Dropping of arm supporting gyroscope

The increase in θ during precession was analysed. The apparatus used to do this is given in figure 1.

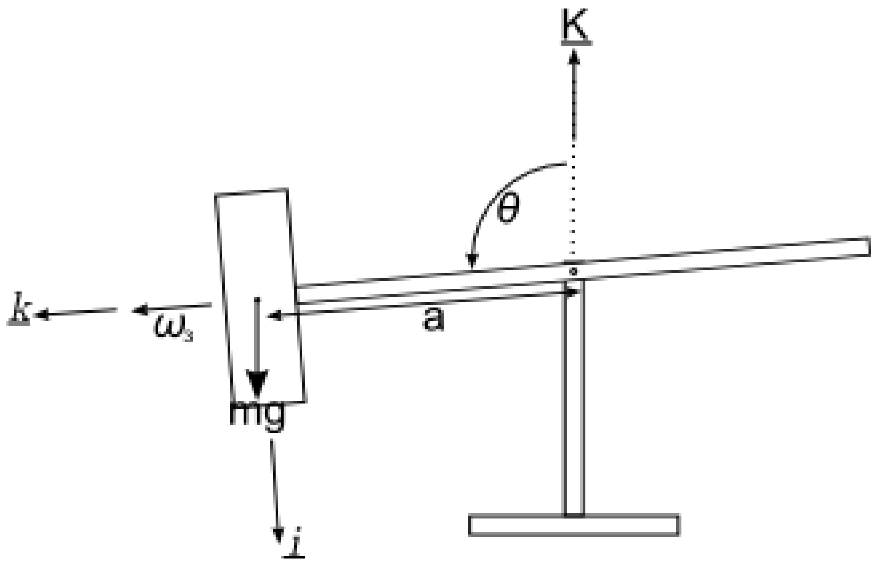

Figure 1: Apparatus used in experiment

A value for the initial spin speed, ω_3 of the rotor was obtained using a strobe gun, and estimates of $\dot{\phi}$, θ and $\dot{\theta}$ were obtained by analysis of three videos of the same motion. Graphs of the change in θ over time are displayed in figure 2.

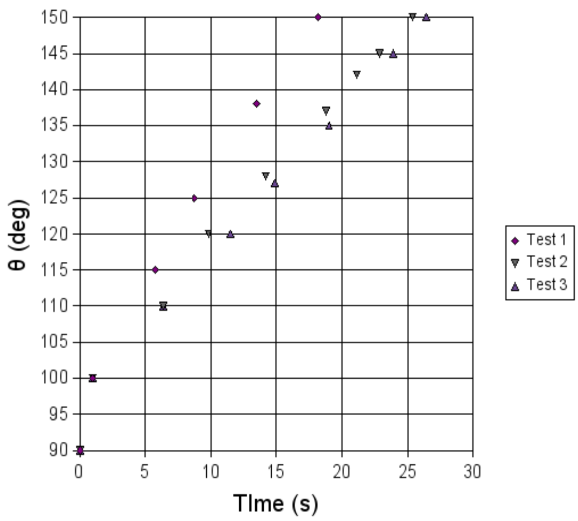

Figure 2: Graph showing the evolution of θ over time in three different cases

Variable	Estimate
$\dot{\theta}$	0.175 rad/s
$\dot{\phi}$	11.9 rad/s
ω_3	1100 rad/s
$\dot{\omega}_3$	5.8 rad/s^2

Figure 3: Estimates of variables from experiment

Since the analysis is to be based upon the initial motion, where $\theta \approx \frac{\pi}{2}$ an average of the initial gradients of these graphs was used to obtain an estimate for $\dot{\theta}$

Estimates of $\dot{\theta}$, $\dot{\phi}$, ω_3 and $\dot{\omega}_3$ are given in figure 3, these estimates relate to the initial motion, where $\theta \approx \frac{\pi}{2}$

From these estimates it was decided fair to assume $\omega_3 \gg \dot{\phi}$ and $\dot{\phi} \gg \dot{\theta}$

The change in θ is caused by the fact that Q_1 and Q_3 are non zero, therefore, the effect of these on $\dot{\theta}$ was investigated. This was done by first considering the effect of Q_F, friction in the bearings of the upright then by investigating the effect of Q_3, friction in the bearings of the rotor.

Q_3 was initially assumed to be zero and the effect of Q_F on $\dot{\theta}$ was established. This analysis applies only to the initial motion as θ is assumed to be approximated by $\frac{\pi}{2}$.

As fast spin is assumed, it is assumed that

$$\mathbf{h} = C\omega_3\mathbf{k} \tag{1}$$

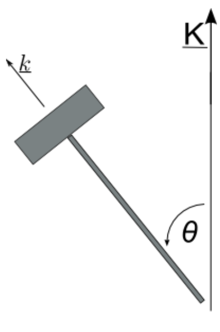

Figure 4: Angle between rotor and vertical

From figure 4 equation 2 is obtained, which is then differentiated to give equation 3.

$$\mathbf{h} \cdot \mathbf{K} = C\omega_3 \cos\theta \tag{2}$$

$$\frac{d}{dx}(\mathbf{h}\cdot\mathbf{K}) = -C\omega_3\dot{\theta}\sin\theta = -Q_F \tag{3}$$

This gives a value for $\dot{\theta}$ due to the friction about the vertical shaft Q_F.

$$\dot{\theta}_{QF} = \frac{Q_F}{C\omega_3\sin\theta} \tag{4}$$

In order to analyse the effect of the couple Q_3 on the motion, Q_F was assumed to be zero.

It is assumed that moment of momentum is conserved about the vertical axis (this was checked and found to be a fair assumption)

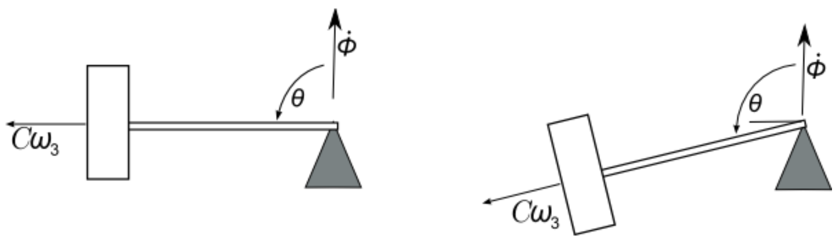

Figure 5: Position at $t = 0$ and after time dt has elapsed

$$\mathbf{h} = C\omega_3\mathbf{k} + A\dot{\phi}\sin\theta\mathbf{K} \tag{5}$$
$$\mathbf{h}\cdot\mathbf{K} \approx C\omega_3(\mathbf{k}\cdot\mathbf{K}) + A\dot{\phi} = C\omega_3\cos\theta + A\dot{\phi} \tag{6}$$

Since moment of momentum is assumed to be conserved this can be differentiated giving

$$\frac{d}{dt}(\mathbf{h}\cdot\mathbf{K}) = -C\omega_3\sin\theta\dot{\theta} + A\ddot{\phi} = 0 \tag{7}$$

An expression for $\ddot{\phi}$ is sought to substitute into equation 7, this is found using the Gyroscope equations. The second of the Gyroscope equations is.

$$A\ddot{\theta} + C\omega_3\dot{\phi}\sin\theta = mga\sin\theta \tag{8}$$

The expression $A\ddot{\theta}$ is small, and so can be assumed negligible. This gives the following expression for $\dot{\phi}$.

$$\dot{\phi} = \frac{mga}{C\omega_3} \tag{9}$$

From this

$$\frac{d}{dt}(\dot{\phi}) = \frac{mga}{C}(\frac{-1}{\omega_3^2})\dot{\omega}_3 \tag{10}$$

$$= \frac{mga}{C^2\omega_3^2}Q_3 \tag{11}$$

Substituting this into equation 7, and using the assumption that θ is approximately $\frac{\pi}{2}$ gives

$$\dot{\theta} = A\frac{mga}{(C\omega_3)^2}Q_3 \tag{12}$$

Therefore $\dot{\theta}$ due to friction in the rotor bearings is given by

$$\dot{\theta}_{Q3} = A\frac{\dot{\phi}}{(C\omega_3)^2}Q_3 \tag{13}$$

An overall expression for $\dot{\theta}$ around $\theta = \frac{\pi}{2}$ is given by

$$\dot{\theta} = \dot{\theta}_{Q3} + \dot{\theta}_{QF} = \underbrace{A\frac{\dot{\phi}}{(C\omega_3)^2}Q_3}_{Change\ in\ \theta\ due\ to\ Q_3} + \underbrace{\frac{Q_F}{C\omega_3}}_{Change\ in\ \theta\ due\ to\ Q_F} \tag{14}$$

This gives

$$\frac{\dot{\theta}_{QF}}{\dot{\theta}_{Q3}} = \frac{Q_F}{Q_3}\frac{C\omega_0}{A\dot{\phi}_0} \tag{15}$$

Due to the fact $\omega_3 \gg \dot{\phi}$ it can be seen that the Q_F effect is much bigger that the Q_3 effect unless $Q_3 \gg Q_F$.