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On the Behavior of Corrugated Plates in Bending

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Abstract

A numerical analysis on the bending behaviour of thin shell, corrugated plates is presented in this study. A finite element analysis is performed to investigate the effect of initial geometry upon the salient features of the moment-rotation response.

Keywords

Shell, morphing structures, corrugated plates.

1 Introduction

Corrugated plates have been recently employed in so-called "morphing" structures, which offer structural integrity and high compliance, in order to impart extra functionality to structures [1]. If the corrugated sheet is pre-stressed, it can become bistable, being flat or tightly coiled, and, in one prototype application, it forms the backing support plate for ultra-thin electronic displays, which can be "rolled up" for portability [2]. Even though many of the practical problems in devising such sheets have been understood generically [3], there are many detailed features of behaviour to explore. Thus, a numerical analysis is required for simulating the morphing capabilities of the corrugated sheet for distilling salient features of performance, which can be compared with the theoretical model in [2]. In this study, a nonlinear finite element analysis is chosen, as the sheet undergoes a large-displacement, complex snap-through buckling, where a specific aim is to investigate the effects of the corrugation geometry upon the bending behaviour of a range of sheets.

2 Moment-Rotation Relationship

Figure 1 indicates the geometry of a corrugated plate with material thickness t, overall length L

and each corrugation subtending angle ϕ at radius R. Here, plates with six half-waves are considered: there are three corrugations facing downwards; two full, and two half corrugations at the edges, facing upwards. The geometrical and mechanical properties are chosen to match a physical specimen, and are listed in Table 1.

Table 1. Corrugated plate: geometrical and mechanical properties

Corrugation radius, R	20 mm
Thickness, t	0.2 mm
Young's modulus, E	131000 N/mm^2
Poisson's ratio, ν	0.3

The commercially available software package ABAQUS [4] is used to model the sheet as a mesh of S4R5 shell elements. The rigidly encased ends of plate are loaded externally by a pair of equal and opposite bending moments, *M,* applied along the *y*-axis, which attempt to curve the sheet in the *x*-direction, with a relative rotation between the ends of θ.

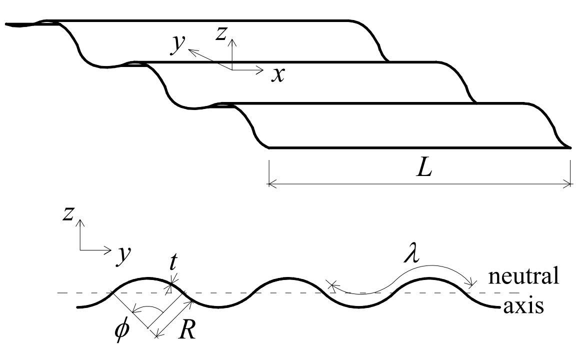

Figure 1. Geometry of a corrugated plate

The resulting large-displacement bending behaviour is highly nonlinear and is explained with reference to the schematic diagram shown in Fig. 2. For small rotations, M initially varies in a linear manner with θ, both for positive and negative moments. For positive moments, which induce tension along the free edges, the cross-section of the plate begins to flatten in the middle as the moment increases. The moment reaches a peak value, $M_+^{\max}$, before the deformation localises in the middle of sheet while the moment decreases quickly. This feature produces a rapid "snap-through" of the sheet in practice, but in the simulation, the localisation first occurs with flattening of the central corrugation, which then spreads transversally and symmetrically to adjacent corrugations before enveloping the entire width of section. In order for the rotation to increase, the moment must increase gently before reaching an approximately constant value, generally called the "propagating moment", denoted by M_+^*; at this stage, a full-width elastic "fold" region of approximately constant radius has formed whose properties do not depend on the relative end rotation. For negative moments, which induce compression along the free edges, the linear behaviour ends slightly sooner as the free edges undergo local buckling. This leads to a smaller value of peak moment, $M_-^{\max}$, compared to the positive $M_+^{\max}$.

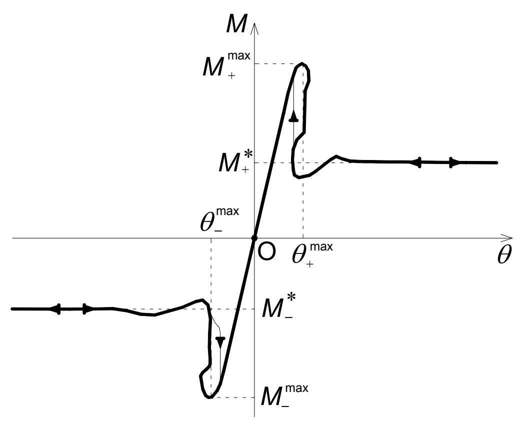

Figure 2. Schematic $M(\theta)$ diagram.

The propagating moments are the same for both directions of bending and, in Fig. 2, both unloading paths are also shown.

3 Parametrical Study

We now consider how varying the geometry of sheet can affect the generic bending response. From earlier work on the bending of so-called "tape-springs" [5], similar to carpenter's tapes, it is known that deeper, shorter sections have higher peak moments but the same propagating moments as shallower, longer sheets, provided the width of cross-section is not changed. Here, we investigate these effects directly using sheets with a range of subtended angles of cross-section and different lengths overall for the same width.

Figure 3 details the bending responses for the indicated subtended angles. The non-dimensional variable $\hat{M}$ is introduced where $\hat{M} = mL/D$, m is the bending moment per unit length and D is the flexural rigidity of cross-section.

Unexpectedly, the peak moments differ for a given plate, with $\hat{M}_{+}^{\max}$ being higher than $\hat{M}_{-}^{\max}$: however, the disparity between values increases as the subtended angle increases. The propagating moments, $\hat{M}_{+}^{*}$ and $\hat{M}_{-}^{*}$, remain the same irrespective of the angle subtended. With the shallowest corrugated plate, in which $\phi = 14^{\circ}$, there is no snap-through response and the moment tends towards the same value of propagating moment at higher values of θ. The angle subtended by the cross-section for this limiting behaviour is also predicted by the analytical model in [2]: as a corollary, the propagating moment does not depend on the angle subtended by the cross-section.

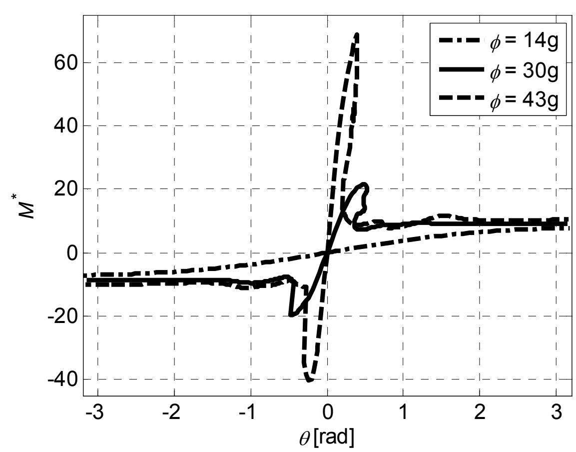

Figure 3. Non-dimensional $\hat{M}(\theta)$ relationship for corrugated plates with $t = 0.2$ mm and $L = 200$ mm

Figure 4 presents some results for different lengths of plate. For convenience, the applied moment is non-dimensionalised by setting equal to $m / D\kappa_{T0}$, where κ_{T0} is the initial corrugation curvature.

As can be seen, the shortest plate has the largest peak moment and, as the length increases, the peak moment decreases and tends towards a constant value. The longer the corrugated plate becomes, the smaller the influence of the constraint applied by the rigid ends to the localisation which follows snap-through. Thus, the corrugated plate becomes softer, and the initiation of the fold takes place at a lower bending moment. The post-buckled behaviour remains essentially the same for all the lengths. When the peak moments are compared to theoretical predictions, Fig. 5, the difference becomes clearer, with the latter always lagging the finite element results.

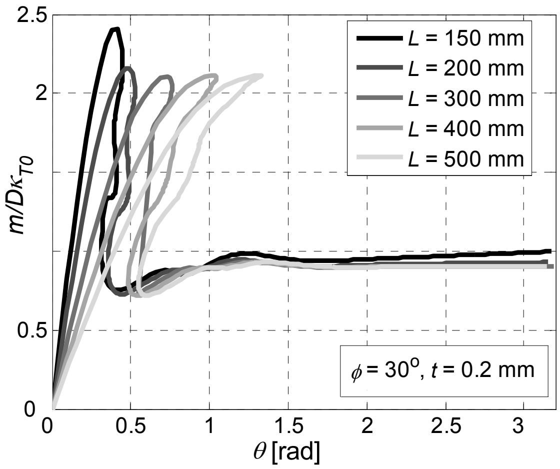

Figure 4. Non-dimensional moment-rotation relationship for corrugated plates of different length

When the plate is shortest, the difference is largest and thus, the constraint applied by the ends is important, which is neglected by theory. Secondly, the analytical model assumes that flattening is a uniform process in which the corrugations remain as circular arcs: this may not be true in practice especially when the localisation, which precedes snap-through, stems from the corrugations flattening in a sequential manner across the section.

In conclusion, the corrugated sheet behaves similarly to a tape-spring: the peak moments are affected by rigid end effects but the propagating moments associated with localised fold formation remain unaffected.

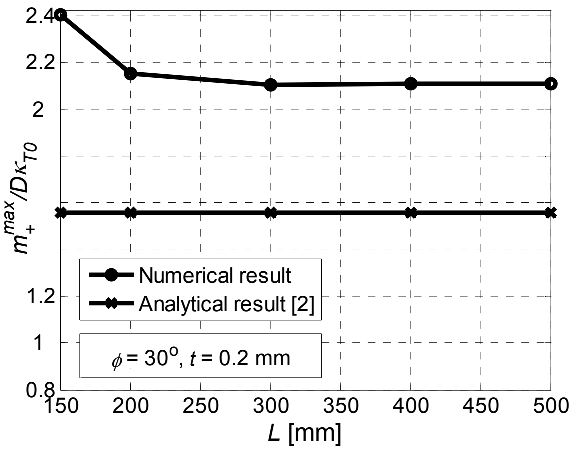

Figure 5. Non-dimensional peak moment against L for corrugated plates of different length

Acknowledgments

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