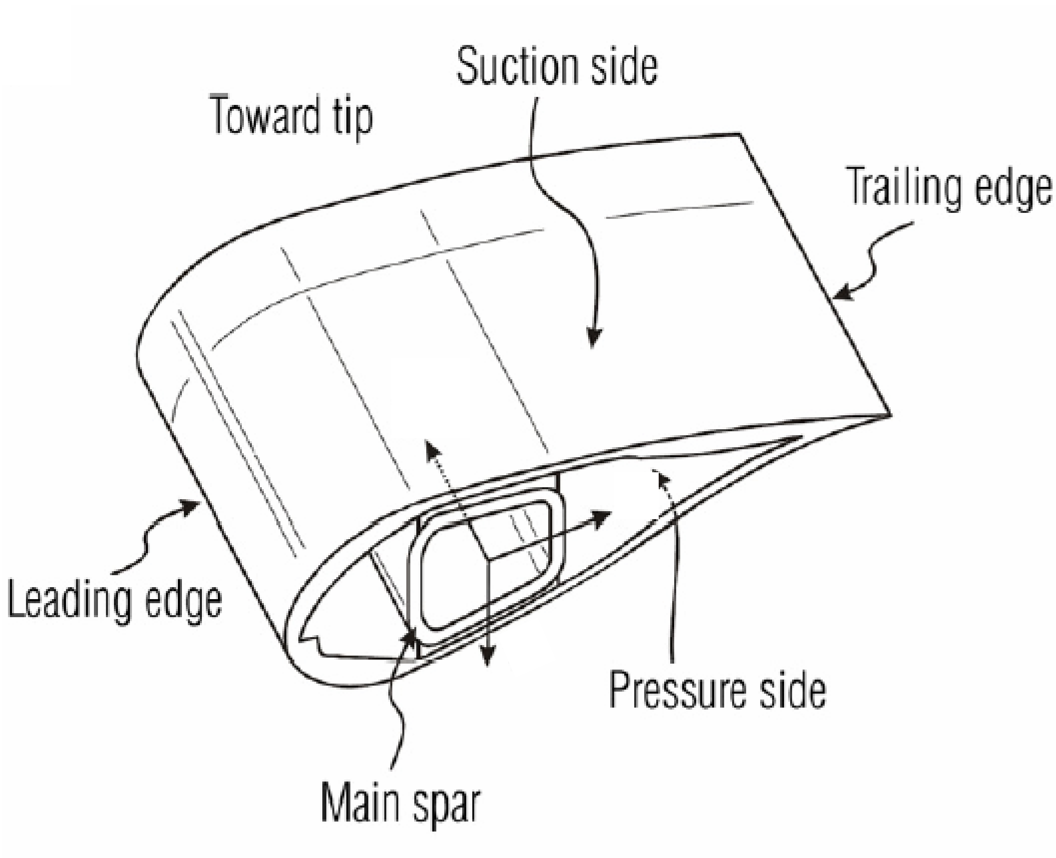
Bending moment at root of blade due to self weight

$$M_{sw} = \int_{r_{\min}}^{R} m(r) g (r - r_{\min}) dr$$

where *m(r)* is the mass of the blade per unit length. This is a tangential (edge-wise) bending moment and therefore the maximum bending stress due to self-weight is given by:

Simplified method: split blade into elements, assume each element has uniform self weight.

## 4.4 Combined Loading




$$\sigma_{\max,aero} = \frac{M_N}{I_{TT}}\frac{d_o}{2} + \frac{M_T}{I_{NN}}\frac{b}{2}$$

$$\sigma_c = \frac{F_c(r)}{A(r)}$$

$$\sigma_{\max,sw} = \frac{M_{sw}}{I_{NN}}\frac{b}{2}$$

Operational maximum stress: $\sigma_{\max} =$

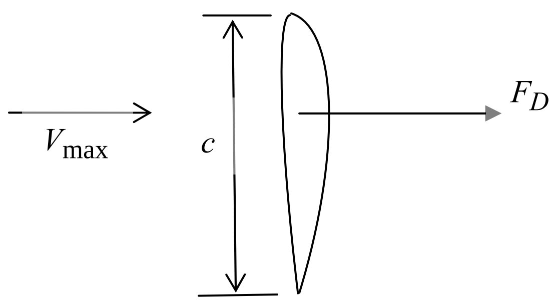
Minimum stress at same location: $\sigma_{\min} =$

## 4.5 Storm Loading

### *4.5.1 Drag force on blade*

Blades parked. Extreme wind speed

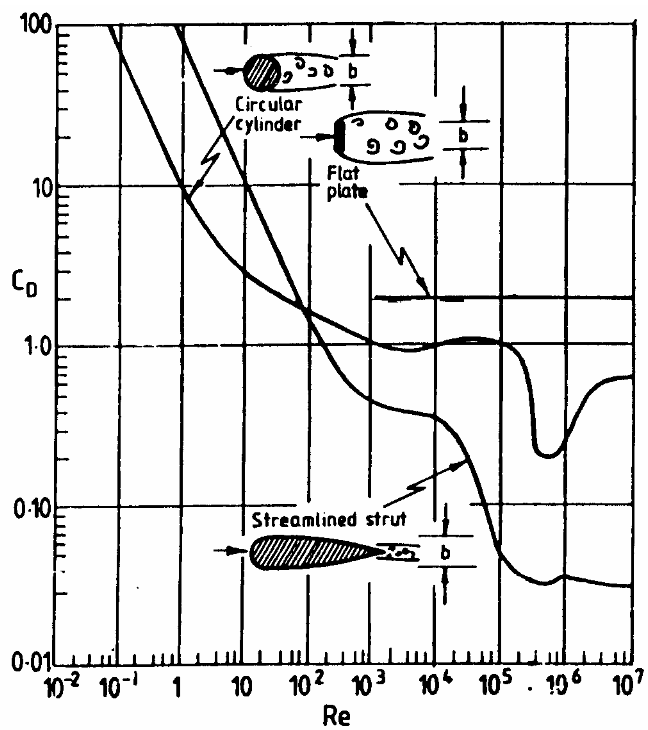
load per unit length




$V_{max}$ = 50 m/s, c = 1.3m

$$\mathrm{Re} = \frac{\rho V_{max} c}{\mu} =$$

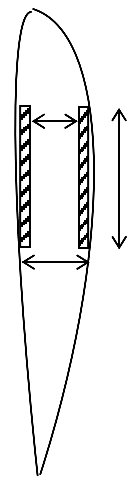
Hence $C_D$ =



### *4.5.2 Bending moment*

Find bending moment at root of blade

$$M = \int_{r_{\min}}^{R} (r - r_{\min}) F_D \, dr$$



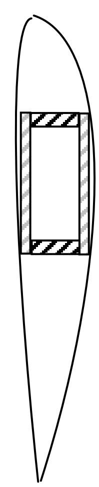
$$\frac{\sigma}{y} = \frac{M}{I} = E\kappa$$

$$\sigma = \frac{M}{I}\frac{d_o}{2}$$

$$I = \frac{b(d_o{}^3 - d_i{}^3)}{12}$$

### *4.5.3 Shear stress*

$$S = \int_{r_{\min}}^{R} F_D \, dr$$



$$q = \frac{S A_c \bar{y}}{I}$$

$$q = \tau t_w$$

Note:

High solidity rotor (multi bladed) gives excessive forces on tower during extreme wind speeds. Therefore use fewer blades.