Engineering **SECOND YEAR**

Part IB Paper 8: Elective (2)

Wind Turbines

Examples Paper 2 Solutions

Guest lecture

1. (a) Micro (<2m) are for top-up charging of boat batteries, roadside electronics, sign illumination and non-power-hungry applications. No use for domestic applications, except for very-low-power homes. A 1m-diameter micro turbine typ 200W (*i.e.* $(1/5)^2$ * 5kW , power proportional to area) but low height == low wind speeds = low capacity. Expect annual average yield of 20W. Small (2 to 10m) can feed a typical home. Around 5kW with a capacity factor of 0.2 gives an annual average yield of 1kW. Rule of thumb: spend £1000 a year on electricity = annual average power of 1kW. Sounds good for homes, but 5m rotor needs to sit on a 12m tower for decent winds, and 100m away from the nearest tall obstacles. Not for the average back yard. Large (>10m) utility turbines definitely not for the back yard. Scaling by area, 50m turbine gives 100 times the power of a 5m turbine, *i.e.* 500kW – but will give more being taller and sited in an especially-windy place. Expect 100kW, with a higher capacity factor. A sensible way to extract wind energy, but some think they're ugly and noisy.

(b) In very high winds need to protect the turbine from over-speeding. Mechanical or electrical brakes ok up to a modest level, but the power that needs to be extracted goes as V^3, so in 60mph winds things get nasty. Best to *spoil* the aerodynamics, done in various ways.

Stall the blades – ie pitch control. Tilt the blades so that a high angle of attack causes stall. Lift is poor, drag is high. Power delivery is low, torque is low. Passive pitch control the blades tilt under the action of centrifugal and aerodynamic forces alone. OK for small turbines. Large turbines are more vulnerable because of their sheer size and unwieldiness (compare operation of a small yacht with an ocean liner). Large utility turbines have actuators at the blade root to control blade pitch in response to electricity demand and wind speed.

Reduce the projected area Furling and Coning, also spoil the aerodynamics. Furling, the turbine rotates about is yaw axis, the rotor does not face square to the wind. Coning: the blades tilt back to sweep a cone. Both generally passive, controlled by aero/centrifugal forces.

Brakes – either mechanical or electrical. The electrical brake option is good because the generator itself can be used. A short circuit with low resistance can offer an enormous torque. Dangerous if applied suddenly – the inertia loads will cause blades to break. Disc brakes are a useful backup. They operate just like car brakes. Best used on the fast-moving generator shaft (if there is a gearbox) because lower torque is needed, *i.e.* smaller brakes.

(c) Noise issues can be subjective, but 100m from neighbours (who don't like noise) is a rule of thumb for small wind. Noise generated by blades creating turbulence and by moving parts (e.g. bearings, generator and gearbox). Can use isolators to reduce vibration and noise.

Materials

2. (a) Need to consider manufacture, structures, materials, operational factors. For example the blade needs to have relatively low weight to keep the dynamic loads down, while its aerodynamic design dictates a complex shape with a good surface finish. Moreover the blades have significant fatigue strength and stiffness constraints. These factors make composites attractive, and in particular GFRP with its lower cost. Finally the good corrosion resistance of GFRP is advantageous.

The tower, however, is static, so weight is not so much of an issue. Steel or concrete are obvious choices, the former easier to make into a slender tower while concrete might benefit from better corrosion properties offshore. It seems that lattice structures are not so preferred for aesthetic reasons, these would tend to dictate a metallic construction.

(b)	Growth exponent n (L^n)
Cross-sectional area $A \sim L^2$	2
Mass $\sim \rho AL$	3
Second moment of area $I \sim Ad^2$	4*
Aerodynamic load $\sim pL^2$	2 (where p is pressure)
Aerodynamic root moment $\sim pL^2 \times \beta L$	3 (where β depends on the shape of the load)
Self-weight root moment ~ weight × γL	4 (where γ depends on the weight distribution)
Aerodynamic stress $\sim dM/I \sim L \times L^3/L^4$	0
Self weight stress $\sim dM/I \sim L \times L^4/L^4$	1

*assuming t << d, (although this will scale with L^4 even if t isn't much less that d)

Dimensional analysis
Aerodynamic stress σ (Nm^{-2}) = fn(L, p, and shape e.g. d, c etc.): σ/p = constant
Self weight stress σ (Nm^{-2}) = fn(L, ρ, g, and shape e.g. d, c etc.): $\sigma/(\rho g L)$ = constant

3. Mass $m = \rho\int_0^L A dx = \rho\int_0^L A_0 \frac{x}{L} dx = \frac{\rho A_0 L}{2}$

Moment $M(x) = \frac{WL}{3}\left(\frac{x}{L}\right)^3$

Second moment of area $I(x) = Ad^2 = A_0 d_0^2 \left(\frac{x}{L}\right)^3$ assuming $t << d$

Peak stress $\sigma(x) = \frac{Md}{I} = \frac{WL}{3}\left(\frac{x}{L}\right)^3 \frac{d}{Ad^2} = \frac{WL}{3A_0 d_0}\left(\frac{x}{L}\right)$ Mass $m_\sigma = \frac{\rho A_0 L}{2} = \frac{\rho WL^2}{6 d_0 \sigma_f}$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} = \frac{WL}{3EA_0 d_0^2} \Rightarrow \frac{dy}{dx} = \frac{WL}{3EA_0 d_0^2}[x - L]$$

$$\Rightarrow \delta(x=0) = \frac{WL}{3EA_0 d_0^2}\left(\frac{L^2}{2}\right) = \frac{WL^3}{6EA_0 d_0^2}$$

Mass $m_\delta = \frac{\rho A_0 L}{2} = \frac{\rho WL^4}{12 E d_0^2 \delta}$

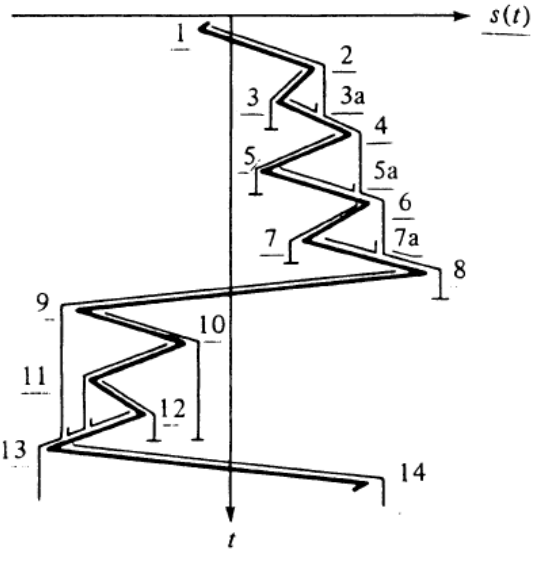
For m_δ/m_σ greater than one, stiffness is critical, while the converse is true for $m_\delta/m_\sigma < 1$. Here, with E = 45 GPa, σ_f = 150 MPa, δ = 5m and $d_0 = 0.05\ L$

$\frac{m_\delta}{m_\sigma} = \frac{1}{2}\frac{\sigma_f L^2}{0.05 \times EL\delta}$ so that the cross-over occurs when $L = \frac{0.1E\delta}{\sigma_f} = 150m$

(L> 150 m $\Rightarrow$ stiffness limited)

4. (a) See notes

(b) 2-3, 3-3a, 4-5. 5-5a, 6-7, 7-7a, 1-8, 8-13, 9-10, 10-12b, 11-12, 12-12a, 13-14



5. For each entry in the table identify the average stress range value S and average mean stress value S_m .
Obtain equivalent values of stress range $\Delta\sigma_0$ using Goodman's rule:

$$S = \Delta\sigma = \Delta\sigma_0\left(1-\frac{S_m}{\sigma_{ts}}\right)$$

with σ_{ts} = 200 MPa.

Number to failure using $N_{fi} = \left(\frac{S}{S_0}\right)^{-M} = \left(\frac{S}{400}\right)^{-9}$

Miner's rule, find lifetime used up in one month block: $\sum_i \frac{N_i}{N_{fi}} = \alpha$

Total life is 1/α, with a factor 1000 to account for numbers of cycles being quoted in thousands, and a factor of 12 to convert to years. See spreadsheet below for calculation.

300	300	200	Number of cycles ('ooo per month)
200	300	200	
200	200	100	
10	20	30	Mean stress
10	20	30	
10	20	30	
20	20	20	Alternating stress
30	30	30	
40	40	40	
21.1	22.2	23.5	Effective stress range - Goodman's rule
31.6	33.3	35.3	
42.1	44.4	47.1	
3.227E+11	1.984E+11	1.186E+11	Number of cycles to failure - Nfi
8.394E+09	5.160E+09	3.085E+09	
6.302E+08	3.874E+08	2.316E+08	
9.297E-07	1.512E-06	1.687E-06	Miner's rule - lifetime used up - N/Nfi
2.383E-05	5.814E-05	6.484E-05	
3.173E-04	5.162E-04	4.317E-04	
Sum N/Nfi	Life (months)	Life (years)	
1.416E-03	706.1	58.8	

5 (b) $\phi(S) = \frac{S}{\bar{S}^2}\exp\left(-\left(\frac{S}{2\bar{S}}\right)^2\right)$

$$1 = \int_0^\infty \frac{N_{Tot}\phi(S)}{N_f(S)}dS = \int_0^\infty N_{Tot}\frac{S}{\bar{S}^2}\exp\left(-\left(S/2\bar{S}\right)^2\right)\left(\frac{S}{S_0}\right)^M dS = \frac{N_{Tot}}{S_0^M\bar{S}^2}\int_0^\infty S^{M+1}\exp\left(-\left(S/2\bar{S}\right)^2\right)dS$$

Make the substitution $t = \left(\frac{S}{2\bar{S}}\right)^2$, $dt = \frac{SdS}{2\bar{S}^2}$

$$1 = \frac{N_{Tot}}{S_0^M\bar{S}^2}\int_0^\infty \left(2\bar{S}\right)^M t^{M/2}\exp(-t)\,2\bar{S}^2 dt = N_{Tot}\left(\frac{2\bar{S}}{S_0}\right)^M 2\,\Gamma\left(\frac{M}{2}+1\right)$$

using $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$

Here, with M =10, we can use the results that $\Gamma(z)=(z-1)!$ for z integer to give

$$N_{Tot} = \left(\frac{S_0}{2\bar{S}}\right)^M \frac{1}{2\times(M/2)!} = 740\times10^6 \text{ cycles} = 61.7 \text{ years}$$

Turbine torque $T_{in} = 52.5$ kNm

Turbine angular speed $w_{in} = 30$ rpm $= \pi$ rad/s

Power In = Power Out $= 52.5 \times 10^3 \times \pi = 165 \times 10^3$ W
$= 165$ kW

Generator angular speed $w_{out} = 180$ rpm $= 6\pi$ rad/s

Generator torque $T_{out} = T_{in} \dfrac{w_{in}}{w_{out}} = 52.5 \times \dfrac{\pi}{6\pi} = 8.75$ kNm

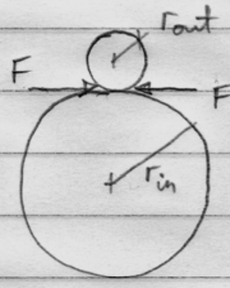
Assume cantilever model for gear tooth bending stress

$\therefore \quad \sigma_b = 2.08 \dfrac{F}{wm}$ where tooth width $w = 20$mm
tooth module $m = 6$mm

and permitted bending stress $\sigma_b = 480$ MPa

Hence contact force $F = \dfrac{\sigma_b wm}{2.08} = 27.7$ kN

a) For a single stage of two parallel gears:



$F = \dfrac{T_{in}}{r_{in}} = \dfrac{T_{out}}{r_{out}}$

$\therefore \quad r_{out} = \dfrac{T_{out}}{F} = \dfrac{8750}{27{,}700} = 0.316$ m

and the number of teeth on the pinion $N_{out} \geqslant \dfrac{2r_{out}}{m} = 105.3$

hence $N_{out} = 106$ teeth and $N_{in} = N_{out} \dfrac{w_{out}}{w_{in}} = 106 \times 6$
$= 636$ teeth

This is too many teeth (standard gears do not exceed 120!)

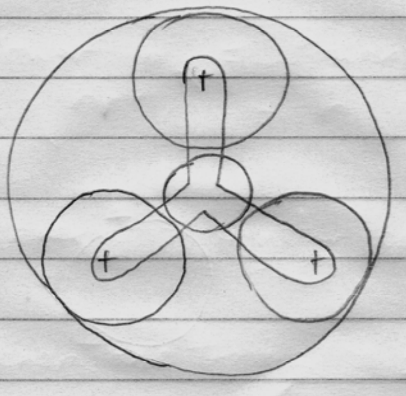
Size of gearbox $= 2r_{out} + 2r_{in} = 2 \times 0.316(1+6) = 4.4$m!

6

b) For a single stage of one epicyclic gearbox with 3 planet gears:

Ring gear stationary $\omega_r = 0$ Planet carrier input $\omega_c = \omega_{in}$

Sun gear output $\omega_s = \omega_{out}$



$$\frac{\omega_{out}}{\omega_{in}} = \frac{\omega_s}{\omega_c} = 6 = 1 + \frac{N_R}{N_s}$$

$$\therefore \frac{N_R}{N_S} = \frac{r_R}{r_S} = 5$$

Torque on output sun gear is shared between 3 contacts

hence $T_{out} = T_s = 3Fr_s$

$$\therefore r_S = \frac{T_{out}}{3F} = \frac{8750}{3 \times 27{,}700} = 0.105 \text{ m}$$

and $\therefore$ the number of teeth on the sun gear

$$N_s \geq \frac{2r_s}{m} = \frac{2 \times 0.105}{6 \times 10^{-3}} = 35.1$$

hence $N_s = 36$ teeth (sun gear)

$N_R = 5N_s = 180$ teeth (ring gear)

and $N_P = \dfrac{N_R - N_s}{2} = 72$ teeth (each planet gear)

Size of gearbox $= 2r_R = 2 \times 5r_s = 1.05$ m

7. (a) (i) $M = \rho\pi tDL = 196$ tonnes

(ii) Effective mass = 0.24 M = 47 tonnes.

Effective stiffness given by matching cantilever and mass on spring stiffnesses:

Clamped cantilever beam model: $\delta = \dfrac{WL^3}{3EI}$. From the spring model: $\delta = \phi L$ and $WL = k\phi$

Hence $k = \dfrac{3EI}{L} = k = \dfrac{3E\pi D^3 t}{8L}$ =2.91×10^9 Nm

(iii) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{0.24L^2 M}}$ = 0.51 Hz.

(b) Simply replace 0.24M by 0.24M + M_h. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{L^2(0.24M + M_h)}}$ = 0.29 Hz

(c) Simply add extra term $\beta\ell M_h\ddot{x}_3$, where β changes with each equation.

Moments about C to tip: $k_3\left(\dfrac{(x_3-x_2)}{\ell}-\dfrac{(x_2-x_1)}{\ell}\right)+\dfrac{\ell}{2}M_3\dfrac{(\ddot{x}_2+\ddot{x}_3)}{2}+M_h\ell\ddot{x}_3=0$

Moments about B to tip: $\dfrac{k_2}{\ell}(x_2-2x_1)+\dfrac{\ell}{2}M_2\dfrac{\ddot{x}_1+\ddot{x}_2}{2}+\dfrac{3\ell}{2}M_3\dfrac{\ddot{x}_3+\ddot{x}_2}{2}+2M_h\ell\ddot{x}_3=0$

Moments about A to tip: $\dfrac{k_1x_1}{\ell}+\dfrac{\ell}{2}M_1\dfrac{\ddot{x}_1}{2}+\dfrac{3\ell}{2}M_2\dfrac{\ddot{x}_1+\ddot{x}_2}{2}+\dfrac{5\ell}{2}M_3\dfrac{\ddot{x}_3+\ddot{x}_2}{2}+3M_h\ell\ddot{x}_3=0$

i.e. $\dfrac{\ell}{4}\begin{pmatrix}0 & M_3 & M_3+4M_h\\ M_2 & M_2+3M_3 & 3M_3+8M_h\\ M_1+3M_2 & 3M_2+5M_3 & 5M_3+12M_h\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\end{pmatrix}+\dfrac{1}{\ell}\begin{pmatrix}k_3 & -2k_3 & k_3\\ -2k_2 & k_2 & 0\\ k_1 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=0$

$M = \rho\pi tDL =$ 78, 67 and 54 tonnes for the three sections M_1, M_2 and M_3

Now for the element we need to use $k_1 = 2EI_1/\ell$, $k_2 = EI_2/\ell$ and $k_3 = EI_3/\ell$ with $I = \pi D^3 t/8$ to give k_1=13.8×10^9 Nm, k_2=4.24×10^9 Nm, k_3=2.37×10^9 Nm

Putting these numbers into a small Matlab code gives a natural frequency of 0.37 Hz

```
%vestas v80 tower examples paper question
E=210e9; rho=7840;LL=78; L=LL/3;t=0.034
D=[4 3.4 2.8]; EI=pi/8*E*D.^3 .*t;D=[3.6 3.1 2.5] ; mm=rho*pi*D.*t
k1=2*EI(1)/L;k2=EI(2)/L;k3=EI(3)/L;
m1=mm(1)*L;m2=mm(2)*L;m3=mm(3)*L;mh=100e3;
mmatrix=[0 m3 m3+4*mh;m2 (m2+3*m3) 3*m3+8*mh;(m1+3*m2) (3*m2+5*m3)
5*m3+12*mh]*L/4;
kmatrix=[k3 -2*k3 k3;-2*k2 k2 0 ;k1 0 0]/L;
[v d]=eig(kmatrix,mmatrix);f=sqrt(d)/(2*pi)
```

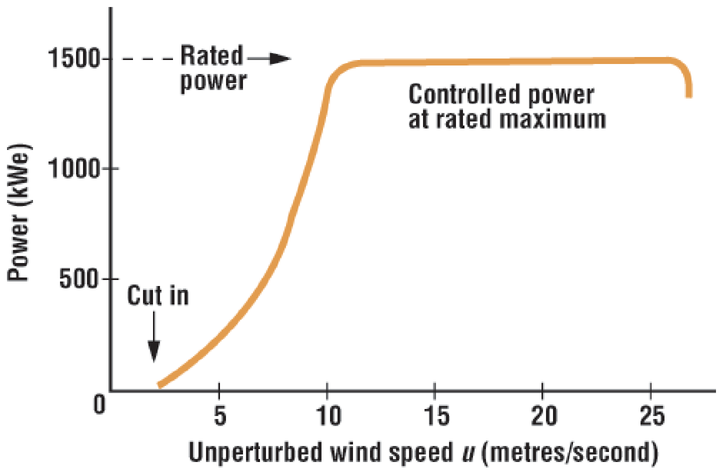
(d) Speed varying from 9 to 19 rpm corresponds to 1P varying from 0.15 to 0.32 Hz and 3P from 0.45 to 0.95 Hz. We need to avoid these frequencies, within 10%. The effective of the towerhead mass is very significant, and brings the frequency close to the maximum 1P frequency. Modelling the change in mass and section area with height in part (d) takes the lowest frequency in the safe area between 0.32 and 0.45 Hz, but more calculations would be needed to verify that.

8 (a) Consider cylinder through which air of density ρ and velocity v flows, interacting with wind turbine blades of swept area A. In time T a mass of air given by ρAvT will have interacted with the turbine blades, and given up a proportion of its kinetic energy (KE). That proportion has an upper limit given by the Betz limit of about 60%, but in practice is around 40% for modern wind turbines. The proportionality factor is the power coefficient, C_p. Using $KE = 0.5mv^2$, substituting for m and including the power coefficient gives the energy extracted in time T:

$KE = 0.5C_p \rho AvTv^2$ and since power is KE/T this gives $P = 0.5C_p \rho Av^3$

(b) Tip-speed ratio is give by $\lambda=\omega R/v$ in which ω is the turbine rotational speed, R is its blade radius and v is the wind speed. Thus, it is the ratio of the speed that the tip of the turbine blade moves at to the wind speed. A typical C_p vs λ characteristic is sketched below. It shows that there is an optimum tip-speed ratio for the turbine which maximises the power coefficient. So, to extract the maximum possible power from the wind, the tip-speed ratio should remain constant at its optimum value, λ_{opt}. For a given wind turbine, R and λ_{opt} are fixed and so using the definition for λ it is easily seen that the turbine rotational speed ω should be maintained proportional to the wind speed v. This, of course, can vary greatly, hence the need for variable speed operation.

(c) The 'cut-in' speed of a wind turbine is the wind speed above which it becomes worthwhile producing power. 'Rated' wind speed is the wind speed at which the turbine-generator produces its rated output power. 'Stall' wind speed is the wind speed above which it becomes unsafe to continue to operate the wind turbine, and so it is stalled. A typical power vs wind speed characteristic is sketched below.



In the range of wind speeds between rated and stall the turbine must be controlled so that it develops no more than its rated power. This can be achieved by furling the blades (twisting them to reduce their power coefficient), by yawing the turbine to reduce the component of wind normal to the blades or by designing the blades to twist in winds which are greater than rated (passive control).

(d) (i) Turbine needs to produce 5MW when $v = 14ms^{-1}$ and with power coefficient 0.45. Using power equation and taking $\rho = 1.23\ kgm^{-3}$:

$5\times10^6 = 0.5\times0.45\times1.23\times A\times14^3$

giving swept area $A = 6584m^2$ and diameter from $A = \pi d2/4$ as d = 91.6 m.

(ii) Tip-speed ratio is 10 and from equation for tip-speed ratio, with R = d/2:

$10 = \omega\times45.8/14$ giving $\omega = 3.1\ rads^{-1} = 29$ rpm

(iii) Discount wind speeds below cut-in and above stall since no power is produced at these. Power produced at 16 ms^{-1} wind speed is rated power of 5MW. At 7 ms^{-1} and 12 ms^{-1} wind speeds use fact that power scales with wind speed cubed, and the system produces 5 MW at a 14ms^{-1} wind speed. Thus, power at 7 ms^{-1} wind speed is $(7/14)^3\times5MW = 0.625$ MW and at 12 ms^{-1} wind speed is $(12/14)^3\times5MW = 3.15$ MW. Now complete table:

Wind speed(ms^{-1})	Power(MW)	Days	Hours	Energy (MWhr)
7	0.625	185	4440	2775
12	3.15	100	2400	7560
16	5	50	1200	6000

giving a total of 16335 MWhr = 16.3 GWhr

The capacity factor is $16335/(365\times24\times5) = 0.373$

9. Large wind turbines rotate slowly, typically 20 - 30 rpm. Unless the output power of the generator is all processed by a power electronic converter, then the generator output will be connected directly to the grid. Thus, it must produce power matched to the grid frequency. The relationship between the generator speed and the frequency of the emfs it produces is $\omega_s = \omega/p$ where p is the number of pole-pair of the generator. Building generators with 200 or more pole-pairs, which is what would be needed to produce 50 Hz electricity at a rotor speed of 30 rpm, is not feasible. Thus, a gearbox is needed to change the high torque, low speed output of the turbine to a high speed, low torque output suited to the generator.

Find the turbine swept area from the power equation, and hence its blade radius:

$2.5\times10^6 = 0.5\times0.38\times1.23\times A\times12^3$

giving swept area $A = 6190m^2$ and diameter from $A = \pi d^2/4$ as d = 88.8 m and R = 44.4 m

Tip-speed ratio is 14 at 12 ms^{-1} wind speed giving:

$14 = \omega\times44.4/12$ giving $\omega = 3.78\ rads^{-1} = 36$ rpm

Synchronous speed of a 10 pole generator connected to a 50 Hz grid is 60f/p = 3000/5 = 600 rpm

Thus, gearbox ratio is 600/36 = 16.7

10. (i) Synchronous speed $\omega_s = \omega/p = 2\pi f/p = 100\pi/4 = 78.54$ rads^{-1}.
Actual speed $\omega_r = (1-s)\omega_s = (1-(-0.03))\omega_s = 1.03\times78.5 = 80.86$ rads^{-1}
Phase voltage is $3.3/\sqrt{3} = 1.9$ kV (star-connected)
From equivalent circuit:

$I=V/((R_1+R_2'/s) + j(X_1+X_2')) = 1905/((0.8+0.65/(-0.03))+j(1.3+1.1)) = 90.7\angle-173^0$

$T=3I_2'^2R_2'/(s\omega_s) = 3\times90.7^2\times0.65/(-0.03\times78.54)=-6808$ Nm

(ii) $P_{out} = 3VI\cos\varphi = 3\times1905\times90.7\cos(-173^0) = -514$ kW
$Q_{out} = 3VI\sin\varphi = 3\times1905\times90.7\sin(-173^0) = -63.2$ kVAr

$P_{loss} = 3I^2(R_1 + R_2') = 3\times90.7^2\times(0.8+0.65) = 35.8$ kW

Input mechanical power = Output electrical power + Power losses = 514 + 35.8 = 549.8 kW
Check: Input mechanical power = $T\omega_r = 6808\times80.86 = 550.4$ kW (agreement within 0.1%)

$\eta = P_{out(elec)}/P_{in(mech)} = 514/549.8 = 93.4$ %

(iii) The generator produces 514 kW of power at a slip of -0.03. Assuming that the power factor doesn't change greatly, then the generator current will increase by the factor 1./0.514 = 1.94 and so the generator losses will increase by the factor $1.94^2 = 3.79$. So, we now have input mechanical power given by:

$P_{in(mech)} = 1000 + 3.79\times35.8 = 1136$ kW

Because of the steepness of the torque-speed characteristic the torque is approximately given by

$T = P_{in(mech)} /\omega_s = 1136/78.54 = 14.5$ kNm

On the steep part of the torque-speed curve, torque is proportional to slip and so new slip will be 14.5/6.81 times old slip of -0.03 = -0.0639.

Speed = $(1-s)\omega_s = (1-(-0.0639))\times78.54 = 83.56$ rads^{-1}

MPF Sutcliffe/DD Symons/TJ Flack May 2014