Gyroscope on arm with second pivot point

The apparatus is represented in figure 1.

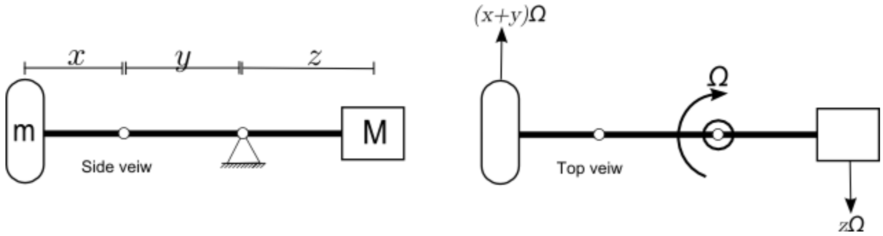

Figure 1: Apparatus viewed from the side and from above

To find an expression for the moment of momentum, the moment of inertia about the stand is required. The arm can be approximated as being light and so the moment of inertia can be expressed from figure 1 as.

$$I = m(x+y)^2 + Mz^2 \tag{1}$$

From this the initial angular momentum, L can be expressed as

$$L = m(x+y)^2\Omega + Mz^2\Omega \tag{2}$$

Figure 2 is a general representation of the position during the subsequent motion.

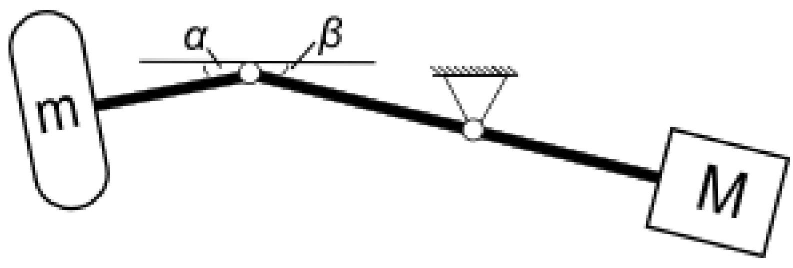

Figure 2: General position assumed during motion

From figure 2 using the principles of conservation of moment of momentum the following is obtained.

$$C\omega \sin\alpha = (m(x+y)^2 + Mz^2)\Omega \tag{3}$$

An energy conservation argument gives.

$$P.E + K.E = 0 \tag{4}$$
$$K.E = \frac{1}{2}I\dot{\theta} \tag{5}$$
$$\frac{1}{2}(m(x+y)^2 + Mz^2)\Omega^2 - Mgz\sin\beta - mg(x\sin\alpha - y\sin\beta) = 0 \tag{6}$$
$$\frac{1}{2}(m(x+y)^2 + Mz^2)\Omega^2 + \sin\beta(mgy - Mgz) - mga\sin\alpha \tag{7}$$

The assembly is balanced when in the position given by figure 3

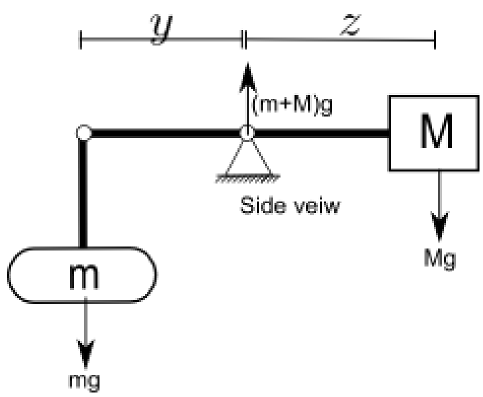

Figure 3: Position assembly initially balanced in

From moment equilibrium of figure 3 the following is obtained

$$ymg = zMg \tag{8}$$

Substituting this expression into equation 7, it can be seen that the terms with β in cancel and the following expression is obtained.

$$\frac{1}{2}(m(x+y)^2 + Mz^2)\Omega^2 - mgx\sin\alpha \tag{9}$$

By substituting in from equation 3 this can be re-written as

$$\frac{1}{2}C\omega\sin\alpha = mgx\sin\alpha(m(x+y)^2 + Mz^2) \tag{10}$$

It can be seen that β cancels in the analysis, and so a conservation of energy and angular momentum argument is satisfied regardless of the value of β. It is the value of β that is the difference between Laithwaite's demonstrations and the repeated demonstrations.