Engineering Tripos Part IB SECOND YEAR

Part IB Paper 8: - ELECTIVE (2)

MECHANICAL ENGINEERING FOR RENEWABLE ENERGY SYSTEMS

Examples Paper 2 – Wind Turbines: Materials, Mechanics and Electrical Power

All questions are of Tripos standard, though not necessarily of Tripos length.

Guest lecture and noise.

1. (a) Discuss the main differences between "micro wind" (<2m diameter), "small wind" and "large wind" (>10m diameter) with reference to their applications and economic favourability. As a point of reference assume that a typical 5m turbine is rated at 5kW.

(b) Discuss over-speed protection and methods of shedding surplus power making sure to include furling, active pitch regulation and mechanical brakes.

(c) Discuss how noise issues can affect wind turbine installations.

Materials

2. (a) Discuss the importance of materials choice in wind turbine blade and tower design.

(b) Consider a series of self-similar blades in which the plan form and cross-sectional shape scale with length L. The blades are subject to a uniform pressure loading along the length of the beam (i.e. a storm loading condition). Use simple beam theory to show that the peak root bending stress associated with this aerodynamic load does not depend on L, while the stress associated with self-weight scales as L. The self-weight stress you should consider is due to edge-wise bending when the blade is horizontal. Confirm your results using dimensional analysis.

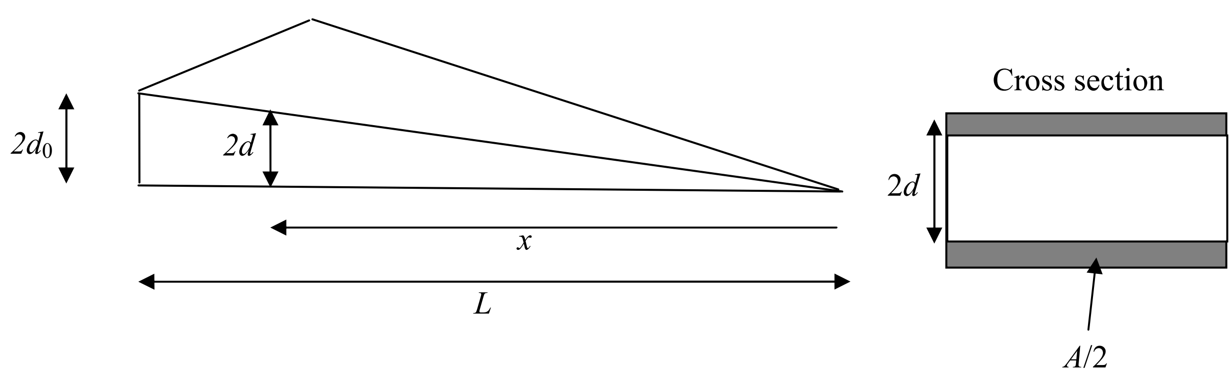
3. Consider the design of a spar of length L as illustrated below with a linear variation of spar depth d with distance x from the tip, and a linearly tapering change in spar cross sectional area A, i.e. $d/d_0 = x/L$ and $A/A_0 = x/L$, where the subscript 0 refers to conditions at the root. A total aerodynamic load W is uniformly distributed as a pressure acting on the triangular plan form. Derive the following expression for the mass required to give a tip deflection δ:

$$m_\delta = \frac{\rho W L^4}{12 E d_0^2 \delta}$$

The spar material has Young's modulus E and fatigue strength σ_f. Assume that spar skin thickness is much less than its depth in calculating the second moment of area.

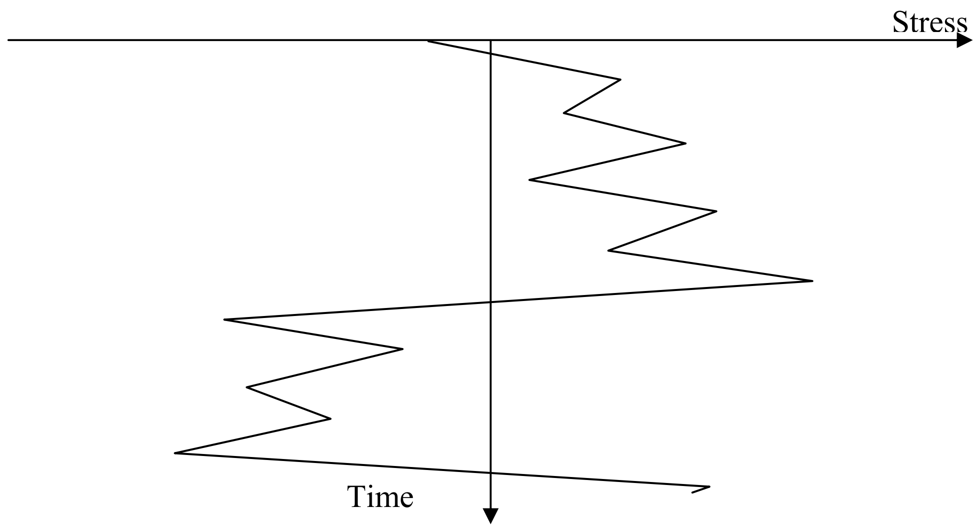
The corresponding mass to meet a strength constraint is given by $m_\sigma = \dfrac{\rho W L^2}{6 d_0 \sigma_f}$

Explain how these results can be used to identify whether stiffness or strength is the critical constraint for this type of design, and identify at what length L the crossover point lies, taking values of E = 45 GPa, σ_f = 150 MPa, δ = 5m and d_0 = 0.05 L.



4. (a) Explain carefully, with a sketch, how rainflow counting is used to identify cycles of loading in a random signal.

(b) Identify all the half-cycles present on the diagram below of stress as a function of time.



5. A blade made of GFRP has fatigue properties which can be fitted by the expression

$$N = \left(\frac{S}{S_0} \right)^{-M}$$

where N is the number of cycles to failure under a given applied cyclic stress range S, with M = 9, $S_0 = 2\sigma_{ts}$ = 400 MPa.

(a) The wind loading is estimated to give stresses in the critical area of the blade following the table distribution given below. Deduce the expected lifetime of the blade.

	Mean Stress S_m (MPa)			
		5-15	15-25	25-35
Alternating stress range S (MPa)	15-25	300	300	200
	25-35	200	300	200
	35-45	200	200	100

Number of cycles (in thousands) in a one month block

(b) An alternative model fits the stress data by a Rayleigh probability density function ϕ,

$$\phi(S) = \frac{S}{\overline{S}^2} \exp\left(-\left(\frac{S}{2\overline{S}} \right)^2 \right)$$

with 10^6 loads per month and a mode stress range $\overline{S}$ = 15 MPa. Calculate the expected lifetime with this model of the loading, taking S_0 = 400 MPa and M = 10.

NB: $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the Gamma function, equal to (z-1)! for positive integers.

Mechanics: gears and dynamics

6. Determine the required minimum numbers of teeth on the gears and the approximate overall dimensions of two alternative designs of gearbox for the wind turbine of questions 7 to 10 in Examples Paper 1. Under the operating conditions given in question 8 of EP1 the turbine is to drive a generator at 180 rpm.

Design a single stage gear box consisting of either:

(a) a pair of parallel gears, or

(b) a single stage epicyclic gear box with three planet gears; where the ring gear is held stationary, the low speed input shaft is connected to the planet carrier and the high speed output shaft is connected to the sun gear.

Gear data
Permitted bending stress = 480 MPa
Gear module m = 6 mm
Gear width w = 20 mm

7. The tower sway vibration mode of a Vestas V80 tower is to be modelled. The operating speed (1P) for the turbine varies from 9 to 19 rpm. The tower is 78 m high and has a circular cross section varying in diameter from 4 m at the base to 2.3 m at the top.

(a) The tower is idealised as a circular tube of uniform cross section made of steel (ρ = 7840 kg/m^3, E= 210 GPa) of uniform wall thickness 34 mm, diameter 3 m and height 78 m. The tower is to be modelled as a single degree of freedom system with a point mass equal to 24% of the tower mass M at the top of the tower, see figure below. Find:

(i) the mass M of the tower

(ii) the effective spring stiffness required to model the bending behaviour of the tower, using the structures data book coefficient for a clamped cantilever (NB $I = \pi D^3 t/8$).

(iii) the lowest resonant frequency of the tower.

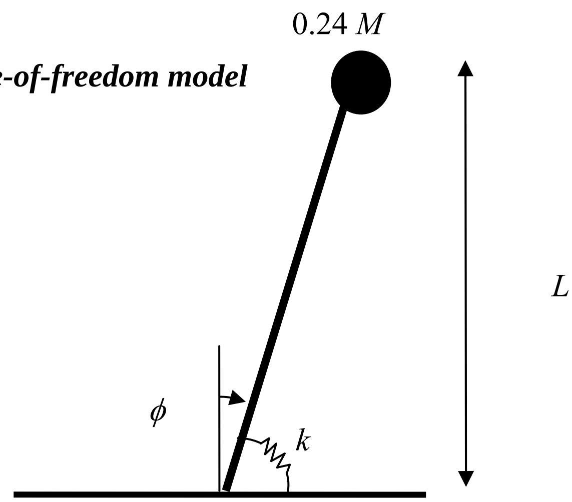
(b) The tower head mass M_h (nacelle, rotor…) equals 100 tonnes. Calculate the lowest resonant frequency of the tower system when this mass is added to the top of the tower, again using the single degree-of-freedom model of part (a), suitably modified.

(c) The tower is to be modelled by a three degree-of-freedom system with three masses and springs representing the tower, as illustrated in the Figure below. The towerhead mass M_h is included at the top of the tower. The beam stiffness properties are given by the expressions $k_1 = 2\dfrac{EI_1}{\ell}$, $k_2 = \dfrac{EI_2}{\ell}$ and $k_3 = \dfrac{EI_3}{\ell}$ where values for I at the relevant locations should be estimated using effective diameters of 4, 3.4 and 2.8 m, respectively. Masses M_1, M_2 and M_3 for each of the beam elements should be calculated from the element volumes, using diameters of 3.6, 3.1 and 2.5 m, respectively with lengths of 26 m and a constant wall thickness of 34 mm.

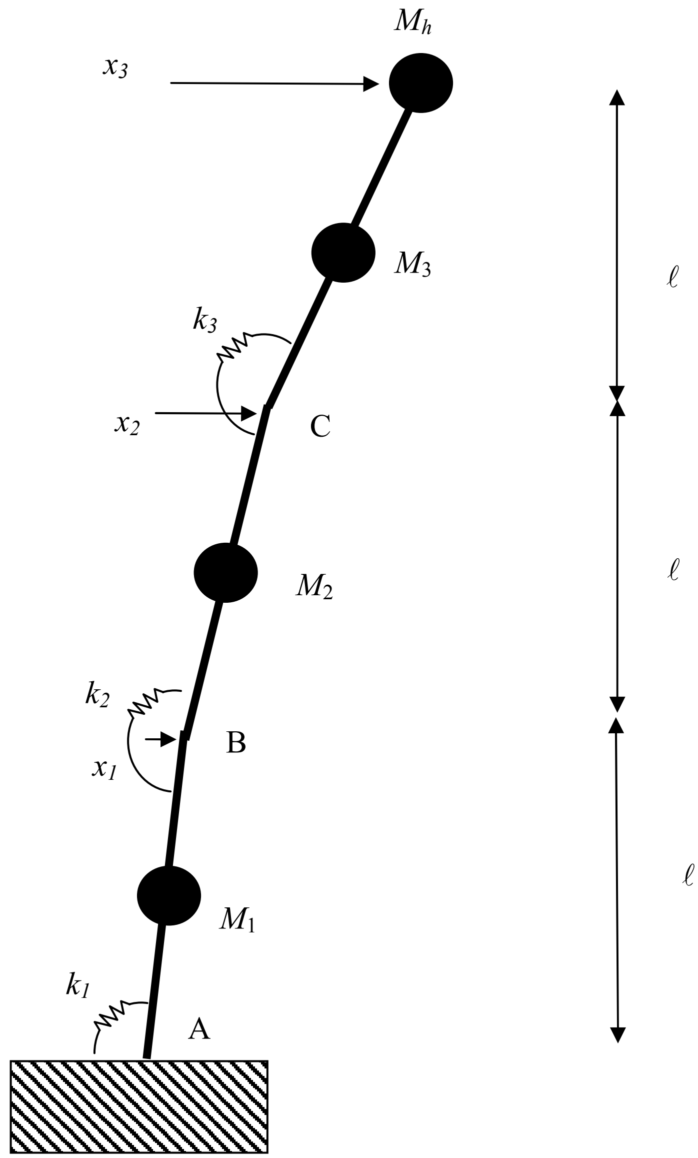
Equations of motion for such a structure, but without the tower head component, are included by the figure. Modify these equations to include the tower-head mass, and hence derive a modified stiffness and mass matrix for the structure. Deduce the lowest vibration frequency of the structure and corresponding mode shape (it is suggested that you use Matlab or Octave to solve the equations).

(d) Comment on the above calculations.

Single degree-of-freedom model



Three degree-of-freedom model



Derivation of equations without towerhead mass:

Moments about C to tip: $k_3\left(\frac{(x_3 - x_2)}{\ell} - \frac{(x_2 - x_1)}{\ell}\right) + \frac{\ell}{2} M_3 \frac{(\ddot{x}_2 + \ddot{x}_3)}{2} = 0$

Moments about B to tip: $\frac{k_2}{\ell}(x_2 - 2x_1) + \frac{\ell}{2} M_2 \frac{\ddot{x}_1 + \ddot{x}_2}{2} + \frac{3\ell}{2} M_3 \frac{\ddot{x}_3 + \ddot{x}_2}{2} = 0$

Moments about A to tip: $\frac{k_1 x_1}{\ell} + \frac{\ell}{2} M_1 \frac{\ddot{x}_1}{2} + \frac{3\ell}{2} M_2 \frac{\ddot{x}_1 + \ddot{x}_2}{2} + \frac{5\ell}{2} M_3 \frac{\ddot{x}_3 + \ddot{x}_2}{2} = 0$

Electrical Power

8. (a)Show that the power extracted from the wind by a wind turbine may be expressed as

$$P = 0.5C_p\rho Av^3$$

and define all the terms in this expression.

(b) Define and explain what is meant by the term 'tip-speed ratio' as applied to a wind turbine. Sketch a typical power coefficient vs tip-speed ratio curve. Using this curve to explain the advantages of variable-speed compared to fixed speed operation.

(c) Define the terms 'cut-in', 'rated' and 'stall' as applied to wind speeds for wind turbines and sketch a typical power vs wind speed characteristic. Give two methods of controlling the turbine power so that it remains fixed at its rated value between rated and stall wind speeds.

(d) Simplified wind data for an offshore wind turbine is as follows:

Wind speed (ms^{-1})	**No of days**
< 4	10
7	185
12	100
16	50
> 25	20

A 'new generation' turbine has a cut-in wind speed of 4 ms^{-1}, a rated wind speed of 14 ms^{-1} at which it produces rated output of 5 MW and a stall wind speed of 25 ms^{-1}.

The turbine operates at variable speed between cut-in and rated wind speeds so as to maintain its optimum tip-speed ratio of 10, at which the power coefficient is 0.45. Find:

(i) The turbine diameter;

(ii) The rotational speed of the turbine at its rated wind speed;

(iii) The annual energy produced;

(iv) The capacity factor.

9. Why do large wind turbine-generators usually incorporate a gearbox ? A 2.5 MW wind turbine has a cut-in wind speed of 3 ms^{-1} and is to be designed to produce rated power at a wind speed of 12 ms^{-1}. At this wind speed it is to operate at its optimum tip-speed ratio of 14, at which its power coefficient is 0.38. The turbine drives a 10 pole synchronous generator which is connected to the 50 Hz grid. Find the required gearbox ratio.

10. A 3 phase, star-connected induction generator has 8 poles and is connected to the 3.3 kV, 50 Hz grid. It has the following equivalent circuit parameters: R_1 = 0.8Ω, R_2' = 0.65Ω, X_1 = 1.3Ω, X_2' = 1.1Ω, R_0 and X_m large enough to be ignored. It is operating at a slip of -0.03. Find:

(i) The generator input current, synchronous speed, actual speed and torque.

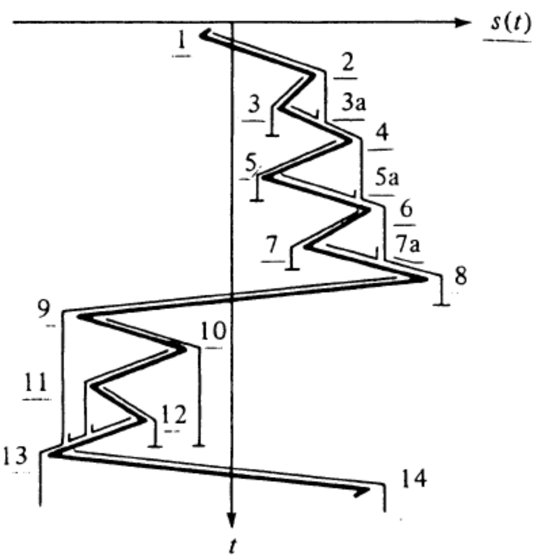
(ii) The generator output real and reactive powers, power losses, generator input mechanical power and efficiency.

(iii) Using these results estimate the slip at which the generator produces 1 MW of real output power.

Answers

3. 150 m

4. (b) 2-3, 3-3a, 4-5. 5-5a, 6-7, 7-7a, 1-8, 8-13, 9-10, 10-12b, 11-12, 12-12a, 13-14



5. (a) 58.8 years (b) 61.7 years

6. (a) 106/636 teeth, 4.4 m diameter (b) 36/72/180 teeth, 1.05 m diameter

7. (a) (i) 196 tonnes, (ii) 2.91×10^9 Nm, (iii) 0.51 Hz, (b) 0.29 Hz, (c) 0.37 Hz

8. (d) (i) 91.6 m (ii) 3.1 $rads^{-1}$ (iii) 16.3 GWhr (iv) 0.373

9. 16.7

10. (i) 90.7 A $\angle$-173^0; 78.54 $rads^{-1}$; 80.86$rads^{-1}$; -6808 Nm

(ii) -514 kW; -63.2 kVAr; 35.8 kW; 549.8 kW; 93.4%, (iii) -0.0639

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