

Cold Gas-Pressure Folding of Miura-ori Sheets

M. Schenk, J.M. Allwood*, S.D. Guest

Cambridge University Engineering Department, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

* Corresponding author: E-mail address: jma42@cam.ac.uk. Tel: +44 1223 338181

Abstract

Folding sheets from flat sheet materials into 3D surfaces provides a way to form textured sheets with a deep relief, without stretching the base material. Manufacturing can therefore be done using only low-energy bending operations along the fold lines. An important challenge to be overcome in the manufacturing process is the significant in-plane biaxial contraction during the folding process. A novel manufacturing process is herein introduced, which uses cold gas-pressure to fold the sheets and requires a minimum of initial tooling. Calculations were done to determine the required forming pressure to fold an example folded sheet, a *Miura-ori* sheet, and were compared with trials.

Keywords: Miura-ori sheets, Gas pressure forming, Folding

Introduction

A promising approach to form textured sheets with a deep relief is to create the product by folding from a flat sheet of material, in a manner analogous to Origami folding. The most common tessellated folded sheet uses the *Miura-ori* pattern shown in **Fig. 1**, but as described by Klett and Drechsler (2010) an almost infinite range of geometries can be designed.

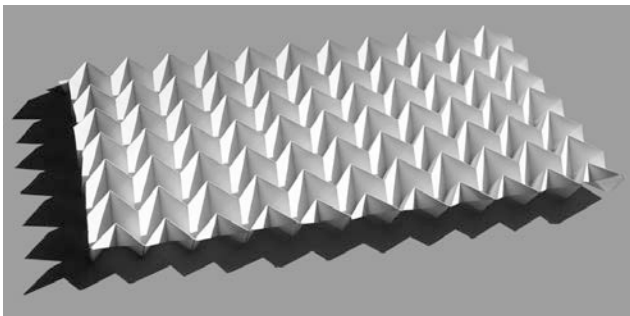


Figure 1. Photo of a *Miura* sheet folded from paper.

The applications of such folded sheets are diverse: Miura (1972) employs them as sandwich panel cores, Basily and Elsayed (2004) investigated their impact absorbing properties, and Schenk and Guest (2009) explored their use as flexible shells for morphing structures. In recent years there has been a revived research interest into the use of folded cores for sandwich panels for aerospace applications, as an alternative to honeycomb structures. Hachenberg *et al.* (2003) list advantages that include the ability to tailor the unit cell geometry to specific mechanical requirements, the possibility to create non-planar cores without distortion of the base material and the presence of open ventilation channels. Furthermore, the folding process allows the processing of a wide range of sheet materials, and large amounts of the core material can be manufactured continuously and cost-effectively. Their mechanical properties, such as compressive strength and impact performance, have been studied using both experiments and numerical models. Heimbs *et al.* (2010) note that an important challenge for correct finite element modelling is

to accurately find the geometry of the folded sheet, including imperfections due to the forming process.

Using conventional forming methods such as stamping for the *Miura-ori* sheets, would lead to thinning and possibly fracture of the facets due to the pronounced folds. An important feature of sheets such as the *Miura-ori* is that they can be folded from a flat sheet of material, without any stretching of the material between the fold lines. Many patterns are also rigidly foldable, and can be folded even without any bending of the facets. These folded sheets can therefore be formed using only low-energy bending operations. There are, however, significant challenges in the manufacturing of folded structures. As the fold lines do not extend across the sheet, there is a strong coupling between the folds, and many of the patterns can effectively be described as 1 DOF mechanisms. This means it is difficult to simultaneously have both folded and unfolded regions in the sheet material. Furthermore, the sheets significantly contract biaxially in-plane and simultaneously expand in thickness during folding. Several manufacturing methods have been developed to overcome these issues, and will be briefly described below.

In addition to bending along the intended fold lines, several undesired deformations may take place during manufacturing of folded sheets. For example, the facets of the fold pattern may bend during folding, fold lines may move through the material, and depending on the process, substantial shear deformation or sign reversal of the fold lines can take place. The choice of process therefore affects the range of materials that can be used for the folded sheet, and the accuracy of the final product. All analysis of the deformation of the sheet material during the manufacturing processes has thus far been qualitative.

Manufacturing Methods for *Miura-ori* Sheets

A range of manufacturing methods for folded sheets has been found in (patent) literature, which may be broadly categorized into synchronous, gradual and pre-gathering techniques.

Synchronous In the synchronous methods, folding takes place along all fold lines simultaneously and these are therefore only suitable for batch processes. Khaliulin and Dvoeglazov (2001) describe a transformable matrix consisting of hinged rigid panels. The flat sheet material is

placed between two partially folded transformable matrices, whose initial volume enables actuation using a vacuum bag. Alternatively, as suggested by Akishev *et al.* (2009), one side may be replaced by a vacuum bag that presses the sheet against the matrix. Gewiss (1968) distinguishes between a ‘molding structure’ that is in contact with the facets of the fold pattern, and a ‘motive structure’ which coordinates the folding motion of the sheet. In one described embodiment, a flat sheet is placed on either side of the ‘motive structure’ and using a vacuum bag both sheets are folded simultaneously. These synchronous methods involve the least amount of deformation of the sheet material, but the size of the final folded sheets is limited.

Gradual In the gradual forming process, the sheet material transitions continuously from a flat to fully folded state. Kling (2007) shows how this could be achieved by gradual deepening through a series of patterned rollers. An alternative approach described by Kehrlé (2005) is to emboss the desired folding pattern onto the flat sheet material, creating a residual stress field which initiates the folding process; the folding is then progressively continued, for example by an array of bristle brush rollers that retard and gather the folded sheet, or tapering guides that fold the sheet to its desired width.

Pre-gathering Another common approach aims to overcome the coupled longitudinal and transverse contraction, by first pre-gathering the sheet material into a singly corrugated sheet, to a width substantially corresponding to the final width of the folded sheet. The double corrugation can then be created row-by-row, feeding the material through to the shaping zone in a stop and burst manner. For example, Hochfeld (1959) describes a machine that periodically inverts the single corrugations and Khaliulin *et al.* (2007) describe a method where the straight corrugations are moved alternately sideways, creating the double corrugation pattern. A continuous manufacturing method is investigated by Elsayed and Basily (2004), which passes the corrugated material through a set of mating patterned rollers that impart the final shape. This modifies the biaxial contraction of the sheet to successive transverse and longitudinal contraction, significantly reducing the amount of slip between the sheet material and the patterned rollers.

The downside of many of these techniques is the cost of the complex tooling required, such as milling the patterned rollers, which limits their suitability to manufacture a range of prototypes of differing geometries. Also, many methods are limited in the thickness and stiffness of the material they can form. Where metals have been folded they have been thin sheets; for example, a folded sheet of 0.1mm thick aluminium foil was investigated by Fischer and Drechsler (2008). The method introduced here enables the production of prototypes of folded sheets with minimal tooling, and can be used to form relatively thick sheet materials.

Miura Pattern

The most common fold pattern for these types of folded sheets is the *Miura-ori* pattern; see Fig. 1. This pattern turns up in a range of applications, and is shown by Ma-

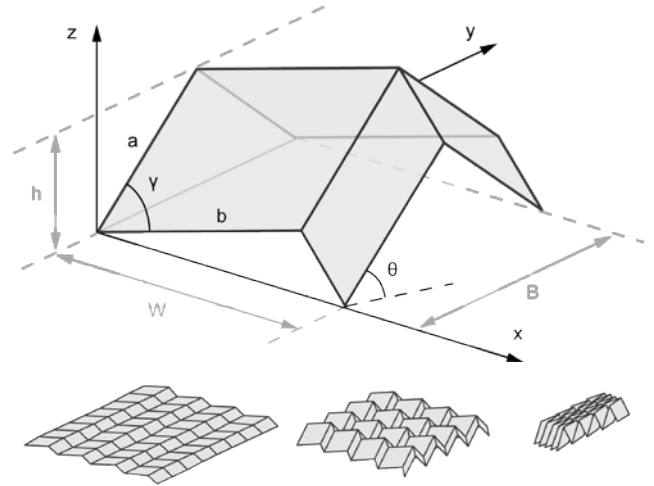


Figure 2. Geometric parameters of a *Miura* unit cell. The values a , b and γ describe the shape of the parallelogram, whereas the angle θ determines the current configuration of the unit cell ($\theta=0,55,85^\circ$ are shown).

hadevan and Rica (2005) to appear as the buckling pattern of biaxially compressed sheets on an elastic substrate. The *Miura* pattern shall be used throughout this report, but the manufacturing process clearly extends to other folding patterns.

For the calculations an analytical description of the folding process of *Miura* pattern is required. The unit cell can be defined by the side lengths a and b , and the acute angle γ of the parallelogram elements; see Fig. 2. The dihedral fold angle θ between the facets and the mid-plane determines the current configuration. The height h , the width W and breadth B of the unit cell are calculated as follows

$$h = a \sin \theta \sin \gamma \quad (1)$$

$$W = 2b \frac{\cos \theta \tan \gamma}{\sqrt{1 + \cos^2 \theta \tan^2 \gamma}} \quad (2)$$

$$B = 2a \sqrt{1 - \sin^2 \theta \sin^2 \gamma} \quad (3)$$

The projected area of the unit cell is then $A_{\text{proj}}=WB$. In their analysis of the *Miura* pattern, Klett and Drechsler (2010) use a different, obviously equivalent, parameterisation which provides additional insight.

Cold Gas-Pressure Folding Process

This section describes a novel forming process to fold a *Miura-ori* sheet, using cold gas-pressure. First the folding pattern is transferred onto the metal sheet, and the material is weakened along the fold lines. This can be achieved through a range of techniques, such as embossing, etching, melting, etc. In our initial implementation a simple perforation pattern is created along the fold lines. Next, two such sheets are packed into a sandwich structure, with the sheets separated by a series of simple ‘spacers’ that are placed into slots along the ‘mountain ranges’ of the sheet; see Fig. 3. The combination is packed into an air-tight bag,

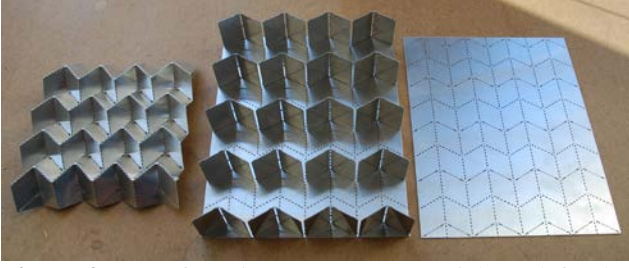


Figure 3. Manufacturing process. From right to left: aluminium sheet perforated along the intended fold lines; sheet with spacer plates slotted into place; the folded sheet after the folding process.

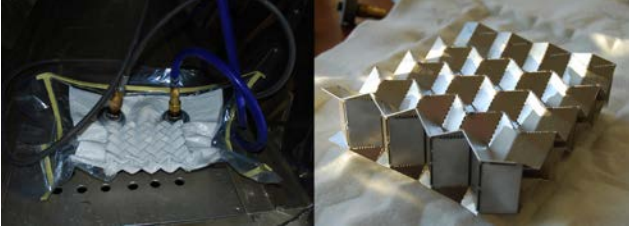


Figure 4. The sandwich of patterned sheets is placed into an autoclave; after creating a sufficient pressure difference the sheet is folded and the vacuum bag is opened to reveal the final product.

which is evacuated to near vacuum, combined with an increase in external pressure. This pressure difference forces the sheets to bend simultaneously along all fold lines; see **Fig. 4**. The biaxial contraction during the folding process is enabled by the freely moving spacers. The fold depth is controlled by the eventual contact between the two sheets, which is determined by the height of the spacers.

The forming process introduced here differs from existing synchronous manufacturing methods for folded sheets in a number of ways. Firstly, it does not require expensive tooling such as folding matrices, and is therefore well suited to create prototypes of different geometries. Also, the spacer plates provide minimal guidance to the folding of the sheet, and the process therefore largely relies on fold lines forming accurately and the sheet contracting uniformly, without external guidance. In contrast, a similar method described by Gewiss (1968) separates two sheets by a more complex ‘motive structure’ that controls the folding motion. Lastly, relatively thick sheet materials can be formed using this method by increasing the external pressure.

Process Calculations

We shall assume that during the folding process a plastic hinge is formed along all the fold lines. For simplicity we ignore many of the subtleties involved in sheet metal bending, such as shifting of the neutral axis and the exact curvature of the fold line (Marciniak and Duncan, 1992). The required forming pressure difference p can be calculated using the principle of virtual work, as follows

$$dW_{\text{ext}} = dW_{\text{int}} \quad (4)$$

$$p \cdot dV = \sum M_p^i \cdot d\alpha_i \quad (5)$$

where $d\alpha_i$ is the change in fold angle i , and M_p the corresponding plastic moment. For the *Miura* sheet we shall assume the compatibility equations of the folding process to be described by the fold angle θ . The energy balance can then be rewritten as

$$p \frac{\partial V}{\partial \theta} d\theta = \sum M_p^i \frac{\partial \alpha_i}{\partial \theta} d\theta \quad (6)$$

which must hold for any $d\theta$ for the sheet to be in equilibrium. The required pressure difference p can subsequently be calculated for every fold depth.

External Work

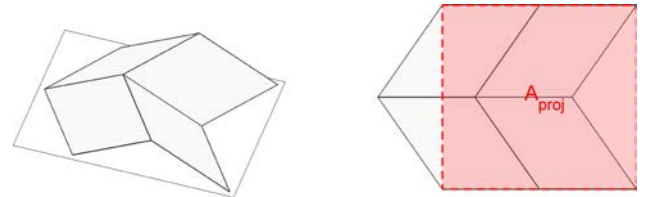
The external work consists of two components: the work exerted by the pressure on the top surface of the sheet and the work it exerts around the perimeter, as the sheet folds up; see **Fig. 5**. The two components of the total displaced volume V are given as

$$V = V_{\text{top}} + V_{\text{side}} = \frac{1}{2} A_{\text{proj}} (h - H) + \frac{1}{2} A_0 H \quad (7)$$

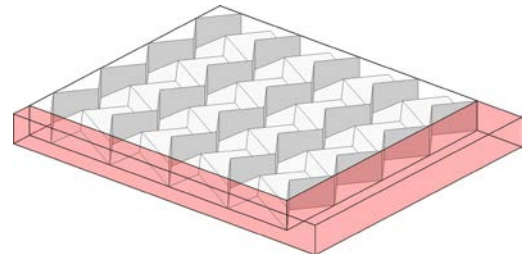
For the virtual work equation, we find dV as

$$dV = \left[\frac{\partial A_{\text{proj}}}{\partial \theta} \frac{(h - H)}{2} + \frac{A_0}{2} \frac{\partial h}{\partial \theta} \right] d\theta \quad (8)$$

It is interesting to note that the component V_{top} is by itself not sufficient to fold the pattern to any desired depth. This is due to the fact that the displaced volume V_{top} reaches a maximum at $\theta=45^\circ$ after which dV_{top} becomes nega-



(a) a *Miura* unit cell in a partially folded state. The volume under the unit cell is given by $V_{\text{top}} = 0.5 A_{\text{proj}} h$.



(b) partially folded sheet with spacers. The displaced volume is indicated by the shaded areas, and is calculated as $V_{\text{side}} = 0.5 (A_0 - A_{\text{proj}}) H$, with H the spacer height.

Figure 5. The total displaced volume V during folding of the sheet consists of the volume V_{top} under each unit cell (a) and the volume V_{side} displaced by the in-plane contraction of the sheet (b).

tive and the pressure on the top surface will actually work to unfold the sheet. In order to fold the sheet further, the effect of the pressure exerting a compressive force along the perimeter of the sheet takes over.

Note that increasing the height of the spacers H decreases the pressure required to fold the sheet; however, in all calculations presented here, we shall assume that the spacer height is double the desired fold depth.

Internal Work

The internal work done by the plastic hinges along the fold lines, during deformation of a unit cell from its initial flat state to a given configuration, is described by:

$$W_{\text{int}} = 2t^2\sigma_y \left(a\kappa\left(\frac{\pi}{2} - \varphi\right) + b\kappa\theta \right) \quad (9)$$

where φ is the dihedral angle between the vertical plane and the facets (see **Fig. 2**), κ the degree of pre-weakening along the fold line and a , b the lengths of the fold lines. The change in internal work dW_{int} then is

$$dW_{\text{int}} = 2t^2\sigma_y \kappa \left(b - a \frac{\partial\varphi}{\partial\theta} \right) d\theta \quad (10)$$

No folding will take place along the perimeter of the sheet, which could be taken into account in the internal work equation. However, assuming the sheet is sufficiently large, the effect along the perimeter becomes negligible and the folding process of an entire sheet can be described using a single unit cell.

Maximum Pressure

Using the equations derived previously, the required equilibrium pressure at every stage of the folding process can be calculated. Of greater practical interest, however, is the *maximum* pressure needed to fold the sheet to a desired fold angle θ_{target} . In order to calculate the required p_{max} for *Miura* patterns of any geometry, the variables governing the forming process have been normalized as follows:

$$\bar{p} = \frac{p_{\text{max}} A_0}{\sigma_y t_{\text{eq}}^2} \quad ; \quad r = \frac{a}{b} \quad ; \quad \gamma \quad ; \quad \theta_{\text{target}}$$

where a , b , γ describe the geometry of the unit cell, A_0 is the initial area of the unit cell, θ_{target} the desired fold angle, and σ_y the material's yield stress, and $t_{\text{eq}} = t\sqrt{\kappa}$ the equivalent material thickness along the fold line. The resulting plot for $r=a/b=1$ is shown in **Fig. 6** and enables quick lookup of the pressure required to fold different patterns.

Folding Experiments

The forming technique was tested by manufacturing several examples of the *Miura-ori* sheet from aluminium. The dimensions were chosen as $a=b=25\text{mm}$, $\gamma=60^\circ$ and $m=n=4$, with a desired fold angle of $\theta=55^\circ$, which corresponds to a depth of 17mm. The sets of aluminium sheets were prepared with the desired perforation pattern $\kappa=0.4$, using a CNC milling machine.

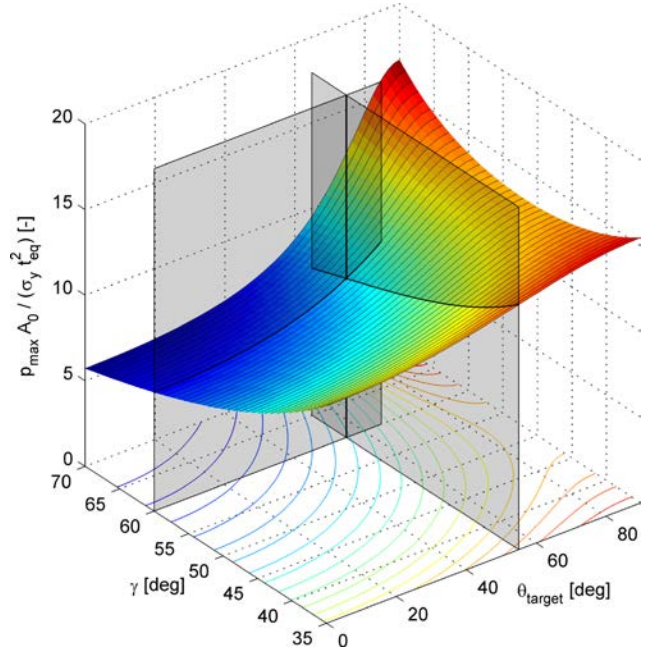


Figure 6. The normalized maximum pressure \bar{p} required during the folding process to attain θ_{target} for varying γ , with the ratio $r=a/b=1$. For example, the surface is here sliced at $\gamma=60^\circ$ and $\theta_{\text{target}}=55^\circ$, yielding a maximum forming pressure of $\bar{p} = 9.3$.

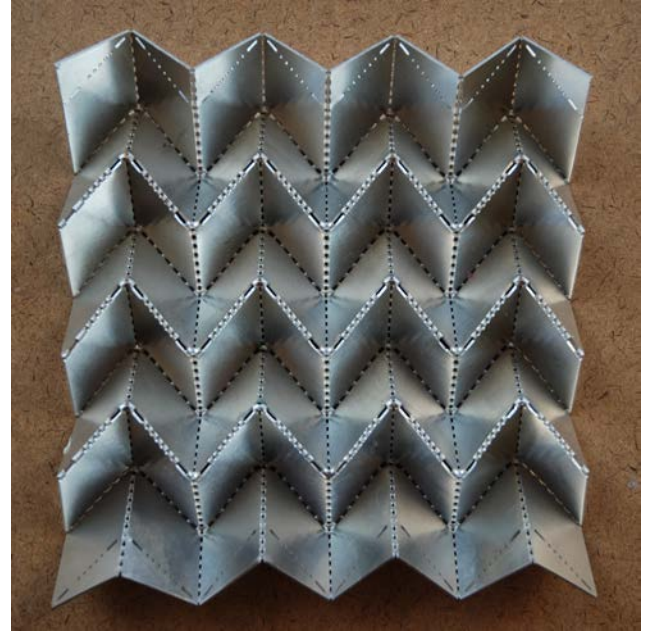


Figure 7. A *Miura-ori* sheet folded from a 22 gauge aluminium sheet, using the novel folding process.

The aluminium alloy used was Al 5251-H22, with an experimentally determined yield stress for the plastic hinge of $\sigma_y=270\text{MPa}$ for the 22 gauge (0.71mm) sheet and $\sigma_y=220\text{MPa}$ for the 24 gauge (0.56mm). These values were found using a modified three-point bending test, with assumed friction coefficient of $\mu=0.3$ between the rig and specimen. The spacer plates with height of 37.5mm were manufactured from 22 gauge stainless steel with a waterjet cutter.

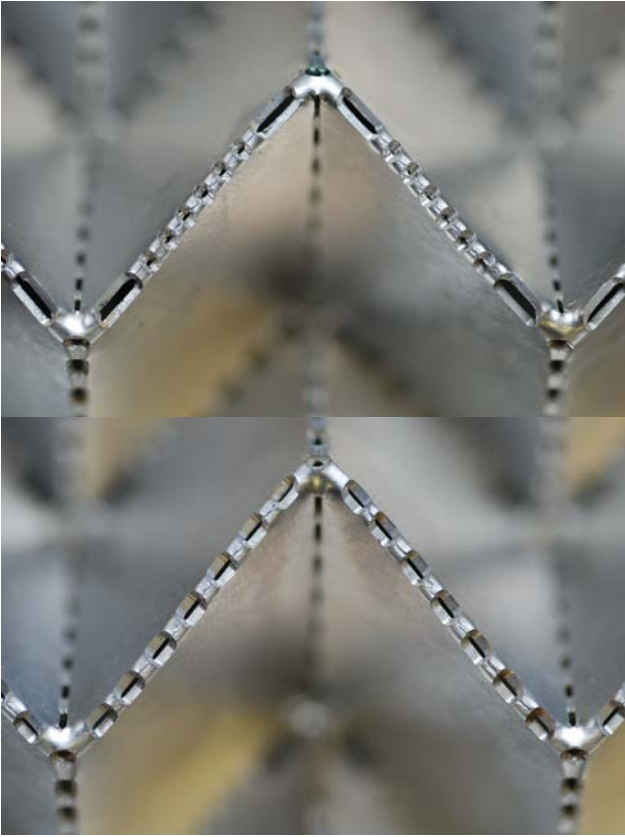


Figure 8. Close-ups of the fold lines for the 22 gauge sheet. Top shows a ridge supported by the spacer plates, and bottom one that is freely formed. Note how the fold pattern avoids the intended vertex, seeking a lower-energy solution.

Results and Discussion

A *Miura-ori* sheet of 22 gauge aluminium created using the new process is shown in **Fig. 7**. The vertices appear to undergo more complex and substantial plastic deformation than assumed in our simple kinematic model. Looking at **Fig. 8** the holes drilled at the intended vertices of the folded sheet are consistently not centred on the actual vertex in the folded state. It is postulated that the curved fold line follows a lower-energy route, and thereby avoids the intended vertex. As a result, the intended fold line with the perforation pattern no longer lies along the actual fold line. The more complex deformation kinematics at the vertices also resulted in small membrane strains in the corners of the parallelogram elements. A closer look at the material along the fold lines in **Fig. 8** also reveals that it has cracked along the outer radius of the fold lines, due to the tight fold angle.

For two sheets the fold depth versus the applied pressure difference was measured, and the results are compared with the calculations in **Fig. 9**. It is to be noted that the sheet thickness was measured after pressure was released (*i.e.* after springback) and the value was converted to fold angle, assuming the ideal kinematics of the sheet. As can be seen, the predicted values lie below the measured pressures required to fold the sheet to a desired depth. This suggests that even in this relatively pure form of folding, *e.g.* no moving fold lines, no bending of the facets during

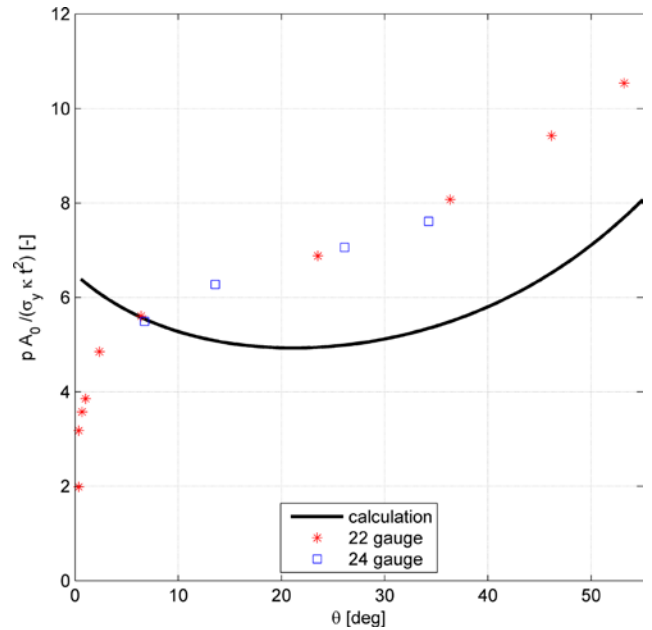


Figure 9. Comparison of the theoretical forming pressure, with the measured values; both are normalized. The measured values were normalized using the experimentally determined yield stress. Given the relatively small dimensions of the test specimen, the absence of folding along the perimeter was taken into account for the calculations.

forming, we cannot yet account for the full forming energy required to fold the pattern.

Some tests were performed to establish the influence of the weakening factor κ along the fold lines. The sheets that were pre-weakened less than $\kappa=0.4$ did not fold as desired and ended up close to being singly-corrugated with the sheet forming an almost straight corrugation along the unsupported ridges. There was also a global instability as the two sheets did not remain parallel, but rather bulged and buckled. This also damaged the spacer plates, which were significantly bent, thereby significantly reducing their buckling load.

Conclusions and Future Work

In this paper we have introduced a novel method to fold a *Miura-ori* sheets, using cold gas-pressure forming. The method requires very little initial tooling, and can therefore be used to cost-effectively generate a range of prototypes of differing geometries. Furthermore, relatively thick sheet materials can be folded by increasing the external pressure.

The calculated forming pressure assumed an ideally plastic material model, and plastic hinges along the fold lines during the folding process. Comparison with experiments showed that the current approach underestimates the required forming pressure, and cannot yet account for the full forming energy required to fold the pattern. This could in part be improved with a more accurate material model, but also the simple plasticity assumptions no longer seem to hold for these large deformations.

Etching and embossing of the fold lines, rather than simple perforations are worth exploring, to alleviate some of

the forming problems at the vertices. Gitlin *et al.* (2003) and Durney (2006) describe more sophisticated methods of ensuring accurate bends in sheet metal folding, and these are worth exploring to reduce the required pre-weakening along the fold lines.

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