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Actuation of kagome lattice structures

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The kagome lattice has been shown to have promise as the basis of active structures, whose shape can be changed by linear actuators that replace some of the bars of the lattice. As a preliminary examination, this paper examines the effect of the actuation of a single bar in a large two-dimensional kagome lattice. Previous work has shown that interesting properties of the kagome lattice depend on the bars that are co-linear with the actuated bar being straight, but has also shown that actuation causes these bars to bend; this paper therefore explores the geometrically non-linear response of the structure. Numerical results show that due to geometrically non-linear effects, the actuation stiffness is reduced from that predicted by linear models, while the peak elastic strain in the structure is increased.

I. Introduction

Recent work has shown the feasibility of manufacturing structures that on a micro-scale are repetitive trusses, and there is now interest in using these repetitive trusses as the basis for *active* structures.[1,2] Individual actuators replace some of the members of the truss: altering the length of these actuators changes the macroscopic shape of the structure.

In two dimensions, the kagome truss (Fig. 1) has been shown to be a promising solution for these structures.[3] It has been shown to be one of the few periodic, planar, single length scale lattice topologies that has optimal passive stiffness. At the same time, if considered as pin-jointed, any bar can be actuated without resistance. Although practical micro-scale structures will necessarily be rigid-jointed, the additional resistance to actuation from bar bending is small providing that the members are slender. The rigid-jointed planar Kagome lattice therefore has the required properties for use in high authority shape morphing structures; namely passive stiffness and low resistance to actuation.

As a preliminary investigation, this paper will examine the effect of the actuation of a single bar in a large two-dimensional kagome lattice, and will consider how the *stockiness*, s, of the members affects the response. The stockiness is a non-dimensional measure of the aspect ratio of each bar, defined as the ratio of the in-plane radius of gyration of the cross-section, k, to the length of the member, L. Practical structures have s in the range from 0.005 to 0.05, which is the range investigated here. Previous work[4] has examined the energy required to actuate a single bar in a large two-dimensional lattice by considering various *linear* analytical and computational models. However, the special properties of the kagome lattice are dependent on its geometry, and actuation imposes large geometric deformations. This paper therefore builds on the previous work by considering the *geometrically non-linear* response of the structure. The resistance of the structure to actuation will be considered, as well as the limitation on actuation imposed by material yield; the limiting actuation strain will be calculated for various values of material yield strain.

The paper is structured as follows. Section II will describe the computational model used, and Section III will describe some generic features of the response of the kagome lattice to the actuation of a single bar. Section IV then goes on to explore the build-up of force in the actuated member as the bar is actuated. Section V explores the peak elastic strain anywhere in the structure due to the imposed actuation, and shows how the actuation strain is limited by yielding of the structure for varying values of stockiness and yield strain.

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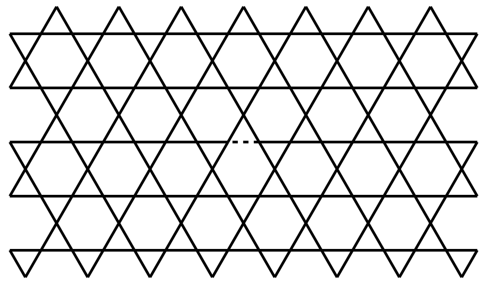

Figure 1: A kagome lattice. This paper assumes that the structure is very large, and considers the effect of lengthening a single bar, shown dashed. The computational model described in this paper has approximately 40 times as many bars as the fragment shown here.

II. Computational model

The finite element package Abaqus[5] was used to model the structure in two dimensions. The lattice considered is rectangular, with a width of 100 times the individual member length L, and a height of $60 \times L\sqrt{3}/2 \approx 52L$; the aim was to make the lattice large enough that edge effects were not significant. To check this, two sets of simulations were carried out, one with all boundary nodes fixed, and one with all boundary nodes but one free; the one fixed node was used to suppress rigid-body modes. The fixed and free boundary cases provide bounds on the true behaviour of a very large lattice. A single lattice member was actuated; it lies at the centre of the structure, and is parallel with the longer dimension of the structure. The bar was actuated by raising its temperature, causing it to elongate. Each bar has a circular cross-section of radius r. The material properties are assumed to remain linear, with a Young's modulus E, giving an axial stiffness EA, where $A = \pi r^2$, and a bending stiffness EI, where $I = Ak^2$, and $k = r/2$; the actuated bar is assumed to have the same elastic properties as every other bar. Each bar of the structure is modelled with four 2-node Euler-Bernoulli beam elements; test cases showed that using a higher number of elements per bar gave results that were indistinguishable from those reported here. No consideration is given to the actual size of a joint between bars, which is assumed to be at a point.

Results were calculated for eleven different bar radii, giving results for stockiness in the range $s = 0.005$ to $s = 0.05$. These stockiness ratios are chosen as they cover the range of interest for practical applications.

In order to check that the two-dimensional analyses were not suppressing an out-of-plane buckling mode, one full three-dimensional analysis was carried out. This was done for the most slender structure, $s = 0.005$, with small random imperfections introduced in the out-of-plane direction. The results were indistinguishable from the two-dimensional case, and we concluded that out-of-plane deformations did not have to be considered.

III. Mode of deformation

The essential features of the deformation of the kagome lattice due to the actuation of a single bar are the same for all values of stockiness and actuation strains that we have considered; Fig. 2 shows the small-deformation mode (greatly magnified) for two different values of stockiness. It can be seen that deformation takes place primarily along a narrow corridor parallel with the member being actuated.

Reference 4 described analytical models where the deformation was *constrained* to lie within a narrow corridor parallel with the actuated bar. These models considered linear-elastic small deformation behaviour, and showed that in this case the deformation dies away exponentially from the actuated bar, with a decay constant proportional to the stockiness. This simple picture is somewhat complicated by the geometric non-linearities considered in this paper. For actuation strains that cause significant deformation, the numerical results show that the decay in actuation with distance from the actuated bar is no longer exponential, and deformation begins to localise close to the actuated bar. However, in none of the cases reported in this paper is the effect large, and the deformation modes would be difficult to distinguish from those plotted in Fig. 2.

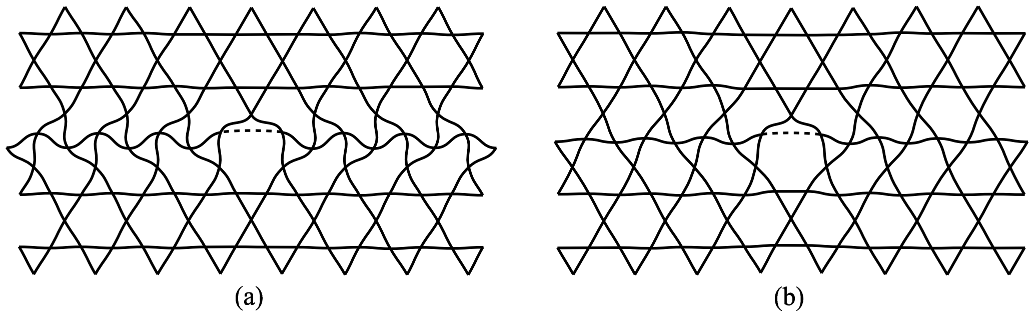

Figure 2: The shape of the central portion of a large kagome truss after a single bar (shown dashed) is lengthened. In this case the calculation assumed small deformations; the deformations shown here are greatly magnified for clarity. The width of the bars is not to scale. (a) Stockiness ratio $s = 0.005$. (b) Stockiness ratio $s = 0.045$. Note that the displacement attenuates more quickly in (b).

IV. Actuation forces

This section will consider the force, F (defined as compression positive), developed in the actuated bar as it is lengthened. For small deformations, when the bar is approximately straight, F is nearly constant along the length of the actuated bar; for larger deformations we define F to be an average of the varying force $F(x)$ along the bar, $F = \int_0^L F(x)\,dx/L$.

In order to present the results in a general format, we define a non-dimensional force $\hat{F}$,

$$\hat{F} = \frac{F}{EA}, \tag{1}$$

where EA is the axial stiffness of the actuated bar. We describe the actuation in terms of an *actuation strain*, ε_a, where ε_a is the strain that the bar would experience if it were unconstrained; it is defined as being positive when the bar gets longer. (In fact, the actuator extends by less than ε_a, as it also compresses elastically due to the force F).

Figure 3 shows the evolution of $\hat{F}$ with the actuation strain, ε_a. As may be expected, a structure made of stockier bars develops a larger axial force in the actuator than a structure made with less stocky bars — the bending stiffness that resists the deformation shown in Fig. 2 is larger for the stockier bars. The $s = 0.005$ case shows considerable non-linearity, with $\hat{F}$ reaching an approximate plateau at small actuation strains of the order of 10%.

We define the actuation stiffness, K, to be the rate of change of force with actuation strain,

$$K = \frac{dF}{d\varepsilon_a}. \tag{2}$$

K has units of force, as it is a stiffness measured with respect to a strain rather than a displacement; we can non-dimensionalise K with respect to the axial stiffness of the actuated bar, EA,

$$\hat{K} = \frac{K}{EA}. \tag{3}$$

$\hat{K}$ is the slope of the graph shown in Figure 3. It is clear that as actuation proceeds, the actuation stiffness decreases, and this agrees with a comment in Ref. 4: as actuation proceeds, the line of bars containing the actuator is bent (see Fig. 2), and this is likely to lead to more bending-dominated behaviour, and hence a geometrically non-linear analysis will show a consequent reduction in stiffness.

Figure 4 shows how the actuation stiffness varies with stockiness, and also how the tangent stiffness evolves as the structure is actuated. As ε_a increases from zero, it can be seen that there is a reduction in stiffness.

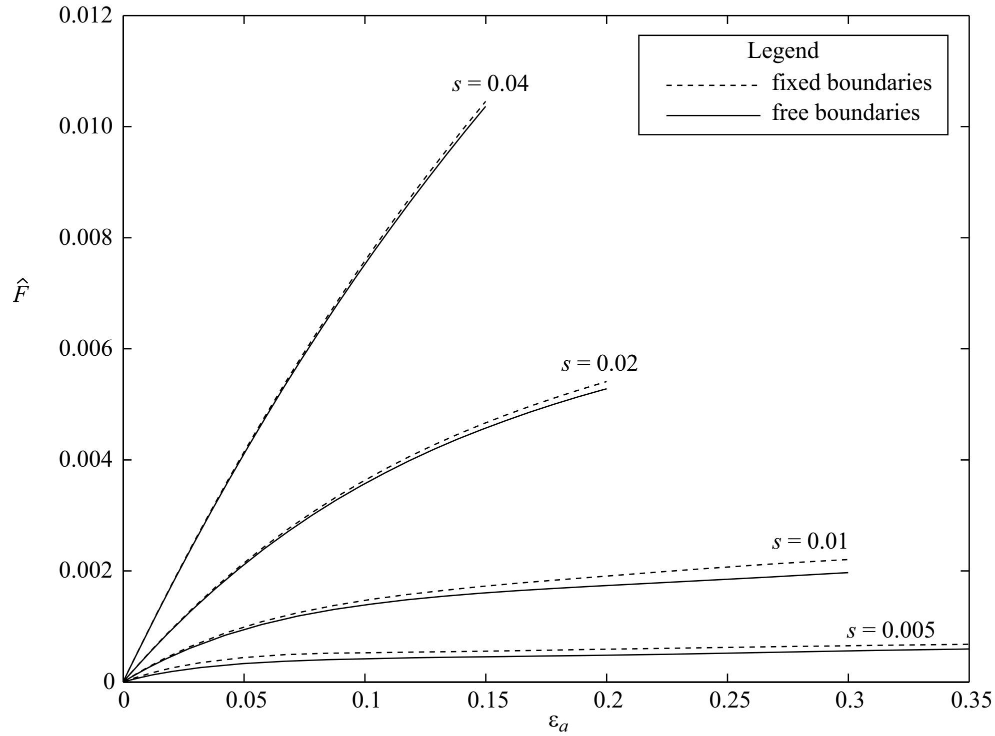

Figure 3: The non-dimensional force $\hat{F}$ developed in the actuated bar, when it has been actuated by a strain ε_a, for four different values of stockiness s. In each case, two sets of results are presented, one where the boundaries are fully fixed, and one where the boundaries are unconstrained

It is straightforward to calculate the work required for actuation if the dimensions of the actuated bar are assumed to remain constant. If an increment of actuation strain, $\delta\varepsilon_a$, is added, the work done by the actuation, δW, is

$$\delta W = FL\delta\varepsilon_a \tag{4}$$

and hence the total work done in actuating the bar from the rest state is

$$W = \int_0^{\varepsilon_a} F(\varepsilon)L \, d\varepsilon. \tag{5}$$

In the linear case, we can substitute $F = K\varepsilon$ into (5), and hence

$$W = \int_0^{\varepsilon_a} KL\varepsilon \, d\varepsilon = \frac{K\varepsilon_a^2 L}{2}. \tag{6}$$

Reference 4 defined a non-dimensional energy penalty $\hat{W}$ by comparing the work done to actuate the bar, W, with a reference energy, the work done by the actuated bar if the rest of the structure was rigid, $W_0 = (AEL/2)\varepsilon_a^2$. For the linear case

$$\hat{W} = \frac{W}{W_0} = \frac{K}{EA} = \hat{K}. \tag{7}$$

Thus, in the linear case, the non-dimensional actuation stiffness, $\hat{K}$ is exactly the energy penalty $\hat{W}$. For $\varepsilon_a = 0$, the results shown in Fig. 4 reproduce the results for $\hat{W}$ reported in Ref. 4: the actuation stiffness scales with s, a result characteristic of the kagome lattice. (A stretching dominated structure would show that $\hat{K}$ did not vary with s, while a bending dominated structures would show $\hat{K}$ scaled with s^2).

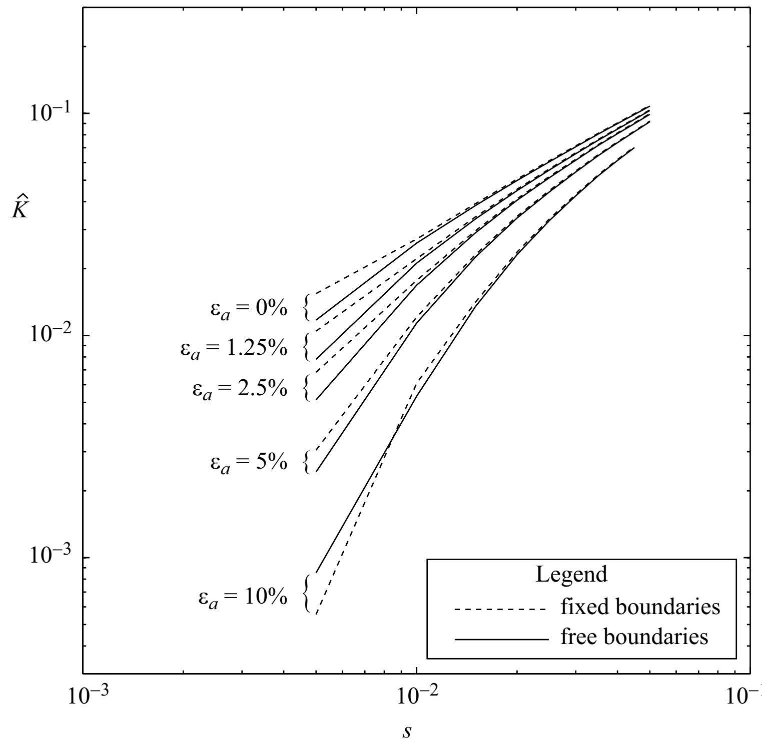

Figure 4: The non-dimensional actuation stiffness, $\hat{K}$ of a large kagome lattice, shown for different values of stockiness s, and different values of actuation strain ε_a.

V. Limits on actuation due to yield

A fundamental limitation on the actuation of a structure is that yielding anywhere in the structure must be avoided. This section reports numerical results for the peak actuation strain that can be achieved in a large kagome lattice.

Figure 5 shows the actuation strain, ε_a, at which yield first occurs for various values of yield strain, ε_y, in the material. In each case, yield first occurred in the bars adjacent to the actuated bar, as shown in Fig. 6.

For larger values of stockiness, and smaller values of yield strain, the curves shown in Fig. 5 would collapse onto a single master curve if $\varepsilon_a/\varepsilon_y$ was plotted. However, significant deviation from this is seen for large strains, $\varepsilon_y = 2\%$, in the most slender case considered, $s = 0.005$. This is the case where the largest geometric changes occur, and hence it is not surprising that the effect of geometric non-linearity is greatest here. The effect of the non-linearity is to reduce the estimate of the peak actuation that can be imposed, and hence it is important to consider these non-linear effects for large actuation of slender lattices.

VI. Conclusions

Many of the important aspects of the mechanics of kagome lattices depend on the line of bars co-linear with the actuated bar being initially straight;[4] however, actuation causes these bars to bend, as may be seen clearly in Fig. 2. This paper has given some insight into the effect that this evident geometric non-linearity has on the performance of actuatable kagome lattices.

The clearest effect of the geometric non-linearity has is on the force that is developed in the actuator. Figure 4 shows the large reduction in actuation stiffness that occurs as ε_a increases. In many ways this could be considered to be a positive effect; relatively, it becomes easier to actuate the structure as deformation increases.

The geometric non-linearity seems to have a lesser effect on the results for the peak strain in the structure caused by the actuation. Only for the largest actuations considered, for the structure made from the most slender bars, does the value of the peak strain depart significantly from the pattern seen for structures made

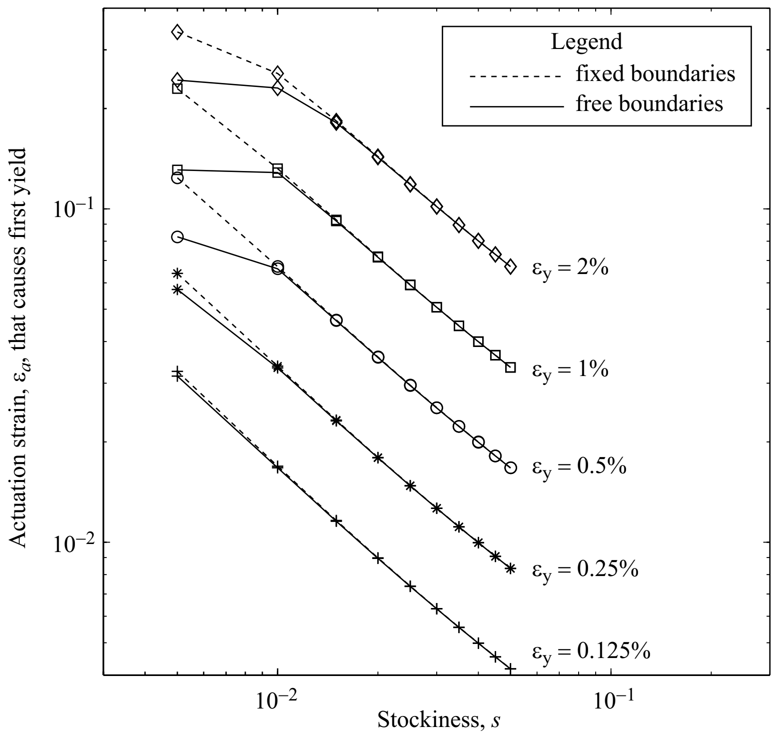

Figure 5: The actuation strain ε_a at which first yield occurs in a large kagome lattice, for various values of stockiness, s, and yield strain, ε_y.

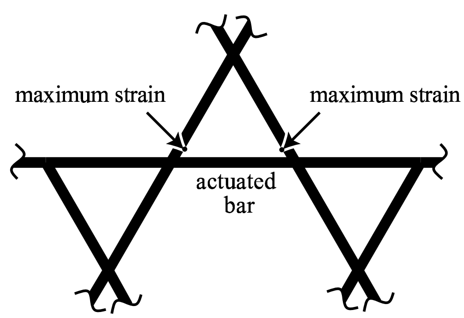

Figure 6: The position of the peak strain in the kagome lattice due to the actuation of a single bar. The peak strain occurs in the bars adjacent to the actuated bar, at the junction with the actuated bar.

from more stocky bars at lower actuation strains. In this case, however, the effect of geometric non-linearity is detrimental, reducing the actuation strain that can be achieved before yield.

There are a number of limitation in the present study that could be improved in future work. All of the results for the case $s = 0.005$ show some divergence between the analysis done with free boundaries, and that done with fixed boundaries. In this case, it is clear that the lattice considered needs to be larger to achieve convergence. Also, there are two detailed aspects of the modelling that are likely to be important but have not been included: no consideration has been given to the actual joint geometry, which may have a significant impact on the peak stresses that occur; also, the effect of imperfections in the construction of the lattice is likely to have a significant impact on the actuation stiffness.

Acknowledgments

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References

[1]dos Santos e Lucato, S.L., Wang, J., Maxwell, P., McMeeking, R.M., Evans, A.G., "Design and demonstration of a high authority shape morphing structure," *International Journal of Solids and Structures* (to be published).

[2]Symons, D.D., Hutchinson, R.G. and Fleck, N.A., "Actuation performance of the kagome double layer grid", *Journal of the Mechanics and Physics of Solids* (submitted for publication).

[3]Hutchinson, R.G., Wicks, N., Evans, A.G., Fleck, N.A. and Hutchinson, J.W., "Kagome plate structures for actuation," *International Journal of Solids and Structures*, Vol. 40, 2003, pp. 6969–6980.

[4]Wicks, N. and Guest, S.D., "Single member actuation in large repetitive truss structures", *International Journal of Solids and Structures*, Vol. 41, 2004, pp. 965–978.

[5]Abaqus, software package, Ver. 6.3, Hibbit, Karlsson & Sorenson, Inc., Pawtucket RI, USA.