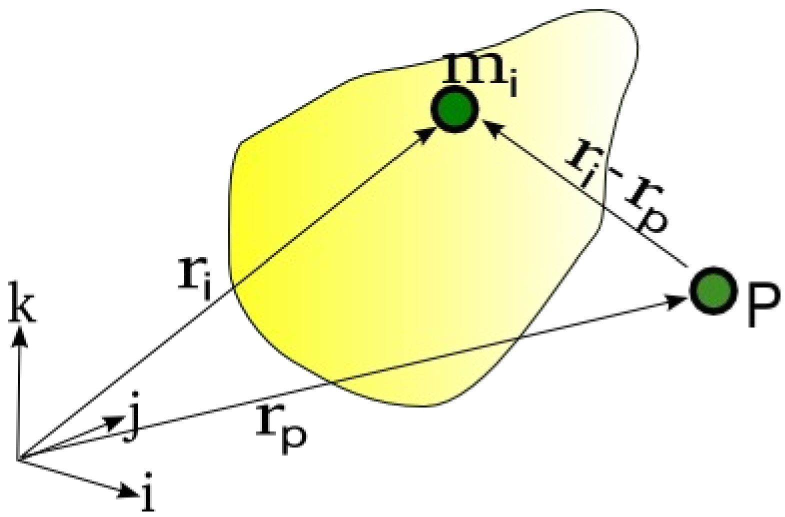
How is $h_p = [I_p]\omega$ obtained?

Consider a body (collection of particles) in 3-D



The moment of momentum, h_p for a collection of particles is given by:

$$h_p = \sum_i (r_i - r_p) \times M_i \dot{r}_i \quad \textit{(P)}$$

If the point P is stationairy and at the origin then $r_p = 0$, $\dot{r}_p = 0$

The angular velocity of the body can be expressed as $\omega = \omega_1 i + \omega_2 j + \omega_3 k$

The velocity of particle, i is given by $\dot{r}_i = \omega \times r_i$

Substituting this into equation (P) along with the fact that $r_p = 0$ gives

$$h_p = \sum_i m_i r_i \times (\omega \times r_i)$$

Using the vector triple product the above can then be written as

$$h_p = \sum_i m_i [(r_i \cdot r_i) - (r_i \cdot \omega) r_i]$$

If r_i is expressed as $r_i = x_i i + y_i j + z_i k$ and substituted into the above equation this gives

$$h_p = \sum m_i [(x_i^2 + y_i^2 + z_i^2) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} - (x_i \omega_1 + y_i \omega_2 + z_i \omega_3) \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}]$$

Putting this into matrix form gives

$$h_p = \begin{bmatrix} \sum m_i(y_i^2+z_i^2) & -\sum m_i x_i y_i & -\sum m_i x_i z_i \\ -\sum m_i y_i x_i & \sum m_i(x_i^2+z_i^2) & -\sum m_i y_i z_i \\ -\sum m_i z_i x_i & -\sum m_i z_i y_i & \sum m_i(x_i^2+y_i^2) \end{bmatrix}$$

Using the fact that

$$\begin{aligned} I_{xy} &= \int xy\, dm \\ I_{xz} &= \int xz\, dm \\ I_{yz} &= \int yz\, dm \end{aligned} \quad \text{and} \quad \begin{aligned} I_{xx} &= \int (y^2+z^2)\, dm = mk_x^2 \\ I_{yy} &= \int (x^2+z^2)\, dm = mk_y^2 \\ I_{zz} &= \int (x^2+y^2)\, dm = mk_z^2 \end{aligned}$$

h_P Can also be expressed as $h_p = [I_p]\omega$