Actuation-softening in kagome lattice structures

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The two-dimensional kagome lattice has been shown to be a promising basis for active shape-changing structures, having both low actuation resistance and high passive stiffness. Linear actuators replace some members of the truss: activation of the actuators results in a global macroscopic shape change. The linear behaviour of the structure, and in particular the high passive stiffness, depends crucially on the straight bars running across the structure; but this straightness is destroyed when bars are actuated. The current paper investigates this behaviour by imposing large actuations to create geometrically non-linearity. A column of actuators is introduced into a kagome lattice, so that for every actuator, the actuator directly above and below is also activated; the horizontal stiffness of the system is then measured. Numerical results show that when the actuators are extended, there is a sudden drop in passive stiffness at a 'critical' actuation strain. This critical actuation strain depends on the stockiness of the bars of the lattice, and the stiffness of the actuator itself. For lattices with contracting actuators, the stiffness degradation is gradual, and less severe.

I.   Introduction

Recent studies have shown the two-dimensional kagome lattice, shown in Figure 1, to be a prime candidate to form the backbone of *active* structures.[1–4] If some members of the kagome truss are replaced by linear actuators, then actuation leads to global macroscopic shape changes.

Features of the planar kagome lattice include high passive stiffness and low actuation resistance, and they fulfil the basic functional requirements for an efficient active structure. These properties are 'inherited' from the pin-jointed kagome truss 'parent';[5] where any bar can be actuated without work, due to the presence of mechanisms. Practical kagome structures, with fixed joints and relatively slender members, will carry the parental characteristics of low actuation resistance, while the straight bars at 60° to one another ensure passive stiffness.

Two previous kagome lattice actuation studies are briefly presented as they provide fundamental understanding to the topic:

A.   Small deformation single-member actuation

Wicks and Guest[6] have investigated a simple actuation case, where one member of an infinite kagome lattice is replaced by a linear actuator, shown in Figure 1(a). Within a small deformation linear regime, the kagome lattice has a low resistance to actuation compared with other passively stiff structures, for instance a triangulated truss. The resistance to actuation is shown to scale with the stockiness of the bars. The resistance to actuation is shown to come equally from bending deformation and axial deformation of the bars of the lattice.

B.   Large deformation single-member actuation

The equal division between resistance to actuation from bending and stretching of a kagome lattice, described above, depends critically on the original geometry, and in particular the straight line of bars in line with the
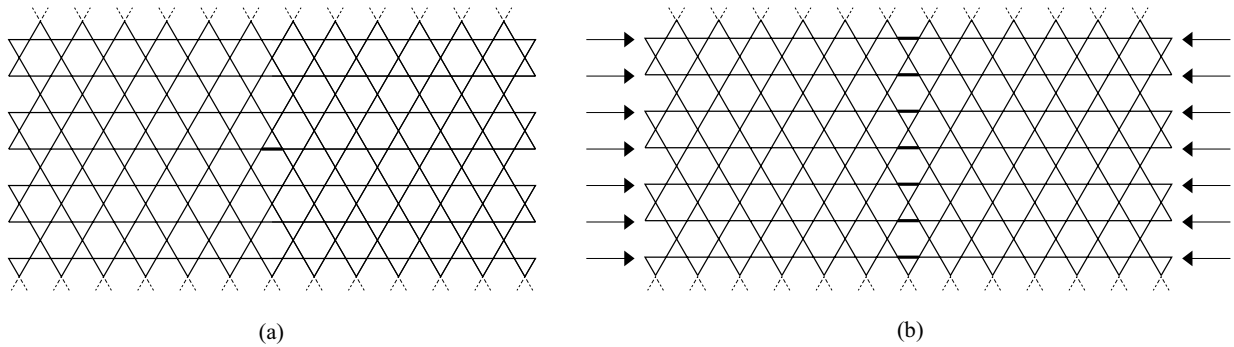
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Figure 1. (a) Part of an infinite kagome lattice model used in single-member actuation analyses, with one thick member in the centre representing an actuator. (b) Part of a semi-infinite kagome lattice model used in multiple-member actuation analyses, with a column of actuators represented by thicker lines. Arrows indicate point loads of equal magnitude applied to the boundary to examine horizontal stiffness.

actuator. However, this geometry is disrupted by actuation. Leung et al.[7] investigated the resistance to actuation in a geometrically non-linear regime. They found that when the actuator lengthened, the resistance to actuation reduced, and the degree of softening was greater for structures made from more slender bars. Conversely, a contracting actuator gave rise to increased resistance to actuation.

The current paper investigates a potentially damaging consequence of non-linearity: its effect on the high passive stiffness of the kagome lattice. To investigate this, the single actuator model is not appropriate, and instead a column of actuators is introduced, as shown in Figure 1(b). The stiffness of the system is measured through the effect of the application of external loadings to examine, numerically, the level of softening, for a range of *actuation strains*, ε_a, and *stockiness*, s. Lattices considered in this paper are of an infinite height and a finite width, with stockiness ranging from 0.001 to 0.05, covering the likely range of practical interest. Actuation strains of up to $\pm 50\%$ are considered, which requires the calculation to be geometrically non-linear.

The paper is structured as follows. Section II will describe the lattice and computational model used, and Section III will describe the influence of stockiness on passive stiffness of the kagome lattice. Section IV will discuss how stiffness of the actuator affects deformation localisation, and Section V will conclude the paper, discussing some salient findings.

II. Computational Model

The finite element package ABAQUS[8] was utilized to model the structure in two dimensions. The model used was designed to represent a semi-infinite lattice, where it has an infinite vertical dimension and a finite horizontal one. The width of the lattice is 80 times the individual member length, L, with each node along the two vertical boundaries subjected to a series of point loads, magnitude P, of opposite directions parallel to the linear actuators. Since it is only the horizontal passive stiffness considered in this paper, the top and bottom boundaries are both mirrored so that a finite portion of grid appears effectively infinite. A vertically infinite model ensures calculations to be free of edge effects. The load-bearing boundary nodes are given one degree of freedom, where they can only displace horizontally. The passive stiffness of the system is calculated by measuring the small displacement due to a small load P.

Excluding the actuators, all bars are of circular cross-section with radius r. The material properties are assumed to be linear, with a Young's modulus E. This gives a bar an axial stiffness of EA, where $A = \pi r^2$, and a flexural stiffness EI, where $I = Ak^2$; k is the in-plane radius of gyration and it equals to $r/2$. The member aspect ratio are quantified by the inverse of *slenderness*, known as stockiness, s, throughout this paper, where $s = k/L$. Stockiness ranging from 0.001 to 0.05 are considered, which covers the likely range of practical interest. Two distinctive types of actuators are considered: *flexible* and *stiff*. A flexible actuator has exactly the same stiffness properties as every other bar in the structure; the stiff actuator is infinitely stiff, and is connected by pin joints to the rest of the structure, so that it does not carry any shear

force or bending moment. Each member in the lattice is modelled with four 3-node Timoshenko (shear flexible) beam elements, apart from the flexible actuator which is modelled with eight elements, and the stiff actuator, which is modelled by imposing an absolute nodal displacement instead of a physical member with any material properties. Numerical experiments showed that this level of discretization is sufficient to capture the behaviour of the system. No consideration is given to the actual size of a joint between bars, which is assumed to be a point.

Actuators are placed in a single column in the centre of the lattices, replacing the ordinary structural members (see Figure 1(b)). Actuation is quantified by the variable ε_a, the actuation strain: we define this strain to be the extension an actuator would experience if unconstrained, divided by its original length. Positive actuation corresponds to lengthening actuators. The actual strain experienced in the flexible actuator, ε, is smaller in magnitude than the actuation strain, as the rest of the structure imposes resistance to create an elastic strain of opposite direction. With a stiff actuator, the actuation strain equals the elastic strain: $\varepsilon = \varepsilon_a$.

III. Stiffness degradation

It is expected that the passive stiffness of a kagome lattice will degrade when the actuators are actuated, as the stiffness will depend more heavily on the bending stiffness of the bars, rather than the axial stiffness. Since the bending stiffness of an individual bar is directly affected by stockiness, we expect stiffness degradation to be most prominent when actuation strain is high and stockiness is low.

In the discussion below, the horizontal passive stiffness, K, is defined as

$$K = P/\delta$$

where P is the magnitude of one externally applied horizontal point load at a boundary node, and δ is the horizontal nodal displacement of the node where P is applied. We define this displacement to be $\delta = x_f - x_i$, where x_i is the post-actuation nodal position and x_f is the post-loading position. δ is therefore the displacement of a boundary node induced by the external loading alone. The measurement of passive stiffness is calculated by a linear step – essentially assuming that the load P is small. To compare results from models of different stockiness, a non-dimensionalised horizontal passive stiffness, $\hat{K}$ is used. It is defined as

$$\hat{K} = K/EA$$

where EA is the axial stiffness of one member in the lattice.

The results will be presented first for the flexible actuator, and then for the stiff actuator.

A. Flexible actuator

For the system with flexible actuators, Figures 2(a) and 3(a) shows how the passive stiffness, $\hat{K}$, of kagome lattices of different stockiness vary with actuation strain, while Figures 2(b) and 3(b) show the boundary nodal displacement response, Δ_x/L, after actuation alone. Figure 2 presents results for positive actuation (lengthening of the actuator), while Figure 3 presents results for negative actuation (shortening of the actuators).

In Figure 2(a), it is visible that under positive actuation there are three phases in the degradation of passive stiffness. The first and last phases consist of gradual lowering of the stiffness with respect to increasing actuation. The second phase is a sudden drop, within a small range of actuation strains. These three phases form a quasi-cubic function, with a calculable point of inflection. These inflection points are marked in Figure 2(a) by asterisk points on the curves. This 'critical' actuation strain indicates a point of vulnerability in the system where a small change in actuation strain severely reduces the passive stiffness. The existence of these critical points can be explained by how the displacement of an end node varies with actuation *before* the external load is applied.

Figure 2(b) plots boundary node horizontal displacement versus actuation strain when no external load is applied. Initially, almost all of the lengthening is transmitted to the boundaries: an actuator lengthening of $\varepsilon_a L$ gives a Δ_x of $\varepsilon_a L/2$ and hence the curve has a slope of $1/2$. However, Figure 2(b) shows an interesting effect under large positive actuation strains. The kagome lattice initially increases in total width with actuation, then peaks at a certain actuation strain, and decreases in width subsequently. These peak values are marked by asterisk points on the curves in Figure 2(b). The peak displacement in Figure 2(b) occurs
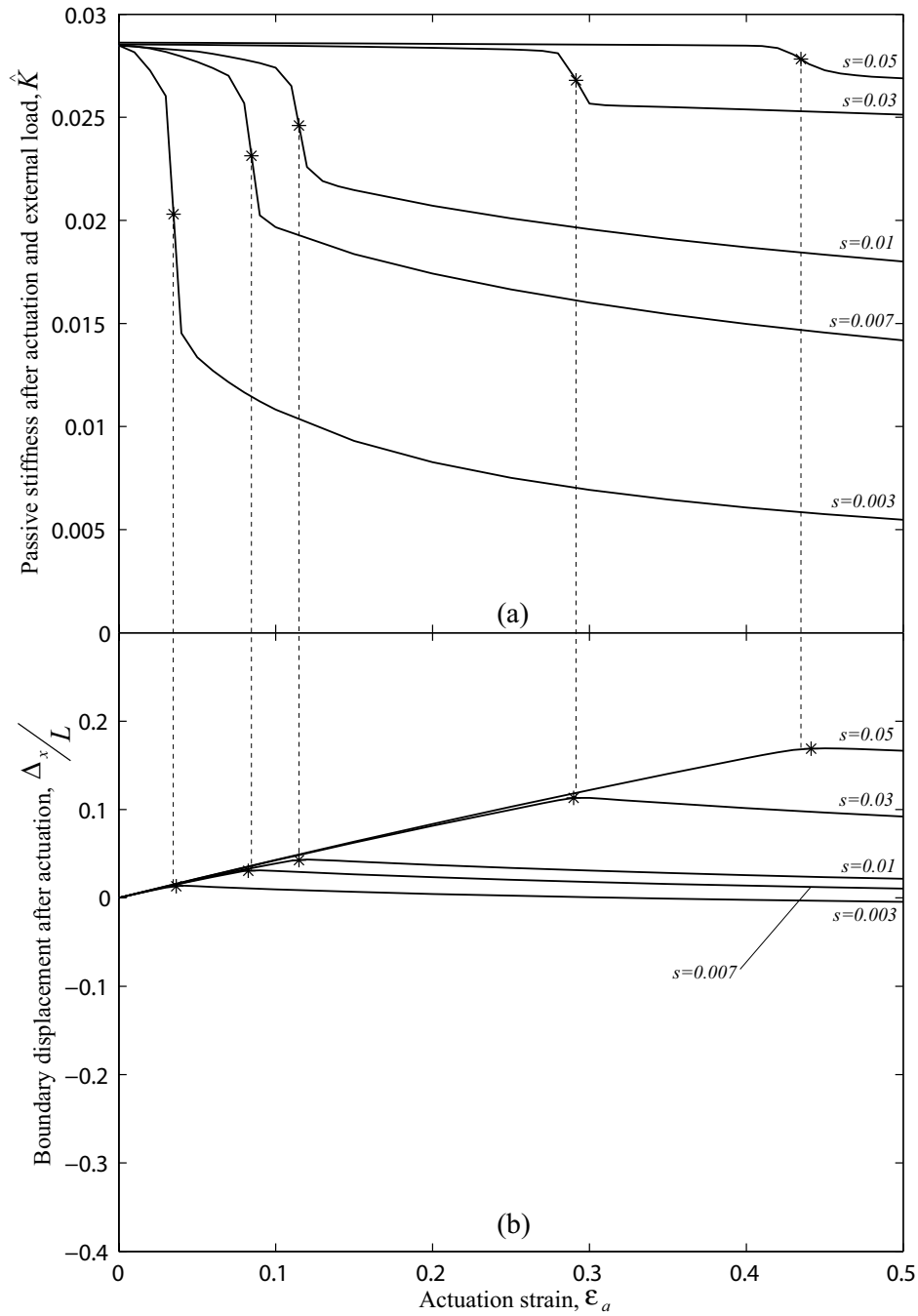
Figure 2. Response of kagome lattices under flexible, expanding actuators for different stockiness s; (a) passive stiffness vs. actuation strain and (b) boundary nodal displacement vs. actuation strain. The asterisks denote points of maximum passive stiffness drop in (a), and in (b) the peak horizontal dimension achieved by actuation. Note that for a specific stockiness, these occurs at the same actuation strain.
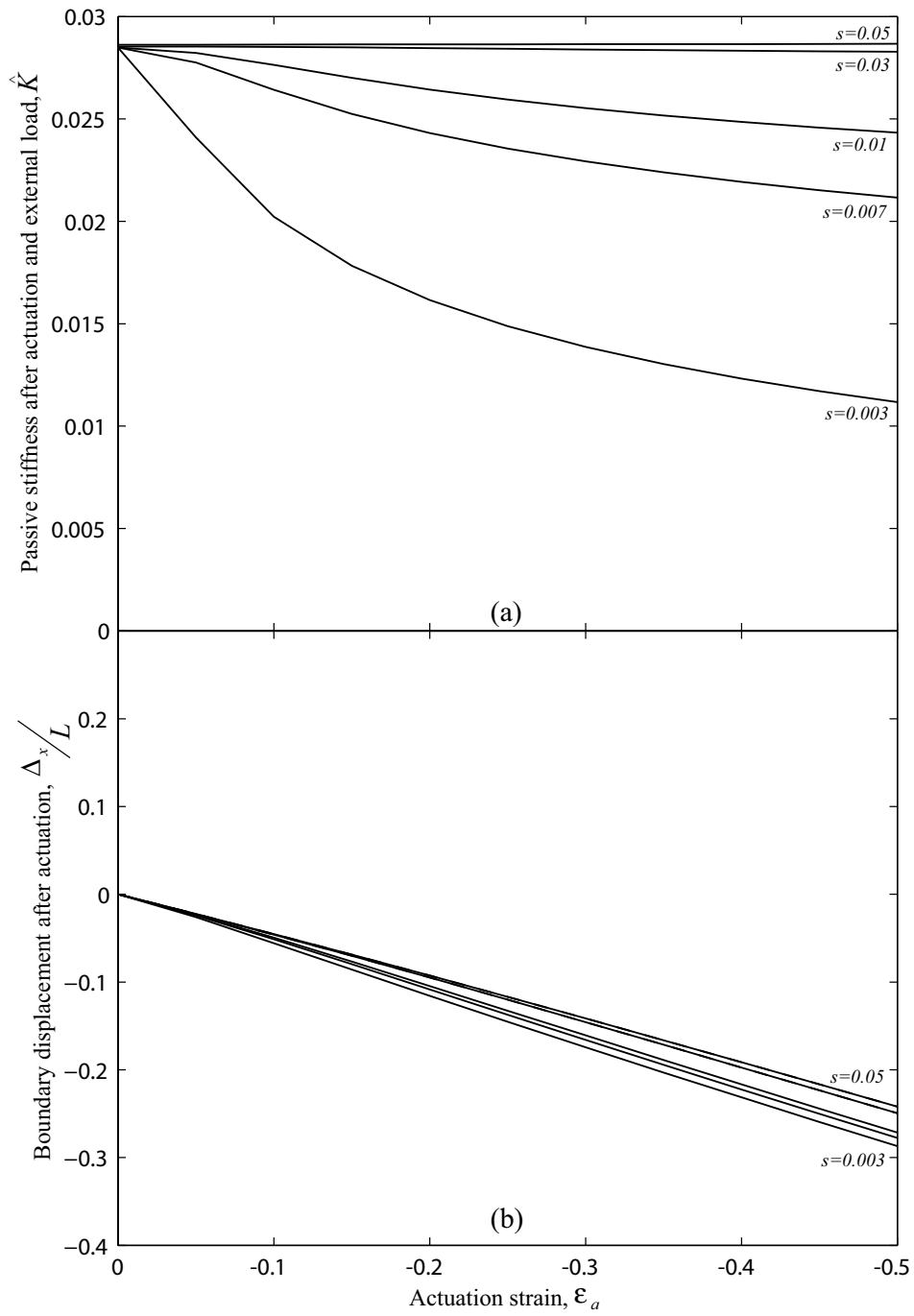
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Figure 3. Response of kagome lattices under flexible, contracting actuators for different stockiness s**; (a) passive stiffness vs. actuation strain and (b) boundary nodal displacement vs. actuation strain.**

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at the critical actuation strain found in Figure 2(a) – and indeed in Figure 2 the two sets of asterisks are connected by dotted lines drawn at constant ε_a. At high actuation, the final width of the lattice can be less than the unperturbed dimension. The unexpected shrinking of the lattice under large actuation is a result of localised flexural deformation, and will be further discussed in later sections.

Figure 3(a) plots the stiffness response under flexible, negative actuation. It can be seen that passive stiffness degrades with increasing actuation strain, but unlike the positive actuation case, softening is gradual. Figure 3(b) shows boundary nodes displace with the contracting actuators in a linear manner, with stockiness having little effect on the response.

B. Stiff actuator

For the system with stiff actuators, Figures 4(a) and 5(a) shows how the passive stiffness, $\hat{K}$, of kagome lattices of different stockiness vary with actuation strain while Figures 4(b) and 5(b) show the boundary nodal displacement response, Δ_x/L, after actuation alone. Figure 4 presents results for positive actuation (lengthening of the actuator), while Figure 5 present results for negative actuation (shortening of the actuators).

In Figure 4(a), it is visible that under positive actuation there is a sudden softening response, similar to that seen for the flexible actuators in Figure 2(a). However, this is only observed in models with stockiness lower than 0.03. For those lattices which soften suddenly, the critical actuation strain is again marked by an asterisk in Figure 4(a).

Figure 4(b) plots boundary node horizontal displacement versus actuation strain when no external load is applied. As in Figure 3(b), initially the slope is approximately $1/2$, but when actuation continues, eventually the boundary displacement ceases to increase and starts to decrease. These peak values are marked by asterisk points on the curves in Figure 4(b).

Figure 5(a) plots the stiffness response for negative actuation for the stiff actuators. Passive stiffness degrades with increasing actuation strain, but unlike the positive actuation case, softening is gradual. Figure 5(b) shows boundary nodes displace with the contracting actuators in a linear manner, with stockiness having little effect on the response.

Comparing Figures 2(a) and 4(a), it can be seen that there is greater stiffness degradation for the stiff actuator case than for the flexible actuator case, although the critical strain occurs at a higher value. This could be because the stiff actuator imposes a larger deformation on the rest of the lattice; although the actuator itself is not bent, there is a larger local zone of deformation as will be seen in the next Section.

Figure 6 shows the 'critical' actuation strain, ε_a^*, at which sudden softening occurs, plotted against stockiness. A slope of approximately one shows that softening resistance of the kagome lattice is approximately proportional to the stockiness of the model. The stiff actuation models show a stronger resistance at the same stockiness compared to the flexible case, with a slope slightly larger than unity.

IV.   Deformation localisation

This section focuses only on the positive actuation results. The sudden drop in passive stiffness observed occurs at the same time as the localisation of deformation imposed by activation. Figure 7 shows the deformed shapes of the two actuation schemes: flexible and stiff, for lattices of stockiness 0.01. It shows the deflected shape immediately before and after the critical actuation strain. It can be seen that within a small range of actuation strain, flexural deflection is quickly developed in a small number of members. For the flexible actuation case, a majority of deformation occurred within the actuator itself, while the stiff actuators induce localised deformation in the members immediately adjacent.

V.   Conclusion

It is observed that as a kagome lattice is actuated, the passive stiffness is reduced. For the contraction of an actuator, this effect is gradual, while for expansion, there is a critical strain where the passive stiffness suddenly drops. This softening effect is more prominent at low stockiness. In all cases, the actuators deform itself and the adjacent co-linear bars. Localised flexural deformation is invoked by activation at high actuation strains: a more flexible actuator creates localisation primarily in itself, while a stiff actuator
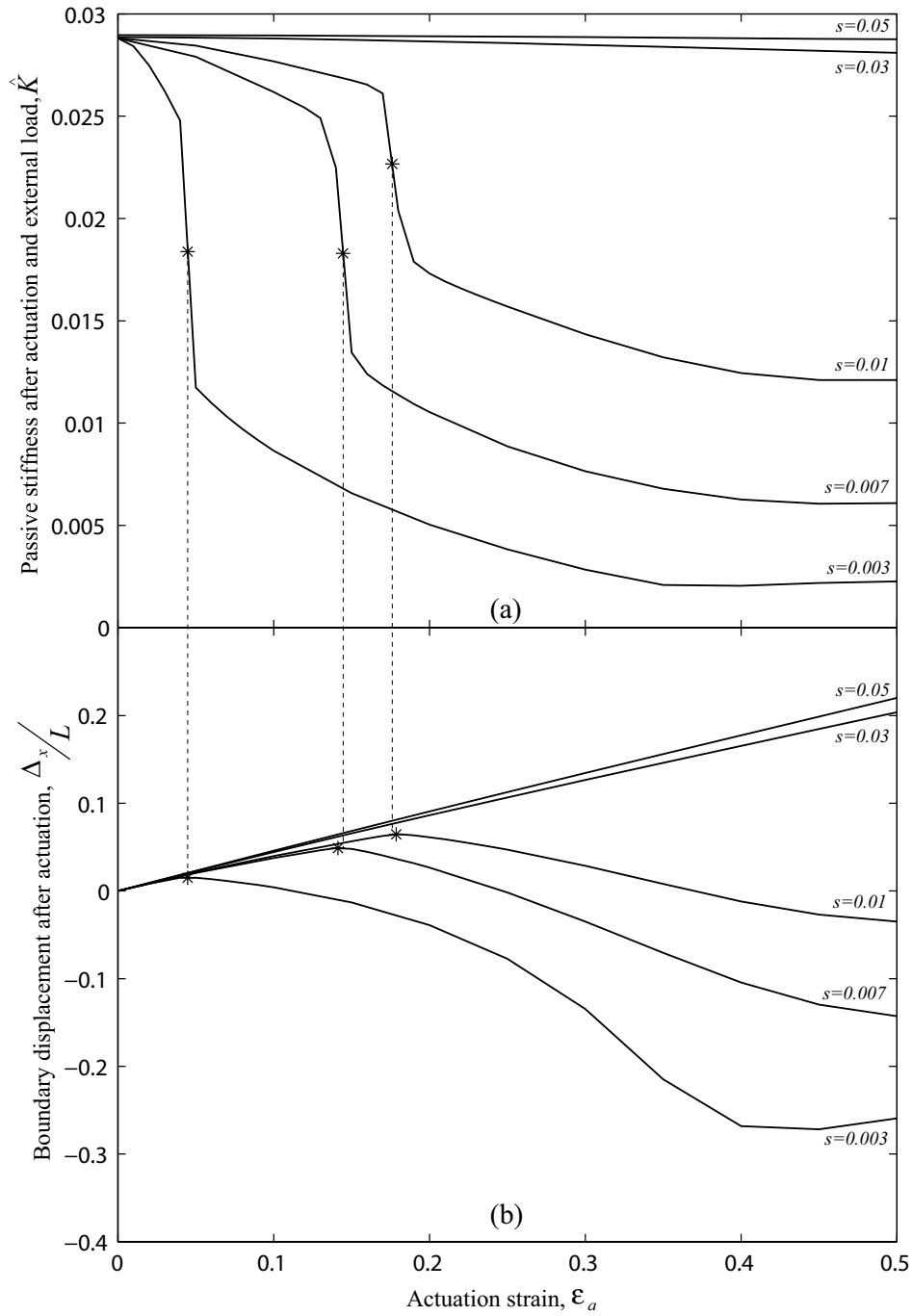
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Figure 4. Response of kagome lattices under stiff, expanding actuators for different stockiness s; (a) passive stiffness vs. actuation strain and (b) boundary nodal displacement vs. actuation strain. The asterisks denote points of maximum passive stiffness drop in (a), and in (b) the peak horizontal dimension achieved by actuation. Note that for a specific stockiness, these occurs at the same actuation strain; such behaviour is absent in models with stockiness 0.03 or larger.
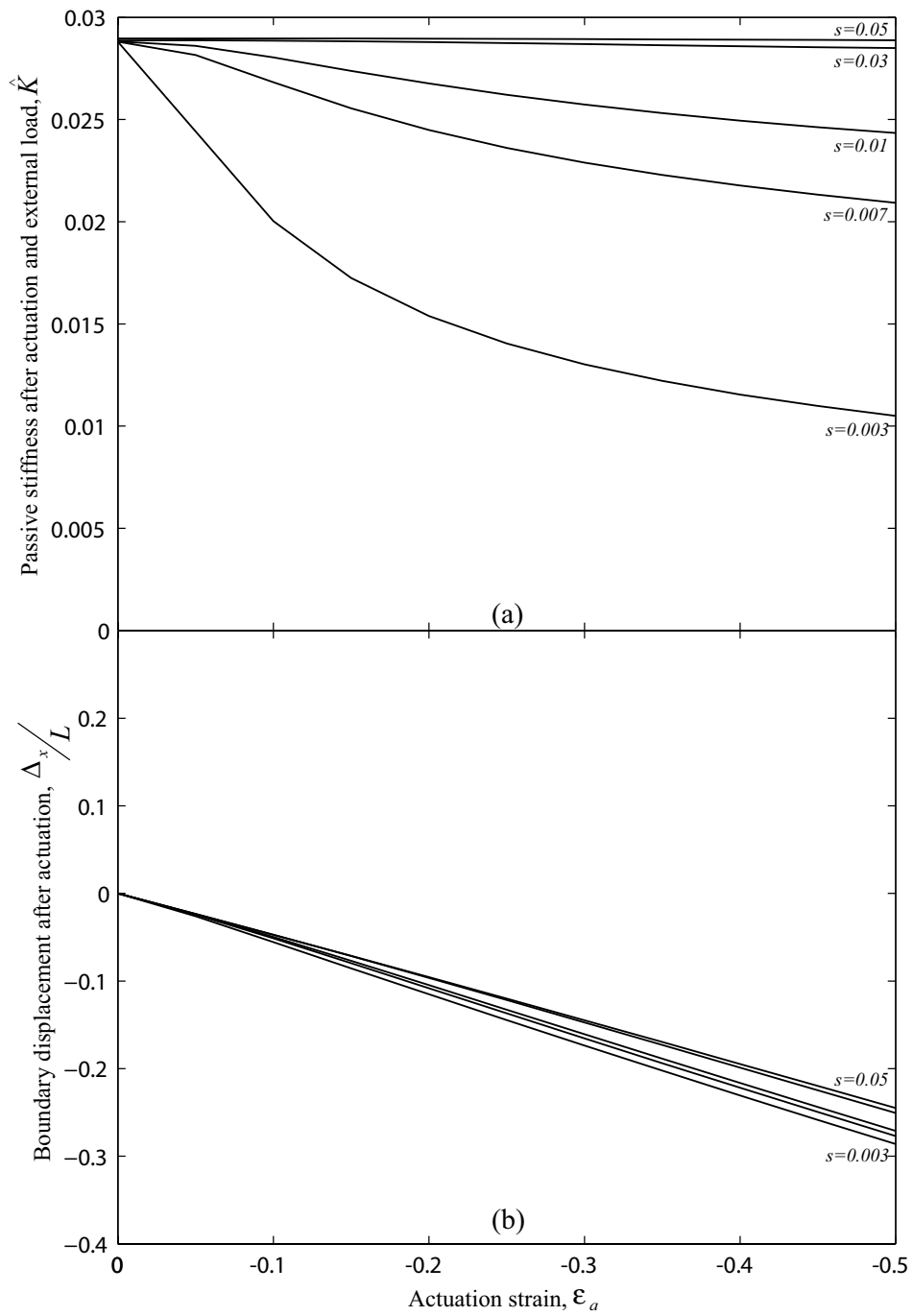
Figure 5. Response of kagome lattices under stiff, contracting actuators for different stockiness s; (a) passive stiffness vs. actuation strain and (b) boundary nodal displacement vs. actuation strain.
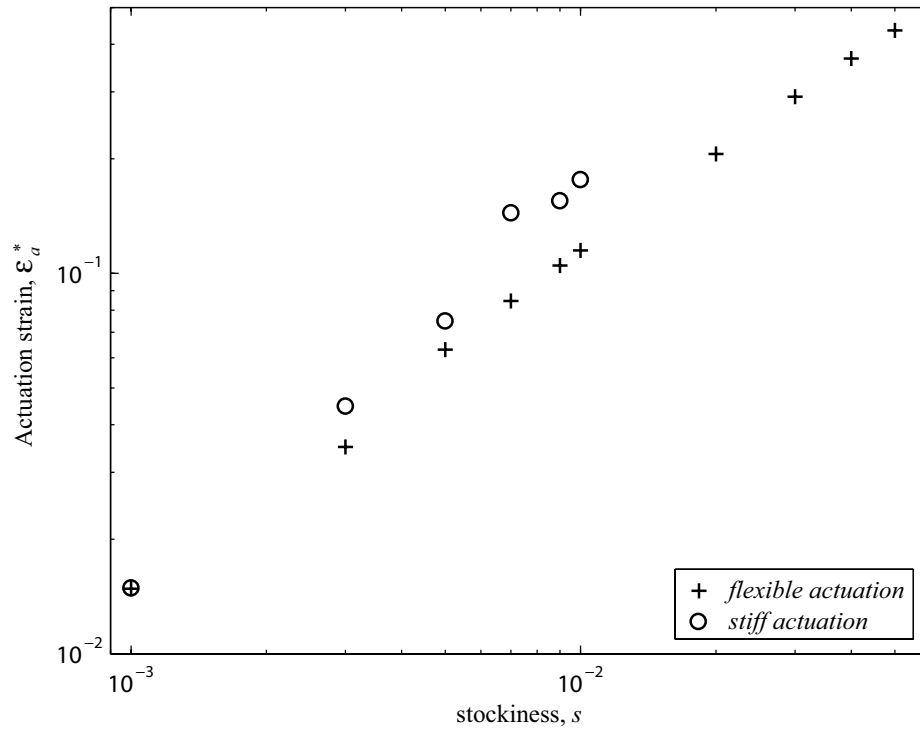
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Figure 6. 'Critical' actuation strain, where sudden stiffness degradation is observed, versus stockiness. Notice the directly proportionality relationship, with a slope of approximately one.
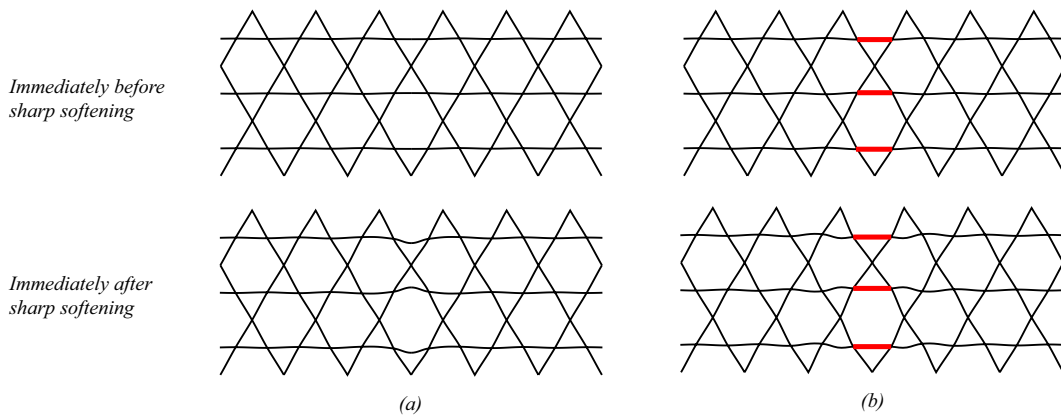

Figure 7. Post-actuation deformation shape of part of the kagome lattices actuated with (a) flexible and (b) infinitely stiff actuators. Localised deformation in (a) is found to be primarily within the actuators, where (b) shows deformation concentrated in the co-linear members immediately adjacent the actuators.

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perturbs the immediate neighbouring bars. For an expanding actuator, localisation occurs at a 'critical' actuation strain which is proportional to the lattice stockiness. At this critical strain, there is a sudden drop in passive stiffness.

In order to present a clear set of results, two detailed aspects of modelling have not been included: no consideration has been given to the actual joint geometry, and no imperfections in the members are considered, although both are important factors that will have a significant influence on the passive stiffness of the kagome lattices.

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