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On the Effect of Prestress on the Bistable Behavior of Tape-springs

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Abstract

The effect of bending pre-stress on the bi- stable behaviour of a tape-spring is investigated by specifying a thermal gradient through its thickness, in order to simulate residual bending. It is shown that the magnitude of the self-stress state which arises influences the bi-stability characteristics of the pre-stressed tape-spring.

Keywords

Pre-stress, elastic spring, bi-stability, shell.

1 Introduction

A tape-spring is a straight, thin-walled beam with a circular arc cross-section. It is well-known [1] that, by introducing prestress to create a self-equilibrating state of bending, these structures can be stable when straight and when coiled up. Simplified analytical models of behaviour can relate information about each bistable state but cannot capture more subtle detail, such as the transition between bistable states, the breaking moments *etc.*, all of which are essential for a robust bistable design.

This study presents a numerical analysis for interrogating the bistable behaviour in more detail. It makes use of a temperature gradient applied through the thickness, in order to simulate forming prestress. It is shown that, for particular values of temperature gradient, bistable configurations are admissible but otherwise, are not. Moreover, it is shown that small differences on temperature gradient modify significantly the energy state of the bistable configurations of the tape-spring.

2 Statement of the Problem

The tape-spring has length L, thickness t, transverse radius R and subtended angle ϕ as represented in Fig. 1. Geometrical and mechanical data are listed in Table 1. The tape-spring is loaded by a pair of equal and opposite bending moments, *M,* such that it undergoes an "opposite sense" bending, where the direction of bending opposes the original transverse curvature. Once the applied moment reaches a maximum value, a localized elastic fold forms.

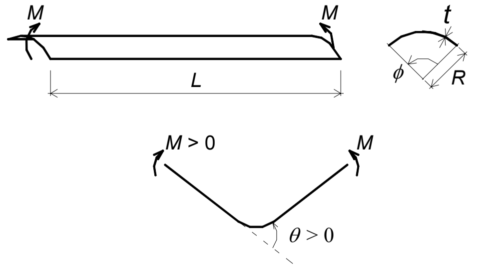

Figure 1. Geometry of a tape-spring and schematic diagram defining positive relative rotation θ

Table 1. Tape-spring: Geometrical and mechanical properties

Transverse radius, R	10 mm
Transverse curvature, κ_{T0}	-0.1 mm^{-1}
Longitudinal curvature, κ_{L0}	0
Thickness, t	0.3 mm
Subtended angle, ϕ	1.57 rad
Length, L	125.6637 mm
Young's modulus, E	131000 N/mm^2
Poisson's ratio, ν	0.3
Thermal expansion coefficient, α	3.68×10^{-6} 1/°C

The bending moment at a well-developed fold is approximately constant and is generally called propagating moment, denoted by M^*, where [2]:

$$M^* = DR\phi\left[\chi_L + \nu\chi_T\right] \tag{1}$$

D is the flexural rigidity equal to $D = Et^3/12(1-\nu^2)$ and χ_L, χ_T are the longitudinal and transverse changes of curvature, respectively. The curvature changes have the following expressions:

$$\chi_L = \kappa_L - \kappa_{L0}, \quad \chi_T = \kappa_T - \kappa_{T0} \tag{2}$$

where subscript '0' refers to initial values of absolute curvatures. In [2], it is shown that the fold can be characterized by zero transverse curvature, κ_T, and constant longitudinal curvature κ_L, denoted by $1/R^*$. In practice, a number of studies suggests that the longitudinal radius of curvature of the fold differs from the transverse radius of curvature of the undeformed tape-spring, *R*, by about 15%.

Now, consider that the shell undergoes a temperature change through the thickness, such that the temperature gradient $\Delta T/t$ is constant. A temperature gradient is used to simulate the effect of bending prestress. In reality, the prestress acts only in the longitudinal direction, while the applied thermal gradient has a biaxial effect. However, due to the relatively small width of the tape compared to its length, this type of thermal loading produces a dominant uniaxial response.

If each element of the shell were free to deform without constraint, then it would deform with a uniform spherical curvature κ_{Th} such that, [3]:

$$\kappa_{Th} = \alpha\left(\partial T/\partial z\right) = \alpha\Delta T/t \tag{3}$$

where *z* is the direction measured normal to the middle surface of the tape-spring.

3 Numerical Analysis

3.1 Temperature effect

In the following, the software package ABAQUS [4] is used to model the tape-spring as a mesh of S4R5 shell elements.

To investigate bi-stability, the straight tape-spring is bent until the relative rotation θ between the ends is π rad (phase 1). From this position, a temperature gradient is applied through the thickness, whilst the relative rotation of the ends is held fixed (phase 2). The ends of the tape move closer or move apart depending on the sign of the gradient. Then, the tape-spring is unbent until the relative rotation between the ends goes back to zero (phase 3). The 3-phase loading history is performed for a range of temperature gradients, and the corresponding moment-rotation responses are plotted in Fig. 2. The loading path (phase 1) is represented by the black solid line. The effect of the temperature gradient is to raise or to reduce the value of the moment M^* at the ends (phase 2).

During unloading (phase 3), a plateau region can be observed where the moment M^* is approximately constant. This phase is characterized by a value that is higher or lower than that value attained during pure bending without heating, depending on the sign of the temperature gradient. Furthermore, the linear part of the diagram has a different slope from the slope of phase 1, since the temperature gradient very slightly changes the transverse curvature of the tape, making it marginally more or less stiff.

An approximate value of the negative temperature gradient can be found when the quasi-horizontal part of the unloading diagram crosses the θ-axis, signifying the existence of an equilibrium state of the prestressed tape-spring under zero external moment.

From [3], the bending moment in the fold taking into account κ_{Th}, Eq. 3, can be modified to give:

$$M^* = DR\phi\left[\chi_L + \nu\chi_T - \left(1+\nu\right)\kappa_{Th}\right] \qquad (4)$$

Setting Eq. 4 equal to zero and computing the resulting temperature gradient, produces: $\overline{\Delta T/t} = -2.4\times10^4$ °C/mm .

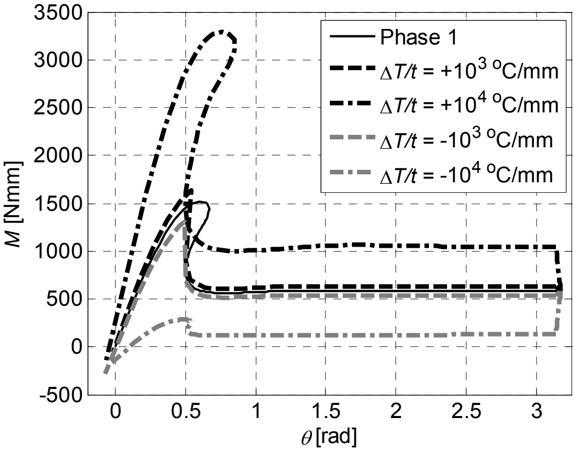

Figure 2. Moment-rotation relationship of tape-spring subject to opposite sense-bending (phase 1), to temperature gradients (phase 2), and then unbent (phase 3).

Finite element simulations reveal a more precise value, where $\overline{\Delta T/t}$ equal to -1.3×10^4 °C/mm .

3.2 Bistable behaviour

The bending behaviours of tape-springs of different lengths subject to the temperature gradient $\overline{\Delta T/t} = -1.3\times10^4$ °C/mm are analyzed and represented in Fig. 3(a). In particular, the unbending path with $-120 < M < 20$ Nmm, is shown in Fig. 3(b). A positive gradient means positive stiffness (stable equilibrium) and vice-versa. Only the shortest tape-spring seems to be bistable, see Fig. 3(b). One of the two stable equilibrium positions corresponds to an almost straight configuration with shallow curvature. The other one corresponds to the "coiled" state. For longer tape-springs, it is necessary to increase the relative rotation between the ends to find the "coiled" equilibrium point ($\theta = 4.7$ rad).

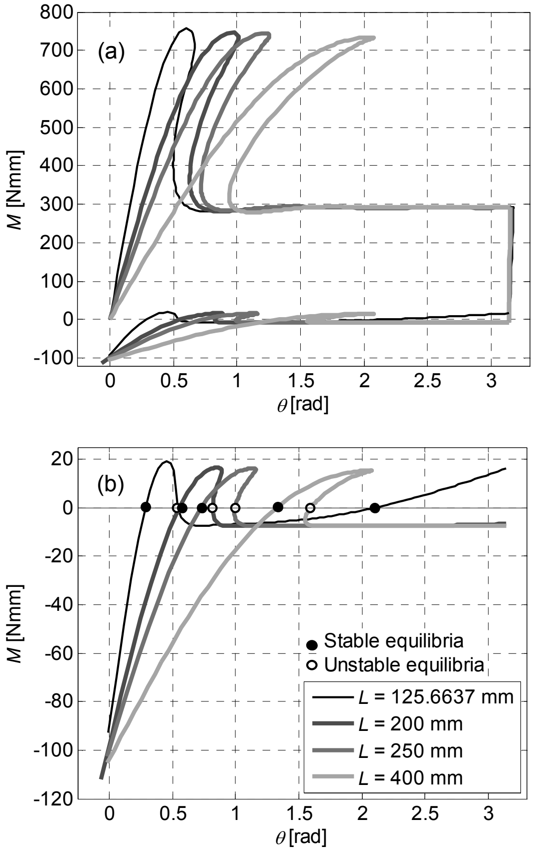

Figure 3. Moment-rotation relationship of tape-springs of different lengths. (a) Tape-springs are subject to opposite sense bending, to a negative temperature gradient and then unbent; (b) unbending paths.

For the shorter tape-spring, it is interesting to consider the variation of the strain energy density with the rotation angle θ in more detail, for this yields information about the bi-stable "preference". In Fig. 4, two different gradients of temperature are applied to same tape-spring of length $L = 125.6637$ mm. As expected, stable equilibrium points correspond to local energy minima and the unstable points correspond to local energy maxima. In particular, for the tape-spring subject to the lowest temperature gradient, $\Delta T / t = -1.3 \times 10^4$ °C/mm, (black lines in the figure) the coiled state is the lowest energy state. In other words, when the tape-spring is held in a position in between the stable and unstable points, it tends to coil up without external loads being applied. The opposite happens for the tape-spring subject to a slightly different temperature gradient, $\Delta T / t = -1.28 \times 10^4$ °C/mm, (grey lines in the figure): the straight state is the lowest energy state to which the tape tends.

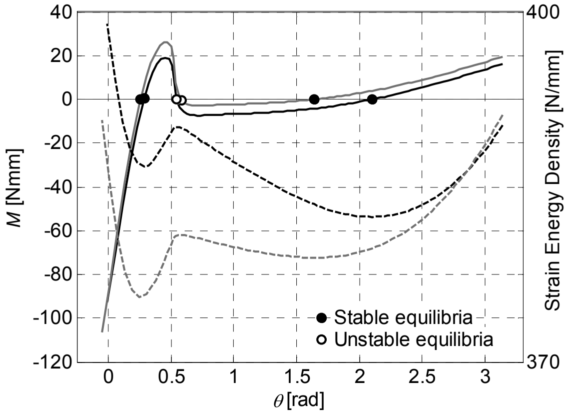

Figure 4. Plot of strain energy density (dotted lines) together with equilibrium plot of moment (solid lines) against relative rotation.

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References

[1] Kebadze, E., Guest, S. D., Pellegrino, S. (2004) Bistable prestressed shell structures,

Int. J. Solids Struct., 41, 2801-20.

[2] Seffen, K. A. (2001) On the behavior of folded tape-spring, *J. Appl. Mech.*, 68, 369-75.

[3] Seffen, K. A., McMahon, R. A. (2007) Heating of a uniform wafer disk, *Int. J. Mech. Sci.*, 49, 230-38.

[4] Hibbitt, Karlsson, Sorenson (1997) *ABAQUS Version 6.1*, Hibbitt, Karlsson, Sorenson, Pawtucket.

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