The Kent Tensegritree project

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Abstract

The development of the design of a new tree-like tensegrity sculpture, the *Tensegritree*, is described. The Tensegritree will provide a permanent memorial celebrating the 50th Anniversary of the University of Kent, and the 10th Anniversary of the founding of Kent School of Architecture. The tree-like conceptual form emerged from a workshop with students at the University of Kent; based on the concept, a final form was developed, simplified, optimized and realized: The final sculpture will have a height of 7 m, and a diameter of 12 m, and is due to be installed in July 2015.

Keywords: Tensegrity, form-finding, design

1. Introduction and Context

Ever since the early experiments in the 1940s by the artist Kenneth Snelson, the remarkable structures known as *tensegrities* have fascinated artists, engineers and architects alike. Taking inspiration from the synergy between compression and tension that characterizes these engineered structural systems, a tree-like tensegrity sculpture, or *Tensegritree*, has been form-found and designed to provide a permanent memorial celebrating the 50th Anniversary of the University of Kent, and the 10th Anniversary of the founding of Kent School of Architecture. A rendering of the sculpture from the design phase is shown in Figure 1: the final structure is due to be installed in July 2015.

The project was first conceived by Don Gray, Head of School at Kent School of Architecture, in early 2013 as a 'Beacon' project, In addition to providing a striking memorial, the Tensegritree sculpture will provide a new out-door teaching space for use by the students and staff at the University of Kent Campus. Under the direction of Don Gray as the client and project manager from the University of Kent, Expedition Engineering are leading the design. The project team also includes the contractor ES Global, and academic consultants Andrea Micheletti and Simon Guest.

During an initial workshop, undergraduate students at Kent School of Architecture were involved, along with the project team, in the physical form-finding of the unique geometry for the Tensegritree, developing new concepts for the sculpture which are now being taken forward to construction.

Figure 1: Tensegritree sculpture render. The structure has a height of 7 m, and a diameter of 12 m

Key criteria that have driven the design are the intention to create a novel and innovative tensegrity form, the design against vandalism and general student "high jinks", and the relatively small budget.

The design team has worked very closely with the contractor appointed to deliver the sculpture, ESGlobal, since the early stages of the feasibility study. This has enabled the project to seamlessly incorporate key considerations of buildability, robustness, safety, minimization of cost of construction, erection and construction sequences from the very beginning. The cross collaborations between contractor, design engineers, academic experts and client have proved very successful in creating a novel type of new single-layer tensegrity structural geometr and translating the design concept into the complete sculpture within a very short time frame, ready for the Anniversary celebrations.

2. The form-finding workshop and concept development

In October 2014, a one-day tensegrity form-finding workshop was organized for the students of the Faculty of Architecture at the University of Kent, see Figure 2. In order to achieve a design solution for the structure that had some of the visual lightness of Snelson's sculptures, the workshop focused on floating-compression tensegrities: those where each node connects just one strut and several cables (called "class-one tensegrities" by Skelton and de Oliveira [9]). Students were taught about the general features of tensegrity, and, in particular, about two basic rules-of-thumb for building a stable assembly of sticks and strings.

Figure 2: Photographs taken during the form-finding workshop.

The first rule-of-thumb taught at the workshop was the so-called "balloon analogy" (Buckminster Fuller [1], Sakantamis and Popovic Larsen [8]), that is, the observation that most stable floating-compression tensegrities present members in tension on the outside of the structure, with members in compression inside: just like an inflated balloon. The second rule-of-thumb taught is that one way to build a modular system is to connect the nodes of one module to the midpoints of the cables of another module, and then to add more cables to stabilize to connection between modules. Students learnt to apply these two rules quickly and were able to produce interesting models by the end of the workshop.

During the workshop, the concept of a tree-like tensegrity emerged. After the workshop the project team met to rationalise the students concepts to a stage where the resultant structure would be within the available budget. In addition, the team used two sources of inspiration from the literature. The first inspiration was one of the tensegrity trees proposed by Pizzigoni et al. [7], a higher-than-class-one structure where at each node one strut branches into two or three other struts, providing a tree-like
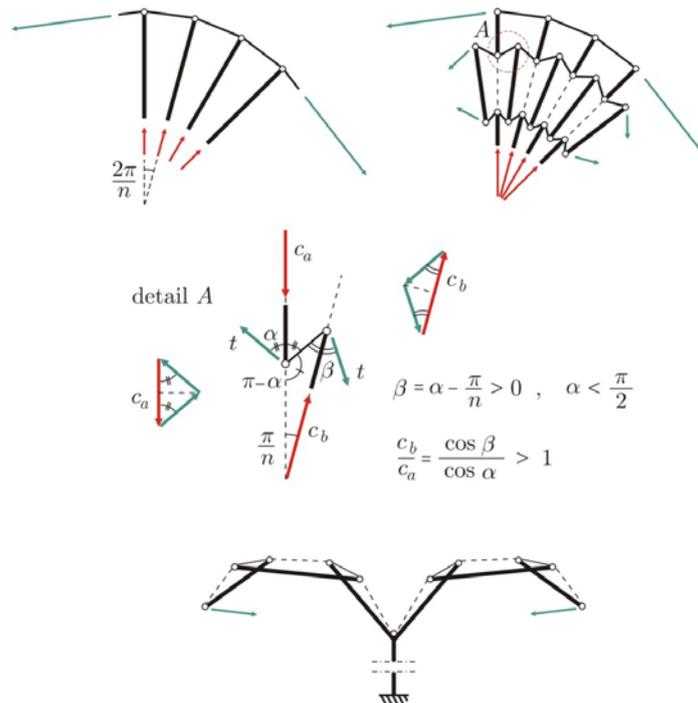
Figure 3: The development of the form: top-left shown the inverted bicycle wheel, with compressive spokes stressed against an out tension ring; top-right shows the "segmented" form, where addition stabilizing cables necessary in 3D to give the elevation shown at the bottom; the middle shows that the compressive force in the spokes is concentrated towards the centre.

visual impression. The second inspiration, that we named the "segmented inverted bicycle wheel concept", was developed after observing the cable dome presented in Tamai and Elnimeiri [10].

In a two-dimensional inverted bicycle wheel, cables form a tension hoop, while struts constitute the radial spokes (Figure 3, top-left) and are connected to a central hub. The segmented version can be generated by starting with an inverted bicycle wheel, adding to it an exterior set of radial struts (each connected to the midpoint of a hoop cable) with a new tension hoop, and then repeating this process to give the segmented pattern shown in Figure 3 (top-right). It is straightforward to show that the self-stress in this structure is characterized by a stress-amplification effect when moving from the periphery to the center of the system. For a fixed value of tensile force in the outer hoop, the compressive forces on the central hub in the segmented wheel shown in the top-right of Figure 3 are much higher than those in the unsegmented wheel shown in the top-left system in the same figure. Each segmentation produces an increment of the compressive force, as shown by the force diagrams in the middle of Figure 3.

The 2D system in Figure 3 (top right) can be considered as the horizontal projection of the 3D system at the base of the current Tensegritree design. Figure 3 (bottom) shows another 2D system (in the vertical plane) to help visualizing the stress pattern of such a 3D system. This projection makes it clear that additional 'stabilizing' cables, shown dashed, are required to maintain the state of self-stress.

To complete the initial form-finding process, a structure based on the concepts described above was developed and two additional set of cables were added: one to obtain a triangulated pattern, in order to eliminate internal motions; another one as a further restraint to avoid snap-through instabilities (e.g., as when an umbrella is flipped up by the wind).

3. Refining the found form

The form-finding workshop, and subsequent discussions, found an approximate basic form for the Tensegritree, but this needed further refinement to give a better resolved geometry that could be used for the final structural design. To do this, a simple pin-jointed analysis model to find self-stressed forms was developed.

3.1 Modelling

For model refinement, the structure is modelled as spherically-jointed, so that each member only carries axial forces. Gravity is ignored, as is the structural connection to the ground. The model has $j = 38$ joints, and $b = 105$ members (cables and struts together). Thus, according to Calladine's [2] extension of Maxwell's [5] rule the relationship between the number of infinitesimal mechanisms m and the number of states-of-self-stress s is given by

$$m - s = 3j - b - 6 = 3$$

Thus, the structure is *kinematically indeterminate*, as it must have infinitesimal mechanisms, which can only be stiffened if a state-of-self-stress exists (Guest, [4]). We needed to ensure that the structure would carry an appropriate state-of-self-stress.

Consider the equilibrium of a typical node i connected to three other nodes j, k, and l, in the absence of external forces, as shown in Figure 4. Equilibrium of the three internal forces carried by the members, t_{ij}, t_{ik}, t_{il} in terms of the coordinates of the points (x_i, y_i, z_i) etc. and the lengths l_{ij} etc. can be written as an equilibrium equation in the *x*-direction,

$$\frac{x_j - x_i}{l_{ij}} t_{ij} + \frac{x_k - x_i}{l_{ij}} t_{ik} + \frac{x_k - x_i}{l_{ik}} t_{ik} = 0 \qquad (1)$$

with similar equations in the y- and z-directions. The difficulty with solving equilibrium equations such as (1) at every node simultaneously is that neither the exact geometry is known, nor are the member forces. To make progress, we adopted an iterative scheme where we find an equilibrium geometry and state-of-self-stress for a structure that is as close to an initially guessed geometry. To do this, we write the equilibrium equations in two ways (Guest, [3]). In the first, we gather the b=105 unknown member forces t_{ij} etc. as a vector **t**, and write the $3j = 114$ equilibrium equations using the *equilibrium* matrix **A** (Pellegrino, [6]), which contains terms such as $(x_j - x_i)/l_{ij}$, as
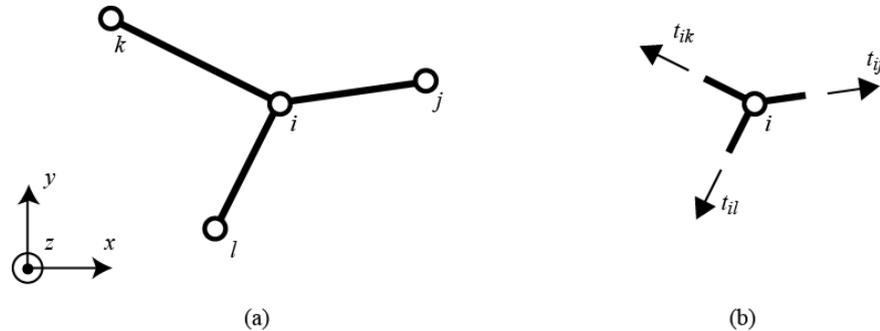
Figure 4: (a) shows four nodes connected by three members; (b) shows a free-body diagram of the central node.

$$\mathbf{A}\,\mathbf{t} = \mathbf{0}. \tag{2}$$

This equation will, in general, not have a solution other than $\mathbf{t} = \mathbf{0}$. However, even if this is the case, we take an approximation for the state-of-self-stress that is required by finding the unit vector that is closest to providing a basis for a one-dimensional nullspace. Using the language of the Singular Value Decomposition (Pellegrino, [6]) what is required is the final right-singular vector, associated with the smallest singular value of $\mathbf{A}$.

We thus take an approximate geometry, and find an approximate state-of-self-stress. The next step is to take this state-of-self-stress to find a more accurate geometry. To do this, we gather the unknown coordinates as three j-dimensional vectors: $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$, and write the $3j = 114$ equilibrium equations in a second way, as three matrix equations that use the *small stress*, or *force density* matrix $\mathbf{S}$ (Guest [3]), which contains terms using force densities, such as t_{ij}/l_{ij}. The equilibrium equations become

$$\mathbf{S}\,\mathbf{x} = \mathbf{0}; \quad \mathbf{S}\,\mathbf{y} = \mathbf{0}; \quad \mathbf{S}\,\mathbf{z} = \mathbf{0}. \tag{3}$$

If the force densities were exactly known, there would be independent solutions to the three equations in (3), which would also be independent of the uniform vector solution $[1\ 1\ \cdots\ 1\ 1]^{\mathrm{T}}$, and the matrix $\mathbf{S}$ would have a four-dimensional nullspace. However, as we are using approximate values of the force-densities, we instead use the closest approximation to a four-dimensional nullspace. Again using the language of the Singular Value Decomposition, we take the space defined by the right-singular-vectors associated with the four smallest singular values. (As the matrix $\mathbf{S}$ is square and symmetric, these vectors are equal to the eigenvectors associated with the smallest *magnitude* of eigenvalues.) We then improve our geometry by projecting the vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ onto our approximation of the nullspace, effectively eliminating aspects of the geometry that don't fit.

Thus, we are able to use an approximate geometry to find an approximate state-of-self-stress, and the approximate state-of-self-stress to find an improved geometry. In practice, following this iterative scheme twice is enough to give an appropriately resolved form of the structure.
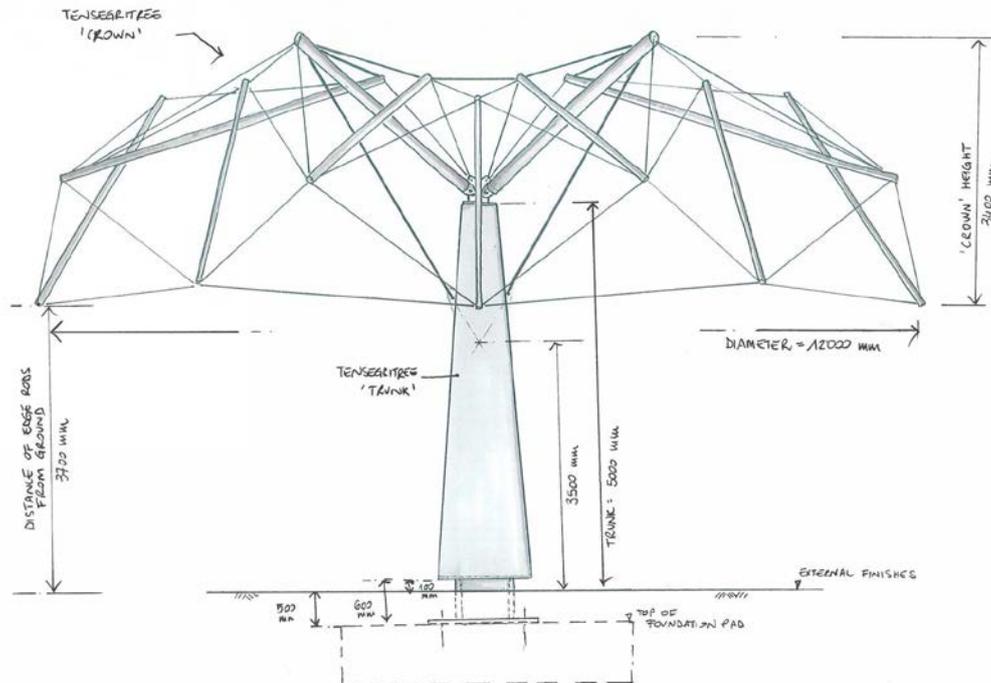
Figure 5: Tensegritree elevation, showing key dimensions.

3.2 Refinement of the form

The resolved version of the geometry associated with the initial found form was found to be poor in a number of aspects. In particular, the ratio of the tension carried by the most highly loaded member to that carried by the least highly loaded member was over 1000:1. Associated with this, the stiffness of modes that correspond to infinitesimal mechanisms, i.e., those that rely entirely on self-stress for their stiffness, was very low. Further, the tension members surrounding many nodes were close to lying in a plane, with the associated potential of having snap-through behaviour of the node when loaded.

The problems described above were solved by an ad-hoc system of adjusting the position of targeted nodes that were problematic, and then re-resolving the geometry as described by the iterative system above. It proved to be straightforward to find a good stressed form of the structure, which could then be fed into the detailed structural design. Sketches of the final form are shown in Figures 5 and 6.

4. Realisation of the Tensegritree

Using the improved resolved form of the structure, Expedition Engineering developed the global structural analysis and the connection details design to produce construction information to allow the fabrication to commence. Detailed structural design of the Tensegritree included: evaluating material options, connectivity within the elements, optimization of the end fitting detailing, prestressing procedures, construction sequence, cost reduction through modular repetition of elements and
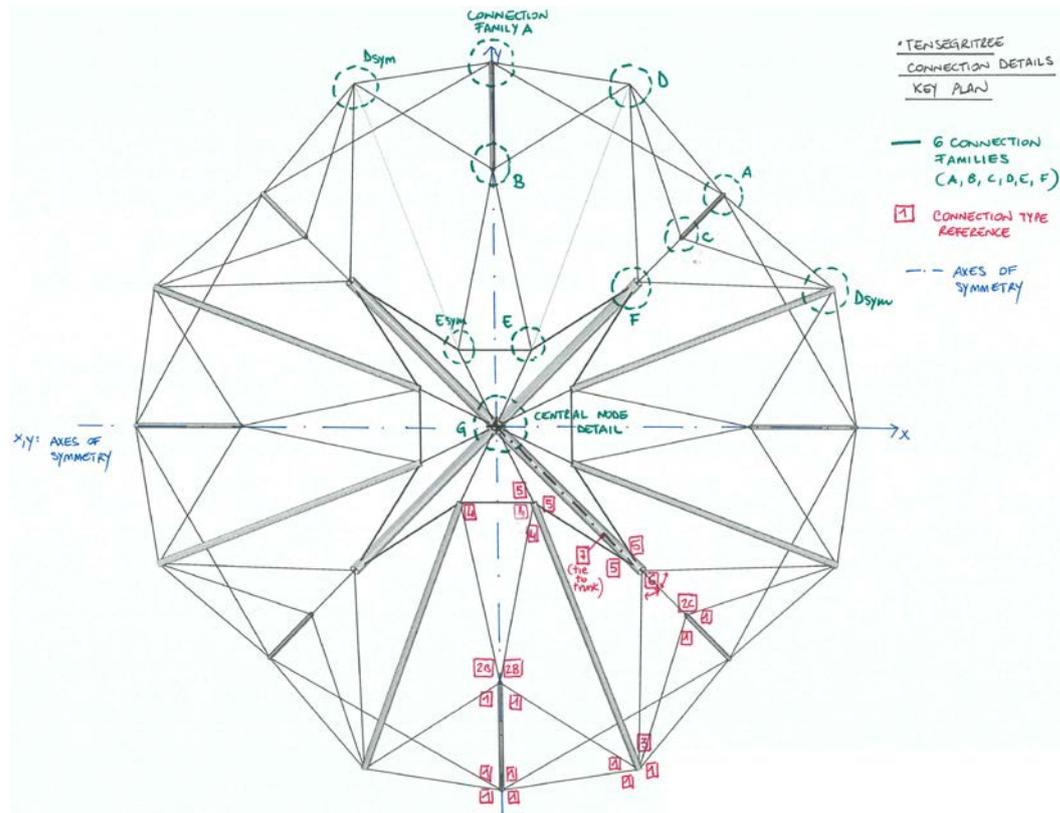
Figure 6: A plan view of the Tensegritree, showing the positions of various groups of end fittings.

minimization of ongoing maintenance, as well as taking into account structural adequacy, robustness and safety. The Tensegritree has been designed as a permanent structure with a life of 25 years, which sets it apart from many other tensegrity structures, which are often temporary installations.

Some key considerations in the design are as follows.

- Materials:

  The material choice has been driven by consideration of structural efficiency and low ongoing maintenance costs. The final design utilizes galvanized steel Circular Hollow Section (CHS) profiles for the compressive "strut" elements, and stainless steel rods for the tensile "cable" element, with solutions to prevent corrosion problems at the bimetallic interface. The selection of element diameters was tuned to give a reduction in size towards the edges of the Tensegritree giving increased lightness and elegance as well as efficiently responding to the structural function. The CHS sections used have a relatively small diameter and large wall thickness, both for aesthetics, and also for the ease of welding the connection details described below. Stainless steel rods rather than cables were used, partly because they will hold their shape when unloaded,
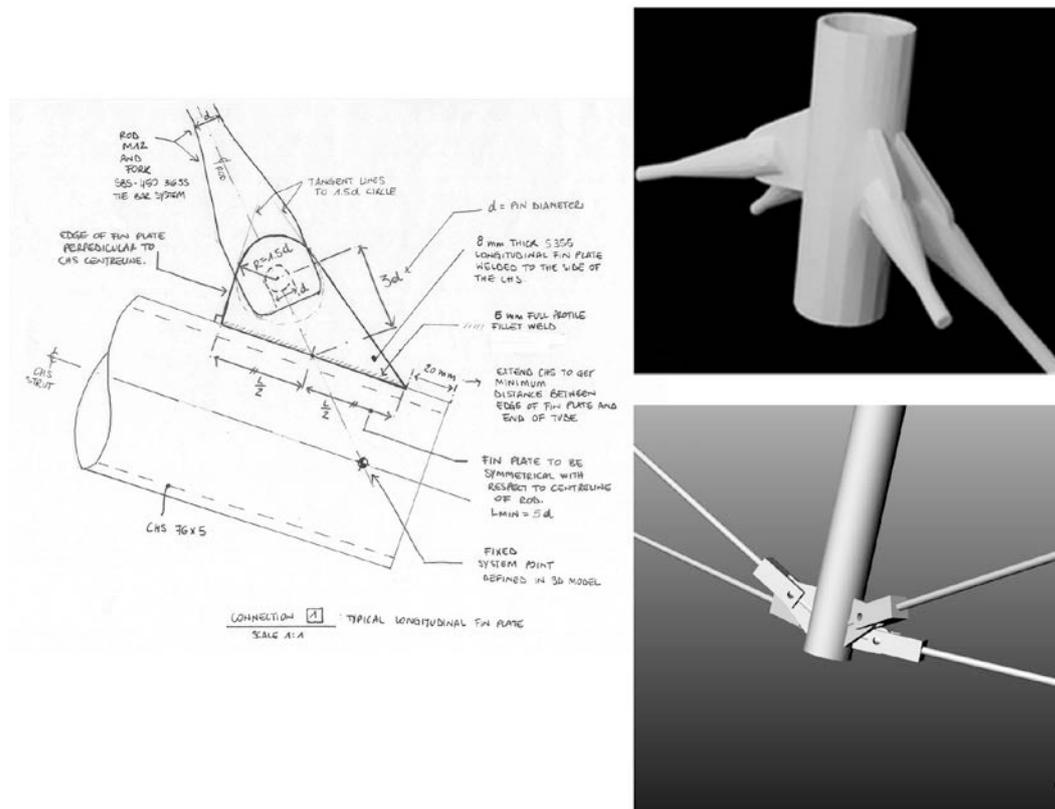
Figure 7: End connection: A typical sketch (left); 3D printed models of connection details (top-right); images of the fabrication model (bottom-right).

helping to make construction easier. Careful connection design ensures that kinks in the "cable" elements, sometimes seen in in sculptural tensegrities, are not required.

- Connection details:

  The rods are connected to struts via fin-plates welded directly onto the face of the CHS tubular sections to allow a simple and elegant connection detail that follows force lines and provides a crisp, clear and consistent connection philosophy. The number of incoming members connections varies from a minimum of 3 to a maximum of 5 at each node, with connections at varying inclinations. To avoid fatigue problems, the welds have been designed to a low stress limit. Various options for the end connections and central node have been explored via use of 3D modelling and 3D printing to investigate feasibility, constraints and visual acceptability. Typical connection details are shown in Figure 7.

- Modularity:

  The geometrical symmetry in plan of the sculpture allows regularity, repetition and economic fabrication of members and end fittings. The final design has six different types of end details, as shown in the sketch in Figure 6.

- Vandalism, robustness and trunk height

  Safety and robustness of the design against vandalism or student "high-jinks" are key priorities for a sculpture on a university campus. These aspects have highly influenced the final Tensegritree geometry shown in Figure 5. For example, most of the tension elements of the structure sit above a pyramidal "trunk" 5 m above ground to reduce accessibility, while the tie-down rods which connect to the trunk at 3.5 m from ground level have larger diameters for robustness against willful damage, for instance deliberate attempts at damage using bolt-cutters. The measures of risk reduction considered in the design phase are in addition to the security monitoring systems on the University of Kent campus.

- Stiffness/compliance

  Compared to conventional structural systems, a kinematically indeterminate structure such as the Tensegritree will be orders of magnitude less stiff and will experience serviceability movements that are larger than the conventional deflection limits appropriate for occupied buildings. The expected deflections are in the order of 20 mm under wind load and in the order of 100–150 mm under the application of the vandal loads. The analysis proved that these large deflections do not result in unsafe non-linear or snap-though type behavior; on the contrary, for a sculpture which acts as a counterpoint to a living, breathing, swaying tree, this greater compliance will add to the aesthetic of the sculpture.

4.1. Structural analysis

Expedition carried out a non-linear analysis of the Tensegritree using the analysis package GSA, to determine both global behaviour and member sizes. Establishing an appropriate level of prestress was essential, as well as considering loading due to wind and aerodynamic excitation, thermal effects, fatigue and dynamic loads. Additionally, there are a series of unusual drivers and parameters influencing the final design.

4.1.1. Prestressing

The prestress in the structure has been evaluated in order to provide the required stability and stiffness to the system. The minimum tensile force in the outer ring elements has been set to 3.5 kN to avoid any risk of slackening in the system, with a consequent resulting peak tension in any element of 137.7 kN. The distribution of member force is shown in Figure 8(b) – it can be seen that forces are concentrated towards the centre, following the initial prediction shown in Figure 3. The design refinement has succeeded in keeping this ratio to less than 40:1, which has been considered to be acceptable.

In the analysis model, the prestress loads were applied using a dummy thermal loading applied to those elements that will likely to be stressed during the erection sequence, helping to simulate the effects of the real construction phase. During this phase of the analysis, the nodal movements predicted during prestressing are of the order of 200 to 250 mm, as shown in Figure 8(a).
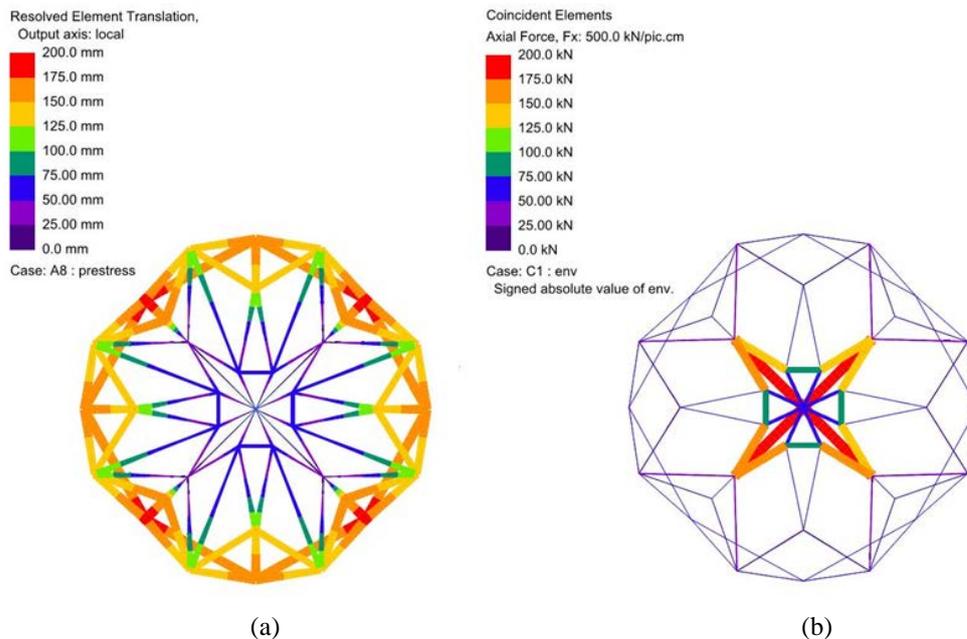
(a) (b)

Figure 8: Structural analysis results showing: (a) deflection due to prestressing at the construction phase; and (b) distribution of tension forces in the tensegrity net of rods.

4.1.2. "Accidental" actions

The team considered hazards such as climbing on the sculpture, vandalism or deliberate excitation of the sculpture, and developed the geometry as described above to reduce the likelihood of this happening. However, the risk of vandalism cannot be fully eliminated, and since maintaining the stress state in a tensegrity is critical to its stability, the design was checked for a person climbing the sculpture and hanging from different positions.

4.1.2. Wind-induced aerodynamic excitation of the structure and fatigue considerations

The Tensegritree consists of a large number of cylindrical members with a wide range of natural frequencies – as such there is potential for wind-induced vibrations, including vortex induced vibration. Rather than expensive wind tunnel testing which would not have been suitable for the limited budget of the project, it has been considered appropriate to follow the design approach of assessing the likely stresses induced by wind effects and to ensure that the stress ranges remain below the constant amplitude fatigue stress limit, giving the steel structure an effectively indefinite fatigue life.

5. Conclusion

The development of the Tensegritree has shown how an enlightened client, engaging early in a project with consulting engineers, contractors and academics, can help bring about successful outcomes in short timescales. The erection and installation of the Tensegritree is expected to be completed in July 2015. Key aspects of the fabrication and construction process will be presented at the conference.

Acknowledgements

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