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A NEW VARIABLE STIFFNESS SPRING USING A PRESTRESSED MECHANISM

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ABSTRACT

A novel design of a semi-active variable stiffness element is proposed, with possible applications in vibration isolation. Semi-active vibration isolators usually use variable dampers. However, it is known from the fundamental vibration theory that a variable spring can be far more effective in shifting the frequencies of the system and providing isolation. Geometry change is a common technique for building variable springs, but has disadvantages due to the complexity of the required mechanism, and slow response due to the inertia of moving parts.

In the variable spring introduced here (VS), the stiffness is changed by force control in the links which corresponds to infinitesimal movements of the links, and does not need a change of geometry to provide a change of stiffness. This facilitates a fast response. The proposed VS is a simple prestressed cable mechanism with an infinitesimal mechanism. Theoretically the level of the prestress in the cables can be used to control the stiffness from zero to a maximum value that is only limited by the strength of the links. In this work, the statics, kinematics and stability of the VS are studied, the stiffness is formulated, and possible configurations of the VS are found.

INTRODUCTION

Stiffness elements (springs) and dampers are the main components in any vibration isolator or absorber. Damping and stiffness are two essential complements in passive vibration control solutions. When passive isolators do not provide the desired vibration control, semi active isolators might be considered. Most of the semi-active isolation solutions are based on variable damping (e.g. magneto rheological materials). However it is known from fundamental vibration theory that stiffness change can be more effective in shifting natural frequencies and hence providing isolation. Variable optimum operation of an isolator variable damping are considered in [1,2]. In [1], Jalili explains that the unpopularity of the available variable stiffness spring is mostly due to their high energy requirements. He also mentions that low power designs of variable stiffness elements suffer from limited frequency range, complex implementation and high cost. The main reason for these disadvantages is that the stiffness in most of these variable stiffness springs changes by changing the geometry [1,3]. Geometry change requires finite motions of mechanical links which limits, the frequency shift that can be achieved, and make these systems slow.

An alternative approach for stiffness change was introduced by the authors in [3]. This method is based on force control in a prestressed mechanism. Since the force change corresponds to infinitesimal motions, the response is much faster than the above mentioned geometry based solutions.. The resulting spring is a semi-active variable stiffness spring in which the stiffness is controlled and changed by the level of the prestress. The concept was first presented in [3] using prestressable cable-driven mechanisms in their singular configurations. In [4], tensegrity prism mechanisms were investigated as the basis for a variable stiffness spring. The present work introduces a new pin jointed mechanism as a variable stiffness spring. This spring (called the VS here) is a prestressable mechanism where the mechanism is infinitesimal. This design has advantages over the tensegrity prism due to its simpler design and application.

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The design of the proposed semi-active variable stiffness spring (VS)

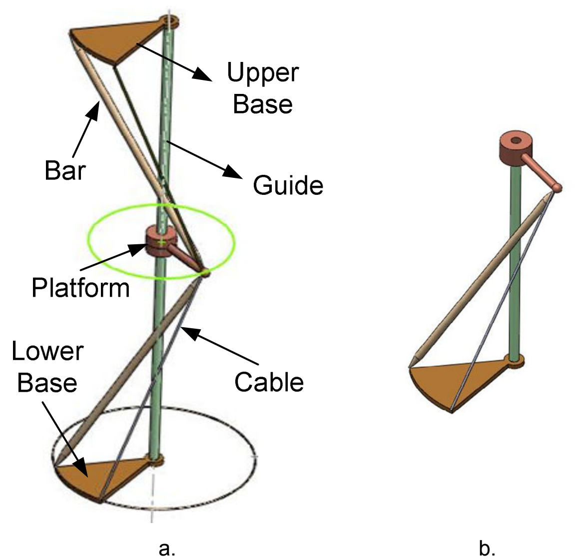


Figure 1 a. VS b. Half of the VS (HVS)

Figure 1.a shows the simplest form of the proposed semi-active Variable stiffness Spring (VS) with a minimum number of components. It consists of two bars, two cables, upper and lower bases, a guide and a platform in the middle. The lower base is fixed to the guide, while the upper base and the platform have cylindrical joints with the guide. The joints between the rods and the bases, and between the rods and the platform, are all spherical joints. Cable ends are assumed to be connected to the center of the spherical joints.

The bars of VS are considered as rigid links with constant lengths but two cables are elastic with variable lengths. As a result of considering one degree of freedom (DoF) for each cable, the VS finds two DoF. Therefore, the upper base can move along the guide and can also rotate about the guide respect to the lower base. However, it will be clear later that because of the resistance of the cables to extension and also because of the effect of prestress level in the VS, translational motion and rotational motion of the upper base require axial load and rotational moment, respectively. For this reason. the VS acts as a translational spring under axial load along the axis of the guide (Figure 6). Desirably, throughout the translational deflection, the two bases move along the axis of the guide and do not rotate. The middle platform rotates and moves along the axial direction. The VS also acts as rotational spring under pure rotational moment applied on the bases (Figure 7). In this case, while the upper base is rotating, the height of the VS remains constant. It will be shown that the VS is a prestressed mechanism, and the prestress level in the cables and rods can provide significant control of the total stiffness. As a result the VS is a semi active variable stiffness spring, with prestress as the control variable.

Before formulating the stiffness of the VS, its kinematics and the statics are first studied. Because the geometry of the upper part of the VS is the mirror image of the lower part shown in Figure 1.b, the upper part and lower part have similar statics and kinematics. For the sake of clarity and simplicity, a general configuration of one half of the VS (Figure 2), described here as the HVS, is studied first and the results will be then extended to the entire VS.

Parameters L_b, L_c and h are the length of the bar, the length of the cable and the height of HVS, respectively. The Cartesian coordinate system XYZ is set such that the Z axis is along the guide and the X axis connect the Origin O_1 to the lower end of the bar (A_1). Point (A_1) and the lower end of the cable (A_2) are in the same distance (r_1) from the Z axis. The upper ends of the bar and cable are connected to the platform at point B_1. The distance between the A_1 and the Z axis is r_2. Angles θ and φ are the angles between the arms of points A_1 and B_1 and the XZ plane. $\boldsymbol{\hat{u}_b}$ and $\boldsymbol{\hat{u}_c}$ are the unit vectors along the bar ($\overrightarrow{B_1A_1}$) and the cable ($\overrightarrow{B_1A_2}$), respectively. $\boldsymbol{\hat{u}_r}$ is the unit vector along the arm of the platform ($\overrightarrow{O_2B_1}$). $\boldsymbol{\hat{u}_b}$, $\boldsymbol{\hat{u}_c}$ and $\boldsymbol{\hat{u}_r}$ are expressed in terms of the parameters as:

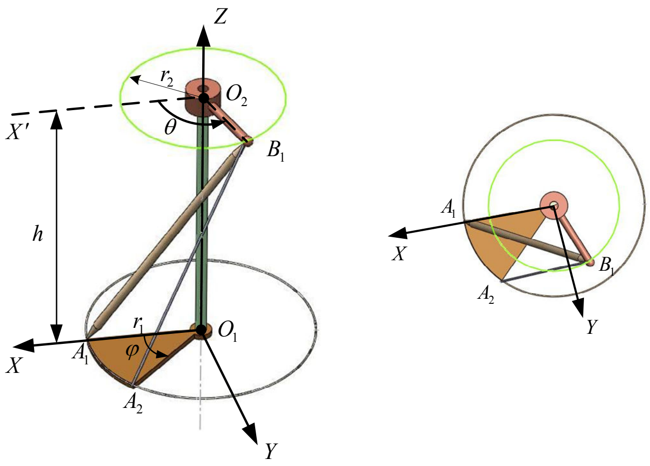
$$\begin{aligned}\boldsymbol{\hat{u}_b} &= \frac{\overrightarrow{B_1A_1}}{\left|\overrightarrow{B_1A_1}\right|} = \frac{1}{L_b}\left(\overrightarrow{OA_1} - \overrightarrow{OB_1}\right)\\ &= \frac{1}{L_b}\left(\begin{Bmatrix} r_1 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} r_2\cos\theta \\ r_2\sin\theta \\ h \end{Bmatrix}\right)\\ &= \frac{1}{L_b}\begin{Bmatrix} r_1 - r_2\cos\theta \\ -r_2\sin\theta \\ -h \end{Bmatrix}\end{aligned} \tag{1}$$

$$\begin{aligned}\boldsymbol{\hat{u}_c} &= \frac{\overrightarrow{B_1A_2}}{\left|\overrightarrow{B_1A_2}\right|} = \frac{1}{L_c}\left(\overrightarrow{OA_2} - \overrightarrow{OB_1}\right)\\ &= \frac{1}{L_c}\left(\begin{Bmatrix} r_1\cos\varphi \\ r_1\sin\varphi \\ 0 \end{Bmatrix} - \begin{Bmatrix} r_2\cos\theta \\ r_2\sin\theta \\ h \end{Bmatrix}\right)\\ &= \frac{1}{L_c}\begin{Bmatrix} r_1\cos\varphi - r_2\cos\theta \\ r_1\sin\varphi - r_2\sin\theta \\ -h \end{Bmatrix}\end{aligned} \tag{2}$$

$$\boldsymbol{\hat{u}_r} = \begin{Bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{Bmatrix} \tag{3}$$

Statics of HVS:

Here, equilibrium of the platform, shown as a free body in Figure 3, is considered. $\boldsymbol{F_z}$ and $\boldsymbol{M_z}$ are the external loads applied on the platform. The internal forces in the bar and cable are shown by $\boldsymbol{\tau_b}$ and $\boldsymbol{\tau_c}$, respectively. $\boldsymbol{F_x}, \boldsymbol{F_y}, \boldsymbol{M_x}$ and $\boldsymbol{M_y}$ are the forces and moments applied on the platform from the guide.



a. b.

Figure 2 a. Right hand HVS b. Top view of right hand HVS

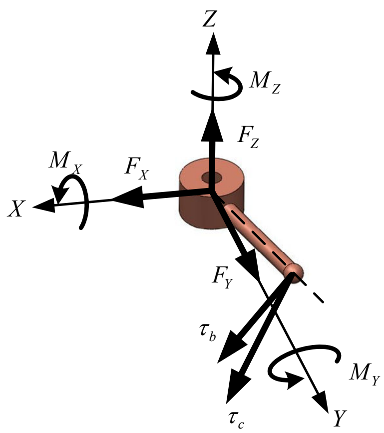


Figure 3 Free body diagram of Platform

The static equilibrium equations of the platform become:

$$\Sigma \boldsymbol{F} = 0,$$
$$F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}} + F_z\widehat{\boldsymbol{k}} + \tau_b\widehat{\boldsymbol{u}}_{\boldsymbol{b}} + \tau_c\widehat{\boldsymbol{u}}_{\boldsymbol{c}} = 0 \tag{4}$$

$$\Sigma \boldsymbol{M} = 0,$$
$$M_x\hat{\boldsymbol{\imath}} + M_y\hat{\boldsymbol{\jmath}} + M_z\widehat{\boldsymbol{k}} + r_2\widehat{\boldsymbol{u}}_{\boldsymbol{r}} \times (\tau_b\widehat{\boldsymbol{u}}_{\boldsymbol{b}} + \tau_c\widehat{\boldsymbol{u}}_{\boldsymbol{c}}) = 0 \tag{5}$$

Equations (4) and (5) gives six algebraic equations. By, eliminating the internal forces and moments acting between the platform and guide *($F_x, F_y,$ M_x and M_y),* two algebraic equations (6),(7) will be obtained which can be expressed as Eqn.(8). These equations relate the internal forces of the bar and cable (τ_b and τ_c) to external loads (F_z and M_z):

$$\tau_b = \frac{L_b(F_z r_1 r_2 \sin(\varphi-\theta) + M_z h)}{r_1 r_2 h(\sin\theta + \sin(\varphi-\theta))} \tag{6}$$

$$\tau_c = \frac{L_c(F_z r_1 r_2 \sin\theta - M_z h)}{r_1 r_2 h(\sin\theta + \sin(\varphi-\theta))} \tag{7}$$

$$\begin{bmatrix} h/L_b & h/L_c \\ r_1 r_2 \sin\theta\, /L_b & r_1 r_2 \sin(\theta-\varphi)\, /L_c \end{bmatrix} \begin{Bmatrix} \tau_b \\ \tau_c \end{Bmatrix} = \begin{Bmatrix} F_z \\ M_z \end{Bmatrix} \tag{8}$$

Prestressability of HVS:

The HVS is partly prestressable if the internal forces of the links such as τ_b and τ_c exist while the external loads ($\mathrm{F_z}$ and $\mathrm{M_z}$) are zero. The existence of prestress in cable and bar can be found by checking Eqn.(8). It is understood from Eqn.(8) that in a specific configurations, where

$$\sin\theta + \sin(\varphi-\theta) = 0 \tag{9}$$

the 2-by-2 matrix becomes rank deficient and hence has a non-zero null-space– there is a non zero prestress vector $\begin{Bmatrix} \tau_{bp} \\ \tau_{cp} \end{Bmatrix}$ in equilibrium with a zero external load vector $\begin{Bmatrix} F_z \\ M_z \end{Bmatrix}$. Thus, the HVS shown in Figure 2 is prestressable when $\sin\theta + \sin(\varphi-\theta) = 0$. The null space $\begin{Bmatrix} \tau_{bp} \\ \tau_{cp} \end{Bmatrix}$ in these configurations give the relationship between the prestress in the bar (τ_{bp}) and prestress in the cable (τ_{cp},). The prestress vector (null space) is found to be

$$\begin{Bmatrix} \tau_{bp} \\ \tau_{cp} \end{Bmatrix} = \alpha \begin{Bmatrix} -L_b \\ L_c \end{Bmatrix}, \tag{10}$$

Mathematically α can be any real number. However, It will be shown that to have stable HVS, the force in link A_2B_1 should be positive (tension). As a result, α is limited to positive values, and since A_2B_1 carries only tension, it can be a cable. Based on Eqn.(10), if the prestress of the cable is set to a tension equal to τ_{cp}, the prestress of the bar will be compressive with a magnitude of ${}^{L_b}/_{L_c}\,\tau_{cp}$.

Discussion on prestressability condition

The prestressability condition found above (Eqn.(10)) is a trigonometric equation with several sets of solutions. In order to find the possible configurations, three possible solutions, shown in Eqn.(11-13), are discussed here.

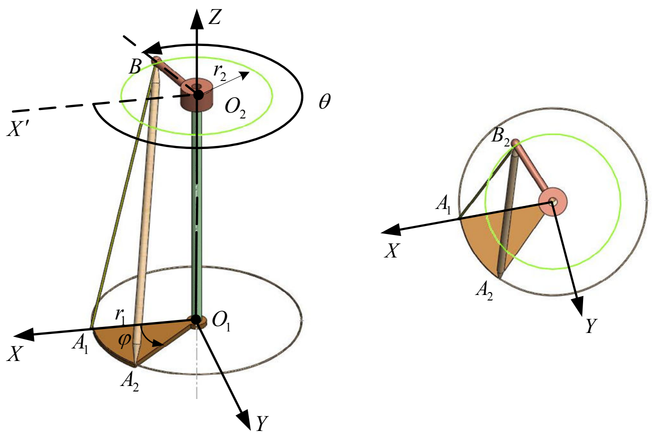
$$\varphi = 0, \ \ \theta\text{: Arbitrary} \tag{11}$$

$$\theta = \frac{\pi+\varphi}{2} \text{ , Right Hand design (RH)} \tag{12}$$

$$\theta = \pi + \frac{\pi+\varphi}{2} \text{ , Left Hand design (LH)} \tag{13}$$

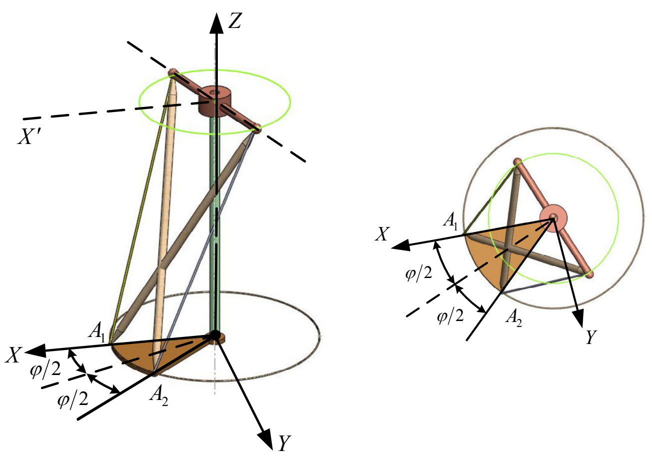
The first solution Eqn.(11) happens when points $A_1$and A_2 coincide and consequently bar and cable are identical. In this case the HVS becomes a finite mechanism and moves under external load. As a result, this configuration cannot carry load and cannot be used as a spring.

The second and third solutions given in Eqn.(12) and Eqn.(13) are useful prestressable configurations. The second solution, similar to the configuration shown in Figure 2, is called Right Hand (RH). The third solution, called Left Hand (LH), is shown in Figure 4. These two solutions are the mirror image of each other with respect to a plane which contains the *Z*-axis, and passes through the bisector of $\angle A_1O_1A_2$ (see Figure 5). It will be shown later that these configurations (RH and LH) when the cables are in tension (links A_2B_2 in RH and A_1B_2 in LH) have positive stiffness and consequently are stable.



a. b.

Figure 4 a. Left Hand HVS b. Top view of left hand HVS



a. b.

Figure 5 Right hand HVS (Figure 2) is the mirror image of Left hand HVS (Figure 4)

Kinematics

In the VS, the bars are chosen to be much stiffer than the cables. For this reason the bars are considered as rigid links with constant length (L_b) and cables as elastic elements with variable lengths (L_c). The length of the cable and the height of the HVS at the original configuration are L_{cp} and h_p, respectively. The original configuration is the configuration in which no external load is applied on the VS and the cable and bar are prestressed. Note that the parameters that describe the original configuration (prestressed configuration) are discriminated by subscript “p”. When external load is applied on the platform of the HVS, the platform rotates while it moves axially up or down. This motion of the platform is called a screw motion here. The screw motion changes the cable length and the height from L_{cp} and h_p to L_c and h. The parameters in a right hand HVS, are found from the followings:

$$h = \sqrt{{L_b}^2 - {r_1}^2 - {r_2}^2 + 2r_1 r_2 \cos\theta} \tag{14}$$

$$L_c = \sqrt{{L_b}^2 + 2r_1 r_2 \cos\theta - 2r_1 r_1 \cos(\theta - \varphi)} \tag{15}$$

At $\theta = \frac{\pi+\varphi}{2}$,

$$h = h_p = \sqrt{{L_b}^2 - {r_1}^2 - {r_2}^2 - 2r_1 r_2 \sin({}^{\varphi}/_{2})} \tag{16}$$

$$L_c = L_{cp} = \sqrt{{L_b}^2 - 4r_1 r_2 \sin({}^{\varphi}/_{2})} \tag{17}$$

In a left hand HVS the geometrical parameters are found from the following formulations.

$$h = \sqrt{{L_b}^2 - {r_1}^2 - {r_2}^2 + 2r_1 r_2 \cos(\theta - \varphi)} \tag{18}$$

$$L_c = \sqrt{{L_b}^2 + 2r_1 r_2 \cos(\theta - \varphi) - 2r_1 r_1 \cos(\theta)} \tag{19}$$

at $\theta = \pi + \frac{\pi+\varphi}{2}$

$$h = h_p = \sqrt{{L_b}^2 - {r_1}^2 - {r_2}^2 - 2r_1 r_2 \sin({}^{\varphi}/_{2})} \tag{20}$$

$$L_c = L_{cp} = \sqrt{{L_b}^2 - 4r_1 r_2 \sin({}^{\varphi}/_{2})} \tag{21}$$

As it was expected, the height and the length of the cable in RH and LH designs are equal (compare Equations (16),(20) and (17),(21). One of the useful differences (resulted from Eqn.(14) & Eqn.(18) between RH and LH designs is that for equal axial motion of the platform, it rotates by the same angle but in opposite directions. This useful characteristic is used in the design of the VS by using one right hand HVS and one left hand HVS attached together at the platform. For example, in the VS (Figure 6), an axial force applied to the upper base moves the upper base upwards but does not rotate it. Consider another example, when a moment is applied on the upper base of the VS while the bottom base is fixed as shown in Figure 7. Under this condition, the upper base of VS rotates but does not move axially. These pure translation or pure rotational motions of the upper base facilitate the use of VS in standard vibration applications.

Infinitesimal mechanism of HVS

Small motions of the platform (differential motion) around the original configuration do not change the length of the cable. This is mathematically shown by noting that the derivative of L_c with respect to θ (Eqn.(15) and Eqn.(19)) vanishes at the original configurations of HVS (When $\theta = \frac{\pi+\varphi}{2}$ for RH and $\theta = \pi + \frac{\pi+\varphi}{2}$ for LH). This implies that in small motions, the elastic stiffness of the cable has minimal impact on the overall stiffness. In other words, the overall stiffness is highly controllable only though changing the prestress.

Small motions, such as this differential motion, that do not change the lengths of the links are known as an “Infinitesimal Mechanism” or “Infinitesimal Flex” in the literature [5].

Stiffness of the VS

Stiffness of the VS (Figure 1.a) is the relation between the external load applied at the upper base and the displacement of the upper base, while the lower base is fixed. Two types of stiffness can be considered for the VS: translational stiffness when $M_z = 0$ and the base moves axially; and rotational stiffness when $F_z = 0$ and the upper base turns under pure moment applied about the axis (Figure 7).

The VS (Figure1.a) is a serial arrangement of a right hand and left hand HVS with equal stiffness. As a result, the stiffness of the VS is half that of the HVS. In the followings, without loss of generality, only the RH design (Figure 2) and its related kinematics equations Eqn.(14),(15) are considered for stiffness derivations.

Translational Stiffness of a right hand HVS

As was shown previously, a right hand HVS is a prestressable mechanism. It means at the original configuration of a right hand HVS (where $\theta = \frac{\pi+\varphi}{2}$), prestress forces τ_{bp} and τ_{cp} can exist in the bar and the cable. It will be shown that this prestresses generate a stiffness called prestress or antagonistic stiffness. Under a pure axial force along the guide (Figure 6), the platform moves down to a new equilibrium configuration and the forces of the bar and cable change from to τ_{bp} and τ_{cp} to τ_b and τ_c, respectively. At this new equilibrium, the translational stiffness of the right hand HVS (K_T) is found by:

$$K_T = \frac{dF_z}{dh} \tag{22}$$

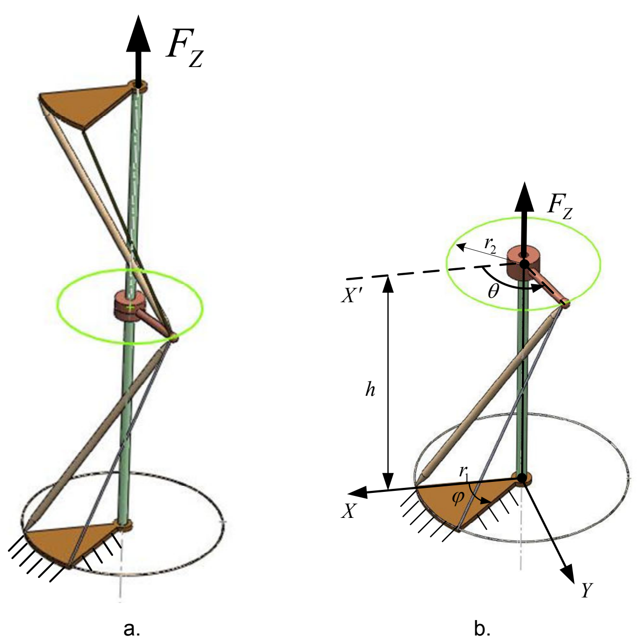


Figure 6. a. Translational VS b. Translational right hand HVS

Where dF_z and dh are the change in the force applied along the axis (z) and the corresponding displacement, respectively. Using the equilibrium equation, the axial force (F_z) can be expressed in terms of the geometrical parameters and the tension of the cable using Eqn.(7).

$$F_z = \frac{h\,(\sin\theta + \sin(\varphi-\theta))}{L_c \sin\theta}\,\tau_c \tag{23}$$

The tension of the cable (τ_c) depends on the elasticity of the cable (k_c), the current length of the cable L_c (Eqn.(15)) and the natural length of the cable L_N:

$$\tau_c = k_c(L_c - L_N) \tag{24}$$

The translational stiffness (K_T) is then found from Eq(22) by using chain rule and using equations (14),(15),(23),(24):

$$K_T = \frac{dF_z}{dh} = \frac{dF_z}{d\theta}\Big/\frac{dh}{d\theta} = K_{T_{\tau_c}} + K_{T_e} \tag{25}$$

where $K_{T_{\tau_c}}$ and K_{T_e} are :

$$\begin{aligned} K_{T_{\tau_c}} = \tau_c \frac{1}{{L_c}^3\, r_1 r_2 sin^3(\theta)} & \{{L_c}^2\, r_1 r_2 sin^3\,\theta \\ -{L_c}^2\, r_1 r_2 sin^2(\theta) sin(\theta-\varphi) &+ h^2 {L_c}^2 sin(\theta)\cos(\theta-\varphi) \\ -h^2\, r_1 r_2 sin^3\,\theta &+ 2h^2\, r_1 r_2 sin^2\,\theta sin(\theta-\varphi) \\ -h^2\, r_1 r_2 sin^2(\theta-\varphi) sin\,\theta &- h^2 {L_c}^2 \cos(\theta) sin(\theta-\varphi)\} \end{aligned} \tag{26}$$

$$K_{T_e} = k_c \left\{\frac{h\big(sin(\theta) + sin(\varphi-\theta)\big)}{L_c sin(\theta)}\right\}^2 \tag{27}$$

$K_{T_{\tau_c}}$, termed the “translational cable force stiffness”, is a function of the cable force (τ_c) and change of the geometry. K_{T_e}, termed the “translational elastic stiffness”, is a function of elasticity of the cables (k_c) and the geometry.

It is notable that at original configuration of a right hand HVS , where $\theta = \frac{\pi+\varphi}{2}$, translational elastic stiffness (K_{T_e}) is zero(Eqn. (29)). This means that at this configuration the stiffness is not a function of elasticity of cable. It only depends on the translational cable force stiffness ($K_{T_{\tau_c}}$). In this case, since the cable force is only the prestress (τ_{cp}), this stiffness is a function of geometry and prestress and is called the “prestress stiffness” or “antagonistic stiffness” K_{T_p}(Eqn.(28)). Following the same argument on the left hand HVS (Equations (18),(21)), similar results are found. The elastic stiffness of the left hand HVS at its original configuration ($\theta = \pi + \frac{\pi+\varphi}{2}$) is zero. In addition, the prestress stiffness of a left hand HVS is the only source of stiffness at its original configurations. It is also equal to the prestress stiffness of a right hand HVS at its original configuration (Eqn.(28)).

$$K_{T_{\tau_c}}\Big|_{\theta=\frac{\pi+\varphi}{2}} = K_{T_p} = \frac{2\sin\left(\frac{\varphi}{2}\right) {h_p}^2}{r_1 r_2 L_{cp} \cos^2\left(\frac{\varphi}{2}\right)}\tau_{cp} \tag{28}$$

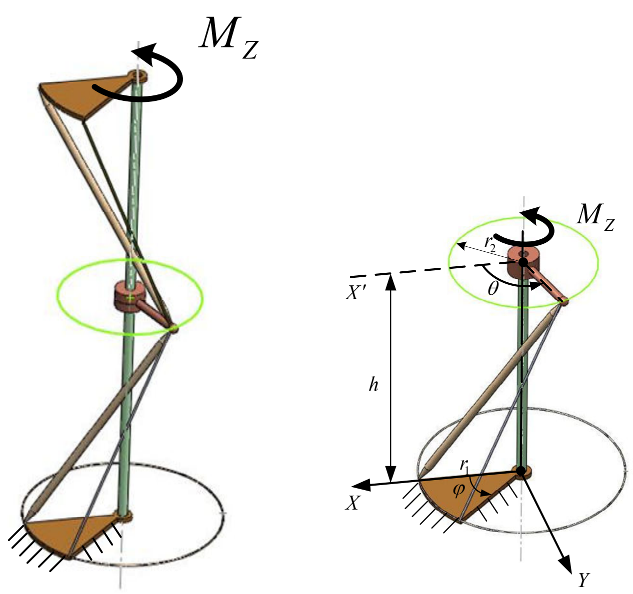
$$K_{T_e}\Big|_{\theta=\frac{\pi+\varphi}{2}} = 0 \tag{29}$$

In summary, at original configuration the stiffness of a HVS comes only from prestress stiffness (K_{T_p}), and this depends on angle φ and the cable prestress. This important result indicates that HVS can be designed as a variable stiffness spring in which the stiffness changes with cable prestress level.

Rotational Stiffness of a right hand HVS

Under a pure moment ($F_z = 0$) about the axial axis, the platform turns to a new equilibrium configuration and the force of the bar and cables change from to τ_{bp} and τ_{cp} to τ_b and τ_c, respectively. At this new equilibrium, the rotational stiffness of the right hand HVS (Figure 7.b) is found by:

$$K_R = \frac{dM_z}{d\theta} \tag{30}$$



a. b.

Figure 7. a. Rotational VS b. Rotational right hand HVS

First, the pure moment (M_z) is found from Eqn.(7):

$$M_z = -\frac{r_1 r_2 \left(\sin\theta + \sin(\varphi - \theta)\right)}{L_c}\tau_c \tag{31}$$

Then by using Equations.(15) and (24), L_c and τ_c are expressed in terms of variable θ. Applying Eqn.(31) to the result gives the rotational stiffness of the right hand HVS (K_R):

$$K_R = \frac{dM_z}{d\theta} = K_{R_{\tau_c}} + K_{R_e} \tag{32}$$

where $K_{R_{\tau_c}}$ and K_{R_e} are :

$$\begin{aligned} K_{T_{\tau_c}} = \tau_c \frac{r_1 r_2}{{L_c}^3} \{ & -{L_c}^2\cos(\theta) + {L_c}^2\cos(\theta - \varphi) \\ & - r_1 r_2 sin^2(\theta) + 2 r_1 r_2 sin(\theta) sin(\theta - \varphi) \\ & - r_1 r_2 sin^2(\theta - \varphi)\} \end{aligned} \tag{33}$$

$$K_{T_e} = k_c \left\{\frac{r_1 r_2\left(sin(\theta) + sin(\varphi - \theta)\right)}{L_c}\right\}^2 \tag{34}$$

$K_{R_{\tau_c}}$, termed the "Rotational cable force stiffness", is a function of the cable force (τ_c) and change of the geometry. K_{R_e}, termed the "Rotational elastic stiffness", is a function of elasticity of the cables (k_c) and the geometry.

It is notable that at original configuration, where $\theta = \frac{\pi+\varphi}{2}$, rotational elastic stiffness (K_{R_e}) is zero and the stiffness originates only from the translational cable force stiffness ($K_{R_{\tau_c}}$). Under this condition, since the cable force is only the prestress of the cable (τ_{cp}), the stiffness is called the "prestress stiffness/ antagonistic stiffness" K_{R_p}.

$$K_{R_{\tau_c}}\Big|_{\theta=\frac{\pi+\varphi}{2}} = K_{R_p} = \frac{2\sin\left(\frac{\varphi}{2}\right) r_1 r_2}{L_{cp}}\tau_{cp} \tag{35}$$

$$K_{T_e}\Big|_{\theta=\frac{\pi+\varphi}{2}} = 0 \tag{36}$$

Translational and Rotational Stiffness of a VS

The translational stiffness and rotational stiffness of the VS are half of the translational stiffness (K_T) and rotational stiffness (K_R) of the left hand side HVS, respectively. The big advantage of the VS over the HVS is the simple motion of the upper base in the VS under pure axial force or pure rotational moment. In the HVS, the motion of the platform is always a combined rotation and translation, while in the VS, the upper base only moves along the axis under translational force and only turns under pure rotational moment. On the other hand, the HVS has a more compact size and higher stiffness, which may be more favorable in particular applications.

In vibration control applications, VS serves as a variable stiffness part. The dead weight of the mass (if exists) can be balanced by the low damping and low stiffness passive mount (e.g. rubber mount). Because of balancing the dead weight, the VS will work at the original configuration. (see case study of [3] as an example) As a result, among the several equation derived above, only two short equations need to be used for design. These equations are the translational and rotational prestress stiffness of the VS (Eqn.(28) and Eqn.(35)).

Stability

The HVS and the VS are stable as long as they have positive stiffness and cables are in tension. At the original configuration (no external loads), as long as the prestress of the cables are tensile ($\tau_{cp} > 0$), the HVS and the VS have positive stiffness (see Equations (28),(35)) and consequently they are stable. Consider an HVS, when external loads are applied and the configuration of the HVS changes from the original configuration (e.g. in right hand HVS $\theta = \frac{\pi+\varphi}{2}$) to a new equilibrium (new θ) found from Eqn.(7). Throughout the change of configuration under external loads the cable force changes from τ_{cp} to τ_c. The stiffness at this new equilibrium is found from general stiffness equations: Eqn.(25) for right hand HVS and from Eqn.(32) for left hand HVS. If at this new equilibrium, stiffness and the cable force are both positive, the HVS is stable. If the cable force becomes negative, the cable becomes slack and the mechanism collapses. Increasing the prestress of the cable postpone the slackness and increase the working range of the HVS and similarly the working range of the VS.

Summary

It was shown that the proposed mechanism (VS) is a force controlled variable stiffness spring with simple translational and rotational motion. The VS consists of two prestressed mechanisms (called HVS) assembled in a serial configuration. These two HVS's are mirror images of one another. They are prestressed mechanisms with one screw-like first order infinitesimal mechanism.

The stiffness of the VS can be controlled by the tension of the cables. At the original configuration (unloaded configuration) of the VS, where the VS has zero elastic stiffness, the stiffness of VS is merely determined by the prestress level in the cables and bars. Therefore force control on these elements, implemented by force actuators, effectively change the stiffness. At original configuration the VS is stable under external loads, as long as the stiffness is positive and the cables are in tension. Prestressing the cables increases the stiffness, and increases the stable working range of the VS.

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