REAL OPTIONS IN PARTNERSHIPS

NICOS D SAVVA & STEFAN SCHOLTES

ABSTRACT. We study partnership contracts under uncertainty but with clauses that admit downstream flexibility. The focus is on effects of flexibility on the synergy set, the core, of the contract. In a partnership context the value of flexibility is captured by the partners who own the right to exercise. On one side, there are cooperative options, which are exercised jointly and in the interest of maximizing the total contract value, on the other side, there are non-cooperative options, which are exercised unilaterally, or by coalitions, in the interest of the option holders' payoffs. We provide a modelling framework that captures the effects of optionality on partnership synergies. We study these effects under a complete markets assumption, based on standard contingent claims analysis, as well as under heterogenous risk-aversion, using a dynamic programming model. The models shows the effect of several strategies on the synergy set and the bargaining position of the partners. It also shows that non-cooperative options, if agreed prior to the negotiation, are powerful bargaining tools but that they can also destroy the partners' incentive to participate in the contract. Finally, the model illustrates how risk sharing provides larger synergies for partners with heterogeneous risk attitudes.

1. INTRODUCTION

Partnerships are a driving force of the modern economy. Joint ventures of car manufacturers, alliances between airlines, co-development contracts between pharmaceutical and biotech companies, production sharing contracts between oil majors and national oil companies, to name but a few industries where partnerships are significant drivers of value.

Partnerships aim to create synergies by combining core competencies of the partners to form a distinctive offering that neither partner could provide alone. Synergies are traditionally thought of in terms of improved efficiency, e.g. through economies of scale or scope. In an uncertain world, however, there are at least two further important sources of synergies: risk sharing and

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Date: February 16, 2006.
Authors affiliation: Judge Business School, University of Cambridge
Cambridge, CB2 1AG, UK. email:n.savva@jbs.cam.ac.uk & s.scholtes:@jbs.cam.ac.uk
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flexibility. Risk sharing is particularly interesting if partners have different risk attitudes, e.g. a pharmaceutical major and a small biotech company, enabling win-win situations by trading off risks. Additional flexibility in a partnership can have significant value by allowing partners to cut downside risk or amplify upside potential as the uncertain future unfolds. In this paper, we will explore such stochastic synergies.

The framework for most partnerships is provided by a legal contract. Two key questions in contract negotiations are: How should the contract be structured to generate significant *total value* at an acceptable level of risk? How should this total value and the associated risks be *shared* amongst the partners? These challenges are exacerbated when long-term partnerships are negotiated in volatile commercial environments. Mitigating clauses tend to be included in contracts to avoid lock-in and enable the partners to react, either jointly or unilaterally, when uncertainties unfold, without the need to breach the contract. The contract becomes a dynamic frame. What are the implications of contingency clauses for the contract value as a whole? How do contingency clauses change the bargaining position of the partners? These are the core questions that we address in this paper.

In our practical experience, value effects of contingency clauses are often underestimated or even completely discarded. The reply of a senior manager to our question about the rationale for his suggested royalty rate is representative in this regard: "We are contributing 50% of the R&D expenditure. It seems only fair to set the royalties so that we receive 50% of the projected value if the R&D is successful". His argument neglected that the decision to launch the successful product was the partner's and that the manager's company would loose all royalty payment if the partner decided, for whatever reason, not to launch the new product. Flexibility can have significant value for its owner and can take away significant value from the other partners. Only if this value-effect is understood and taken into account in contract design and sharing negotiations can we hope to create robust partnerships that do not go sour when companies exercise flexibilities in ways their partners had not foreseen.

The academic discussion of fair distributions of benefits from cooperation goes back to the seminal work of Nash (1950, 1953) and Shapley (1953), which led to the advent of bargaining theory and cooperative game theory. This literature is largely concerned with the allocation of value, not of risk. The models are mainly deterministic and combinatorial. A first strand of this literature relevant to our work is concerned with cooperative game theory in the presence of stochastic payoffs, see e.g. Granot (1977), Suijs and Borm (1999), Suijs et al. (1999). We build in particular on the work of Suijs and co-workers, using the concept of a deterministic equivalent of a stochastic cooperative game, which turns out to be very useful in our analysis. A second relevant body of work focuses on efficient risk sharing and the formation of syndicates, see e.g. Wilson (1968) and Pratt (2000). We integrate these two strands of literature with elements of the real options literature to study the effect of optionality in partnership contracts, an issue which, to our best knowledge, has not been thoroughly investigated to date.

Two concepts play a crucial role in the study of the value effects of uncertainty: Diversification and optionality. Diversification is essentially a *passive* risk management tool and presumes no direct influence on the management of individual projects. It is therefore particularly appealing to investors. Optionality on the other hand, emphasizes the importance of *pro-active* risk and opportunity management and is therefore particularly appealing to managers. A right without obligation to a potential future action creates value in an uncertain environment. The concept of optionality and its valuation in the context of financial derivatives, originated in the seminal work of Merton (1973) and Black and Scholes (1973), has attracted considerable academic attention and made a significant practical impact. Indeed, the concepts and approaches of financial engineering have moved beyond the design and valuation of financial instruments into the realm of capital budgeting and project valuation.

Myers (1984) was amongst the first to advocate that significant optionality, such as growth opportunities, ought to be included in the valuation of a project or company and that appropriate use of the work of Black, Scholes and Merton might make this possible. Myers saw this as an opportunity to bridge the gap between strategy and finance and coined the term *real options* for this line of thinking. Shortly afterwards, Brennan and Schwartz (1985) illustrated how such real options could be valued with a Black-Scholes approach. These seminal papers, together with the monograph by Dixit and Pindyck (1994) spurred a significant amount of academic work over the past two decades and led to the establishment of *real options* as a distinct area in finance with increasing uptake in the strategy literature, see e.g. Kogut (1991), Rivoli and Salorio (1996), McGrath (1997, 1999), McGrath and Nerkar (2004), Kulitilaka and Perotti (1998), Bowman and Moskowitz (2001), Folta and Miller (2002), Cassiman and Ueda (forthcoming), Burnetas and Ritchken (2005). The advent of real options analysis in the strategy literature has not been without controversy, though. We refer the interested reader to Adner and Levinthal (2004b,a) and the response to their paper by McGrath et al. (2004).

An emerging body of literature is concerned with the relationship between optionality and competition, for example see Grenadier (2002). Little work has been done to date to understand the effect of real options on the synergies created by a partnership and the consequences for fair splits of risk and return. Our aim in this paper is to fill this gap by presenting a framework that allows the investigation of contract design issues with a real options flavor. To this end, we combine concepts from cooperative game theory and real options theory - a combination that has not received the attention in the academic community that, we believe, it deserves. Our main emphasis is on the impact of options on the *core* of a cooperative game. The core of a contract conceptualizes a notion of a negotiation synergy set. It contains those payoff allocations for which no partner, or sub-coalition, can improve upon by going alone.

The paper is structured as follows: We begin by discussing a simple model of a cooperative real options game. In section 4 we determine its core under three sets of assumptions: a) the existence of a complete market, b) risk neutral agents, and c) risk averse agents. In section 4 we develop the model and insights, using a more general multi-agent, continuous time framework. Section 5 concludes with managerial implications. In order to reach a broad readership, we have structured the paper in such a way that the key intuition, illustrated by the simpler model of section 4 can be grasped without a detailed understanding of section 4.

2. Options contracts: Cooperative vs. non-cooperative options

An *options contract* is a contract with significant future flexibility. Options are rights but not obligations to future actions. In a partnership this raises the question, who has the right to the action? There are two types of options in partnerships, depending on who has this exercise right.

A contract clause may specify that an exercise decision on an option is taken jointly. We call such flexibilities *cooperative options* and assume they will be exercised in the interest of maximizing the total value of the contract. A typical example is a decision to jointly market a product after a successful R&D effort.

Flexibility may also be owned by a single partner or a sub-group of partners, who have the right to exercise it and will, we assume, do so in the interest of their own payoff, rather than the sum of payoffs resulting from the contract. We call such flexibilities *non-cooperative options*. A generic non-cooperative option on a contract is the option to breach the contract if circumstances do not unfold as anticipated, accepting possible litigation costs as the price of exercise.

The notion of a cooperative option emphasizes the collaborative nature of partnerships, whilst non-cooperative options acknowledge the transient nature of contracts and regard them as part of competitive strategies of firms who will ultimately act in their own interest. Non-cooperative options can be tacit, such as the option to breach the contract, or explicitly acknowledged in a partnership contract. For example, a clause in a co-development contract between a biotech and a pharmaceutical company may allow the biotech company to opt out of further co-development and receive agreed milestone and royalty payments instead. Formally, a non-cooperative option in a partnership can be thought of as a cooperative game followed by a non-cooperative game. The cooperative game, i.e., the contract negotiation, sets a framework for later non-cooperative behavior, which has to be taken into account in the negotiation.

Our main focus in this paper is the effect of flexibility, cooperative or non-cooperative, on the synergies created by the partnership. To this end we employ the notion of the *core* of a cooperative game, which is the set of all allocations of payoffs to the partners that will make *all* partners better off than without the contract. Following Sharpe (1995), we illustrate option effects in the context of a very simple one time period model with simple flip-of-the-coin uncertainty. The model extends to more complex lattice-based and continuous-time models. A continuous time version is developed in Section 4.

3. The core of an options contract: An illustrative model

Assume a biotech company has a drug under development, which has successfully passed all clinical trials and is now awaiting final approval by the regulator. The company estimates the present value of cash flows from the drug to be C_B for a launch investment of $I_B < C_B$. The biotech company has limited production capabilities and its sales and distribution network is rather inefficient compared to pharma majors. The company is therefore negotiating a co-marketing contract with a large pharmaceutical company. The cash flow projection for the co-marketed product is C_{B+P} and the launch investment will be I_{B+P}. How should the value $(C_{B+P} - I_{B+P})$ of the contract be shared in a fair way?

3.1. The core of the deterministic game.

The core of this cooperative game is the set of payoff allocations that make both partners better off than going alone. Denoting by ϕ_B and ϕ_P the share of the contract value $C_{B+P} - I_{B+P}$ for the biotech and pharmaceutical company, respectively, and neglecting costs of capital considerations for simplicity, the core is defined by

$$\begin{aligned} \phi_B &\geq C_B - I_B \\ \phi_P &\geq 0 \\ \phi_B + \phi_P &= C_{B+P} - I_{B+P}. \end{aligned}$$

In other words, the biotech's payoff share ϕ_B is in the core if

$$C_B - I_B \leq \phi_B \leq C_{B+P} - I_{B+P},$$

with the residual payoff $\phi_P = C_{B+P} - I_{B+P} - \phi_B$ being allocated to the pharmaceutical company.

3.2. Uncertain payoffs.

To introduce uncertainty we assume that a competitor is developing a drug that will treat the same indication. If the competitor is successful in developing this drug, the revenue potential of the biotech's drug will be reduced. We assume that p is the probability of failure of the competing drug. In the upside scenario of a failure of the competitor's drug, the cash flow projection if the biotech company goes alone is assumed to be $u_B C_B$, with $u_B > 1$; in the downside scenario of competitor success, this cash flow is projected to be $d_B C_B$, with $d_B < 1$. In the partnership the present values of the cash flows are projected as $u_{B+P} C_{B+P}$ in the upside and $d_{B+P} C_{B+P}$ in the downside scenario.

In section 3.3 we will determine the core of this game under the assumption of complete markets. In section 3.4 we replace the assumption of complete markets with risk neutral agents and we see that much of the results for complete markets carry forward. Finally, we investigate the real options cooperative game under the assumption of risk aversion in section 3.5.

3.3. **Complete Markets.** We will first examine the stochastic cooperative game under a complete markets assumption. Complete markets imply the existence of a portfolio of traded assets that replicates any contract payoffs. Trading in these assets allows the partnership to hedge all risks: partners can individually short sell the replicating portfolio corresponding to their allocation of the contract payoff and thereby offset their payoffs in each state of the market. Hence, the only valuation for the investment opportunity that is consistent with the absence of arbitrage opportunities is the present value of the replicating portfolio. The risk preferences of agents are irrelevant in this situation, for details see Hull (2003).

To illustrate this effect, assume there is an asset which closely tracks the success or failure of the competing R&D project. In the case of success the price of the tracking asset will increase from P to P_u, if the competing project fails, it will decrease to P_d.

If an asset has payoffs (X_u, X_d) in the two states of the market, then this payoff can be replicated with a portfolio of the tracking asset and the risk free asset, which we assume to have return $r = 1$ for simplicity. This replication is done by solving $\psi_B P_d + \theta_B = X_u$, $\psi_B P_u + \theta_B = X_d$, where ψ is the number of bought shares in the tracking asset and θ is the amount invested in the risk free asset. Simple algebra shows that the value x of this replicating portfolio has the form

$$x = \psi_B P + \theta_B = q x_u + (1-q) x_d, \tag{3.1}$$

where q is given by

$$q = \frac{P_u - P}{P_u - P_d}. \tag{3.2}$$

Because $0 \leq q \leq 1$ it is often interpreted as a probability, although its correct economic interpretation is in terms of forward prices, see Sharpe (1995). The reference to q as a probability is convenient because (3.1) allows the interpretation of the non-arbitrage value as an expectation under this measure. The measure q is typically referred to as the *risk-neutral measure* or equivalent martingale measure in the finance literature.

3.3.1. *The contract without options.* Equation (3.1) allows us to calculate the no-arbitrage value

$$x_B = q u_B C_B + (1-q) d_B C_B - I_B \tag{3.3}$$

of the contract for the biotech alone and the no-arbitrage value

$$x_{B+P} = q u_{B+P} C_{B+P} + (1-q) d_{B+P} C_{B+P} - I_{B+P} \tag{3.4}$$

for the partnership. Since under the complete markets assumption both agents would agree on these values (or allow for arbitrage opportunities), the game is reduced to a deterministic cooperative game. The value share ϕ_B of the biotech is in the core if

$$x_B \le \phi_B \le x_{B+P}$$

with the residual $\phi_P = x_{B+P} - \phi_B$ going to the pharma company. The core is similar to the cooperative game without uncertainty, with the difference that the deterministic payoffs are now replaced by the no-arbitrage value of the stochastic payoffs. Effectively, trading in complete markets reduces the stochastic game to a deterministic game.

It is important at this point to make a distinction between *payoff* sharing and *value* sharing. Payoff allocations, which we will denote by $(\Phi_B(\omega), \Phi_P(\omega))$, are functions that allocate the ultimate payoffs, here $u_B C_{B+P} - I_{B+P}$ or $d_B C_{B+P} - I_{B+P}$ to the biotech and the pharma in *each* state of the world $\omega \in \{u, d\}$. In each such state, the sum of the partners' payoff allocations equals the total payoff for the partnership. The value shares (ϕ_B, ϕ_P), in contrast, are the private values which the partners assign to their payoff allocations. In our complete markets setting the payoff shares are related to the value shares by

$$\phi_i = q\Phi_i(u) + (1-q)\Phi_i(d), \ i \in \{B, P\}. \tag{3.5}$$

Since each firm can hedge all risks through trading, the agents are indifferent between two payoff sharing rules that have the same risk-neutral value ϕ_i. In the complete markets case, the relationship $\phi_B + \phi_P = x_{B+P}$ holds.

3.3.2. *A cooperative option.* We will now introduce a cooperative option. Suppose the companies can wait with the launch investment until they know the result of the trials for the competing drug and therefore the cash flow scenario. For simplicity we assume that there is a deterministic cost involved in waiting. Such costs may involve actual costs, such as labour costs, as well as opportunity costs, such as cost of lost sales or finite patent life etc. The waiting cost is k_B for the biotech alone and k_{B+P} for the partnership.

The biotech alone as well as the partnership have two possible project designs to choose from. The first is to pay k_B or k_{B+P}, respectively, and postpone the decision to launch until uncertainty is resolved. This will provide the option to abandon the project if the competition is successful. The alternative is not to pay the costs and as a result they will not have the option.

The method developed in the previous section can be used to price the project with the option to wait. If the biotech goes alone the no-arbitrage value (3.1) is[1]

$$x_B^O = q(u_B C_B - I_B)^+ + (1-q)(d_B C_B - I_B)^+ - k_B, \tag{3.6}$$

[1] We are using z^+ as a shorthand for $\max\{z, 0\}$.

while the value for the partnership becomes

$$x^O_{B+P} = q(u_{B+P}C_{B+P} - I_{B+P})^+ + (1-q)(d_{B+P}C_{B+P} - I_{B+P})^+ - k_{B+P}, \tag{3.7}$$

were q is the risk-neutral probability given by equation (3.2). If the biotech alone or the partnership decides not to wait the values is given by (3.3) and (3.4), respectively.

Knowing the values of the design alternatives, the agents choose the optimal design. If $x^O_B > x_B$ the Biotech would prefer to pay the costs involved with waiting and set up the option. If that is the case, the company would exercise the option in the states of the world where $i_B C_B - I_B > 0, i \in \{u, d\}$. Similarly for the partnership. Again, the stochastic game is reduced to a deterministic game as in the previous section, with the additional complication that the agents need to also choose the optimal design and the optimal exercise policies for their options. The agents will choose the design with the highest risk-neutral value. Since this is the only value that is consistent with no-arbitrage assumption, there will be no disagreement over the value of the different designs or the optimal exercise policy. We shall see that this is not the case in the absence of markets and the presence of risk-aversion.

The condition for the biotech's value ϕ_B to be in the core is now

$$x^*_B \leq \phi_B \leq x^*_{B+P}$$

where x^*_B is the value of the optimal design for the biotech subject to optimal options exercise:

$$x^* = \max\{x_B, x^O_B\}. \tag{3.8}$$

x^*_{B+P} is similarly defined for the partnership.

3.3.3. *A non-cooperative option.* To illustrate the effect of non-cooperative options, let us assume that, in addition to the setting so far, the contract gives the biotech company unilateral flexibility to opt out of co-marketing of the drug before committing to the launch cost, whilst the pharma company is locked in the contract. Suppose the biotech would receive a fixed amount Z, deducted from the contract value, if it exercised the opt-out option. We treat Z as an exogenous parameter and we investigate its effect on the synergy set of the contract.

The first question is how, if at all, the existence of this option changes the total value of the contract? Suppose the agents have agreed a payoff sharing rule $(\Phi_B(\omega), \Phi_P(\omega))$ if the drug is launched jointly. If the upside scenario occurs then the payoff to the biotech will be $\max\{\Phi_B(\omega), Z\}$, taking the option into account, whilst the residual payoff $u_{B+P}C_{B+P} - \max\{\Phi_B(\omega), Z\}$ goes to the pharma company. Whatever the sharing rule, the total payoff in the upside is $u_{B+P}C_{B+P}$. Similarly, in the downside the total payoff is $d_{B+P}C_{B+P}$, i.e., the total value of the contract is not affected by the existence of the option. Since the preceeding cooperative option is exercised in the interest of the total deal value we are back in the former situation of the cooperative option alone.

If the payout Z is larger than the total contract value without the unilateral option, $Z > x^*_{B+P}$, see (3.8) with q replaced by p, then the pharmaceutical company, who will have to pay the amount Z if the option is exercised, will have no incentive to participate in the deal and the core will be empty. So let us assume that $Z \le x^*_{B+P}$.

A first observation is that the biotech payoff will always be at least Z therefore the value-core will be of the form

$$\max\{Z, x^*_B\} \le \phi_B \le x^*_{B+P}.$$

If $Z \le x^*_B$, then the value-core is unchanged by the non-cooperative option. If $x^*_B < Z \le x^*_{B+P}$ then the value core is reduced in favor of the option owner, who gains increased bargaining power. If $Z > x^*_{B+P}$ then the core is empty; the presence of the option makes it impossible for the pharma to agree to the deal.

All the arguments can be made without reference to the payoff allocation Φ, although the options exercise will actually depend on the payoff allocation. It can be shown that any value allocation ϕ in the core can be realised with a payoff allocation Φ for which the option is never exercised (see Proposition 4.4).

3.4. **No traded assets and risk-neutral agents.** The complete markets assumption can be replaced without difficulty by a consistency argument and the assumption of risk-neutral agents. This is done by replacing the standard contingent claims analysis by an equally standard stochastic dynamic programming analysis. Much of the analysis of the previous section remains valid under the risk-neutrality and consistency assumptions.

As before, we assume that the competitor will fail to bring the competing drug to market with probability p. However, we will now make explicit use of the failure probability p. An important, albeit often tacit, assumption in this context is that both agents agree on the value of the probability p. As before, the project value for the biotech and partnership is projected to be $d_B C_B$ if the competing project succeeds and $u_B C_B$ if the competing project fails.

We will require that the initial future cash flow projection C_B is *consistent* with the scenario assumptions in the sense that

$$C_B = p u_B C_B + (1-p) d_B C_B$$

were we assume zero returns on investment. This consistency assumption plays the role of the existence of a suitable martingale measure in the previous section, which is equivalent to the assumption of a complete arbitrage-free market. It links our analysis below to the contingent claims analysis above. The consistency assumption hold if and only if there exists s_B such that upwards and downwards rates are of the form

$$u_B = 1 + s_B\sqrt{\frac{1-p}{p}},\ d_B = 1 - s_B\sqrt{\frac{p}{1-p}}, \tag{3.9}$$

where s_B can be thought of as a measure of volatility[2].

In the partnership contract the present values of the cash flows are as before projected as $u_{B+P}C_{B+P}$ in the upside and $d_{B+P}C_{B+P}$ in the downside scenario. Again, under risk-neutrality, this scenario assumption is consistent with the foregoing valuation of C_{B+P} if the upwards and downwards multipliers u_{B+P}, d_{B+P} have the form (3.9) with a possibly different volatility s_{B+P}.

In contrast to the deterministic and the complete markets case the core is now a set in the two-dimensional $(\Phi_B(u), \Phi_B(d))$-space, as depicted in Figure 1. The width $(C_{B+P} - I_{B+P}) - (C_B - I_B)$ can be regarded as a metric for the size of the synergies involved.

Note that under the risk-neutrality assumption company i's value share is $\phi_i = p\Phi_i(u) + (1-p)\Phi_i(d)$, i.e., the partners are indifferent between payoff allocations on a line $p\Phi_i(u) + (1-p)\Phi_i(d)$ and value them at their expected value ϕ_i. Therefore we can reduce the two dimensional core of the stochastic game again to a one dimensional value core defined by

$$C_B - I_B \le \phi_B \le C_{B+P} - I_{B+P}.$$

The computation of the value core for cooperative and non cooperative options is analogues to the complete markets.

Note that the option values x_B^* and x_{B+P}^* increase, in this model linearly, with the volatilities s_B and s_{B+P}, respectively, while the synergy set increases in size with the difference in volatility $s_{B+P} - s_B$, again linearly. The same argument hold in complete markets.

It is interesting to look at the effect of non-cooperative options in the payoff space Φ_B, Φ_P. We do this in the next section.

3.4.1. *The non-cooperative option.* The biotech payoff will be $\max\{\Phi_B(\omega), Z\}$, where $\omega \in \{u, d\}$ is the observed scenario and $\Phi_B(\omega)$ is the agreed payoff if the drug is launched and the non-cooperative option option is not exercised. The expected payoff for the biotech is

$$p\max\{\Phi_B(u), Z\} + (1-p)\max\{\Phi_B(d), Z\},$$

[2]A geometric Brownian motion with drift ν and volatility σ can be approximated by a binomial lattice with upwards probability p, period length Δt and upwards and downwards multipliers $u = \exp\left(\nu\Delta t + \sigma\sqrt{\Delta t}\sqrt{\frac{1-p}{p}}\right)$ and $d = \exp\left(\nu\Delta t - \sigma\sqrt{\Delta t}\sqrt{\frac{p}{1-p}}\right)$, respectively. The simplified form (3.9) is a first order approximation of the latter formulas for small $s_B = \sigma\sqrt{\Delta t}$ and $\nu = 0$.

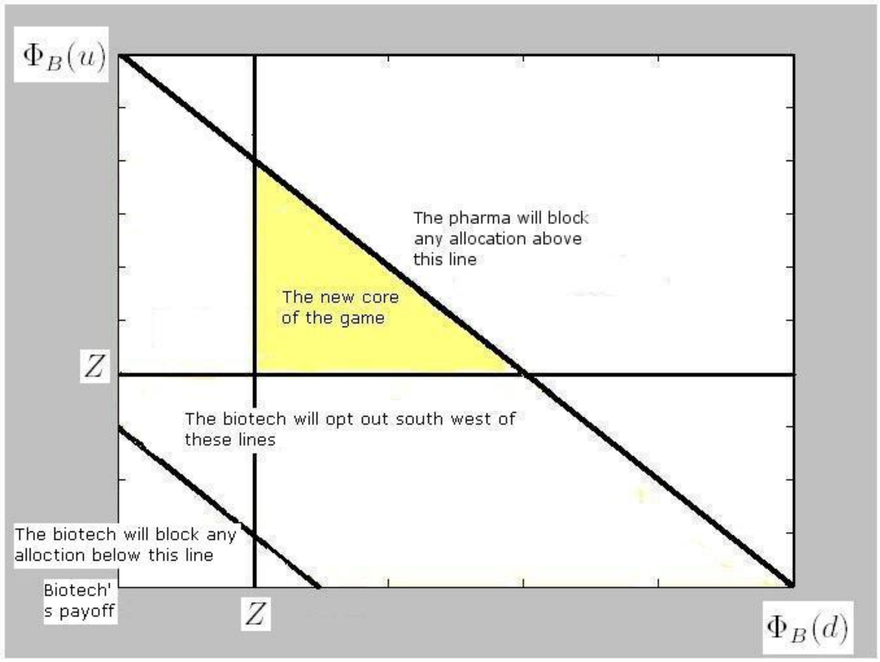

FIGURE 1. The core of the cooperative game with the Biotech having the Opt-out option

the pharmaceutical company receives the residual payoff. The core in the scenario payoff space is given by[3]:

$$x^*_B \leq p \max\{\Phi_B(u), Z\} + (1-p)\max\{\Phi_B(d), Z\} \leq x^*_{B+P}.$$

Figure 1 illustrates the options effect on the core of the game. The core is now the shaded area. Three distinct cases can arise:

(1) The opt-out payoff lines intercept below (southwest of) the core. This is equivalent to $Z \leq x^*_B$. In the case the option does not change the set of admissible sharing arrangements in expected values. However, in contrast to the situation without opt-out option, not all payoffs on iso-expectation lines are admissible.
(2) The opt-out lines intercept in the core ($x^*_B \leq Z \leq x^*_{B+P}$) This is depicted in the figure 1. For this to happen Z has to be above the payoff in the downside but less than the payoff in the upside. The change in the core is in favour of the option owner, i.e., the option improves the option holders bargaining position.
(3) The opt-out lines intercept above (northeast of) the core ($Z \geq x^*_{B+P}$). In this case the core is empty. This happens only if the opt-out payoff Z exceeds the payoff in the upwards scenario.

[3] x^*_B, x^*_{B+P} are given by equation (3.8) with q replaced by p.

The above relationships between Z and the payoff core translate to the relationships developed in section 3.3 for the relationship between Z and the value core in the complete markets case.

3.5. **The effect of risk aversion.** Having dealt with the situation of hedging in complete markets and of risk-neutral agents in the absence of markets, we will now discuss an arguably more realistic situation where we assume that the agents are risk averse with possibly different levels of risk aversion. This is a sensible assumption for example in partnership negotiations between well-diversified and well endowed pharma majors and relatively small biotech companies with serious cash constraints and few drug candidates close enough to the market to raise additional equity capital. We assume that there are no partial hedging opportunities. The presence of possibly different levels of risk aversion introduces two interesting issues:

(1) If two players have different levels of risk aversion they should be willing to trade off risk. This brings up the question how can risk be shared in an efficient way?
(2) If two players have different levels of risk aversion, they may well come to different conclusions about the most desirable contract design. A risk averse partner might be willing to pay the costs associated with waiting in order to have an option that will reduce her risk - a less-risk averse partner might not agree. How can these differing preferences be reconciled?

We address the first issue in section 3.6 where the agents have only one possible project design available. We introduce the problem of design choice in section 3.7 where agents can choose to set up cooperative options or not. In section 3.8 we discuss the effect of non-cooperative options.

3.6. **Cooperative games and risk aversion.** We will illustrate the risk sharing issues using our simple two agent partnership contract, without any options. This section, which is largely based on Suijs and Borm (1999), sets the scene for the analysis of options effects under risk-aversion.
We assume, as before, that the stochastic payoff X is modelled as a flip of a possibly biased coin. The two agents have different attitudes towards risk: the biotech is more risk averse than the pharma. We model their payoff preferences via expected utility functions [4]. Since we assume that the agents' perception of risk are fully captured by an expected utility, it is possible to gauge how much a risky gamble would be worth to each player: Given a gamble X, what deterministic payment m_i would make agent i indifferent to receiving m_i for sure or taking the gamble? This value is the *certainty equivalent* of the risky gamble; it satisfies $u_i(m_i(X)) = \mathbf{E}[u_i(X)]$, i.e.,$m_i(X) = u_i^{-1}(\mathbf{E}[u_i(X)])$. For illustrative purposes we choose exponential utility functions: $u_B(X) = -e^{-\frac{X}{\beta_B}}$, $u_P(X) = -e^{-\frac{X}{\beta_P}}$, with $\beta_P > \beta_B$.

[4]Agent i prefers an uncertain payoff X over an uncertain payoff Y if $\mathbf{E}[u_i(X)] > \mathbf{E}[u_i(Y)]$, where u_i is a suitable utility function.

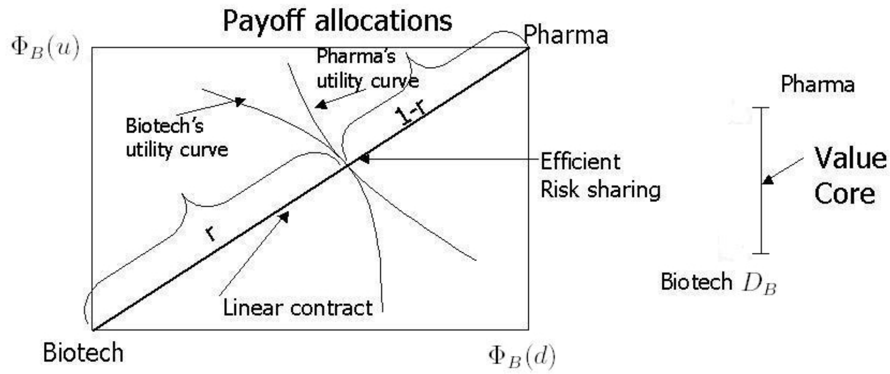

FIGURE 2. Efficient risk sharing

What is the optimal way for the companies to share the risk involved in a joint project? We will focus on linear contracts, i.e., agreements involving a deterministic payment D_i and a share r_i of an uncertain payoff X. The total payoff of agent i from the joint project will be

$$\Phi_i(\omega) = D_i + r_i X(\omega). \tag{3.10}$$

Such simple royalty contracts are commonplace in business, in particular in licensing agreements in the pharmaceutical industry. This type of contract is illustrated in figure 2; we wish to determine an allocation along the diagonal in the space of state-payoffs. The closer the agreement point is to the origin, the more risk is taken up by the biotech and the less risk by the pharma. In the case of were risk-neutral companies, the utility indifference curves are straight lines. If agents are risk averse then the indifference map is no longer linear as can be seen in figure 2.

Both agents prefer to take up as little risk as possible. However, the risk aversion induced by the concave utility function reduces the marginal benefit of a decrease in risk taking. Furthermore, this rate of reduction of marginal benefits will be different for both players, in view of their differing risk aversion levels. The royalty rate r that maximises the perceived total value[5] of the game is achieved where the marginal value of taking up some infinitesimal fraction of the risky project is the same for both agents. If the marginal benefits were different, we would be able to add to the total value by taking away an infinitesimal amount of risk from the player with the smaller marginal benefit and giving it to the player with the larger marginal benefit. This can be seen in figure 2.

Formally, the agents solve the maximisation problem

[5]In the context of a partnership, players cooperate to maximise the total value of the contract. We model this objective by assuming that the players wish to maximise the sum of their certainty equivalents.

$$
\begin{array}{rll}
\overline{X}_{B+P} = & \max & m_B(\Phi_B(\omega)) + m_P(\Phi_P(\omega)) \\
& \text{s.t.} & D_B + D_P = 0 \\
& & r_B + r_P = 1 \\
& & r_B, r_P \geq 0,
\end{array} \tag{3.11}
$$

where (Φ_B, Φ_P) are given by equation (3.10) with

$$X(\omega) = \omega C_{B+P} - I_{B+P},\ \omega \in \{u, d\} \tag{3.12}$$

This is a standard problem in the risk sharing literature Christensen and Feltham (2002), Wilson (1968). Under our assumption of an exponential utility function we have $m(D+Y) = D + m(Y)$ for any deterministic payoff D and stochastic payoff Y. The problem therefore reduces to

$$\overline{X}_{B+P} = \max_{0 \leq r_B \leq 1} (m_B(r_B X) + m_P(1 - r_B)X)) \tag{3.13}$$

with a first order optimality condition

$$\frac{dm_B(r_B X)}{dr_B} = \frac{dm_P((1 - r_B)X)}{dr_B}.$$

For the exponential utility function the optimal share of risk for each player is proportional to their risk tolerance

$$r_i^* = \frac{\beta_i}{\beta_B + \beta_P}.$$

Although this condition specifies how much risk each player will take, it does not determine the total payoff, as the deterministic amount D that agents exchange is not constrained and is determined in negotiation. The stochastic cooperative game is again reduced to a deterministic game with a value core in the standard sense.

The core of the game, subject to optimal risk sharing, is now specified by the following conditions

$$
\begin{array}{rcl}
m_B(r_B X(\omega)) + D_B & \geq & \overline{X}_B \\
m_P(r_P X(\omega)) + D_P & \geq & 0 \\
D_B + D_P & = & 0 \\
r_i & = & \dfrac{\beta_i}{\beta_B + \beta_P},\ i \in \{B, P\}.
\end{array}
$$

where $\overline{X}_B = m_B(\omega C_B - I_B)$ with $\omega \in u, d$. The first two conditions guarantee that each agent's estimation of the value is at least as good as going alone. The third condition is a conservation law: the total amount that changes hands is zero and the forth condition will ensure efficient risk sharing. The value core is again one-dimensional, involving only the deterministic amounts D_i that the two companies will exchange:

$$\overline{X}_B - m_B(r_B X(\omega)) \leq D_B \leq m_P(r_P X(\omega)), \tag{3.14}$$

and $D_P = -D_B$.
It is interesting to consider the special case when $C_B = C_{B+P}$, $I_B = I_{B+P}$ and $s_B = s_{B+P}$. In this case there are no synergies in the traditional sense. It is not surprising that when the agents are risk-neutral the only core allocation for the biotech is $\phi_B = C_B - I_B$. Nothing is gained by a partnership. However, if agents are risk averse, cooperation is valuable. It can be seen from equation (3.14) that the core has non-empty interior. In other words, there are gains to be made by cooperating, because of risk sharing. The corresponding risk sharing set can be thought of as the pure *risk sharing* core of the contract:

$$\overline{X}_B - m_B(r_B[\omega C_B - I_B]) \leq D_B \leq m_P(r_P[\omega C_B - I_B]). \tag{3.15}$$

An interesting observation can be made here. As the risk preferences of the two companies diverge ($\beta_B << \beta_P$), it is optimal for the pharma to take a larger amount of risk ($r_B \to 0$, $r_P \to 1$). As can be seen from equation (3.15), the risk sharing core of the deal is enlarged: Risk sharing synergies become more valuable.
To summarize, the presence of risk aversion does still allow for the reduction of the stochastic cooperative game to a deterministic game, just as in the previous section. Suitable linear sharing rules are Pareto-efficient, see Pratt (2000), and allow an optimal risk sharing. Once the agents agree that they wish to share risk optimally, the cooperative game is played on the deterministic amount that the agents will exchange.

3.7. **Cooperative options and risk aversion.** To illustrate the issue of disagreement on contract design, let us revisit the example of the cooperative option. As before we suppose the agents have to decide whether or not to pay amount k_B, or k_{B+P} in partnership, up front in order to postpone the launch decision until uncertainty is resolved.
In this situation the value of the joint project with the option becomes

$$\overline{X}_i = m_i(\omega C_{B+P} - I_{B+P})^+ - k_{B+P}).$$

Note that this is agent i's personal valuation of the joint project. For different levels of risk aversion, it is possible that the agents will order the two designs differently and disagree which design to choose in a partnership.
In the presence of complete markets the partnership would choose the design with the highest no-arbitrage value. In the present situation it would seem sensible to assume that the coalition will choose the project design that maximizes the total certainty equivalent for the coalition, provided they share the risks optimally.
If the biotech develops the project alone it has to choose between not investing in the option, which has value $\overline{X}_B = m_B(\omega C_B - I_B)$, or investing in the option, which has value $\overline{X}_B^O = m_B((\omega C_B - I_B)^+ - k_B)$. Similarly the partnership has a choice between $\overline{X}_{B+P}$ given in (3.11) and

$$\begin{aligned}\overline{X}^O_{B+P} &= \max_{r_B,r_P} \quad m_B(\Phi_B(\omega)) + m_P(\Phi_P(\omega)) \\ &\quad \text{s.t.} \quad D_B + D_P = 0 \\ &\qquad\qquad r_B + r_P = 1 \\ &\qquad\qquad r_B, r_P \geq 0,\end{aligned}$$

where (Φ_B, Φ_P) are given by equation (3.10) with $X(\omega) = (\omega C_{P+B} - I_{B+P})^+ - k_{B+P},\ \omega \in \{u, d\}$ and the probability of event u is p.
Assuming that it is optimal for the partnership to set up the option, i.e. $\overline{X}^O_{B+P} > \overline{X}_{B+P}$, the core of the game becomes a negotiation over the payments D_B, D_B[6]

$$\begin{aligned} m_B(\Phi_B(\omega)) + D_B &\geq \max\{\overline{X}_B, \overline{X}^O_B\} \\ m_P(\Phi_P(\omega)) + D_P &\geq 0 \\ D_B + D_P &= 0 \\ r_i &= \frac{\beta_i}{\sum_j \beta_j}. \\ \Phi_i(\omega) &= r_i((\omega C_{B+P} - I_{B+P})^+ - k_{B+P}) \end{aligned}$$

As before, the core of the cooperative options game can be expressed in terms of the fixed amounts D_B that is exchanged:

$$\max\{\overline{X}_B, \overline{X}^O_B\} - m_B(\Phi_B(\omega)) \qquad \leq D_B \leq m_P(\Phi_P(\omega)).$$

Similarly to the risk-neutral case, the *asset value* of the contract is the core of the contract in the absence of flexibility and the *option value* is the added value from flexibility. Each of these values has a risk sharing component in the sense that if the partnership has no synergies other than risk sharing, both the asset and the option cores are non-empty.

3.8. **Non-cooperative Options.** Let us now revisit our example of unilateral flexibility in the context of risk-aversion. To simplify the exposition we assume that there are no cooperative options. As before, we assume that the biotech has the right to opt out of co-marketing. However, here we will consider a more general opt-out agreement where the biotech receives a fixed amount Z (milestone payment) and royalties $r_R C_{B+P}$ on the revenue C_{B+P} if the option is exercised. The biotech company can exercise this option after uncertainty is resolved and, assuming a linear risk-sharing agrement as before, would do so if and only if $r_B(\omega C_{B+P} - I_{B+P}) + D_B \leq Z + r_R \omega C_{B+P}$

[6]For our example of exponential utility functions, the risk sharing rule only depends on the risk tolerance β_i of each player and not on the distribution of the gamble payoffs. Therefore we do not need to solve the optimal risk sharing problem again; it is the same as before with solution $r_i = \frac{\beta_i}{\sum_j \beta_j}$. For other forms of HARA utility functions, such as logarithmic or power law utilities, the optimal share of risk for each agent depends on the payoff at each state and therefore would be different in the presence of flexibility.

where r_B is the biotech's share of the revenues from co-marketing and $\omega \in \{u, d\}$ is the state of the world at exercise. The problem of finding an optimal risk sharing arrangement now becomes:

$$\begin{array}{ll} \max\limits_{r_B, r_P, D_B, D_P} & m_B(\max\{r_B X(\omega) + D_B, Z + r_R \omega C_{P+B}\}) \\ & +m_P(\min\{r_P X(\omega) + D_P, X(\omega) - r_R \omega C_{P+B} - Z\}) \\ \text{s.t.} & r_B + r_P = 1 \\ & r_B, r_P \geq 0 \\ & D_B + D_P = 0. \end{array}$$

In this case the computation of the optimal royalty value and he deterministic payoff are not decoupled. The optimization is a non-smooth non-convex problem, due to the max and min terms in the objective function. Furthermore, we are no longer guaranteed to find an optimal linear risk sharing rule.

One way to avoid these complications is to require $r_R = r_B$, i.e. the royalty rate is independent of options exercise and set at the optimal level for the game without the unilateral option, as explained in the previous section. In this way, we allow the biotech to opt-out unilaterally but we do not affect the efficient risk allocation. In this case, the non-cooperative option will be exercised only if the amount Z, the milestone, is higher than the amount D_B the biotech receives from the pharma if it does not opt out. The core now satisfies the following conditions

$$\begin{array}{rcl} m_B(r_B X(\omega)) + \max(D_B, Z) & \geq & m_B(\omega C_B - I_B) \\ m_P(r_P X(\omega)) + \min(D_P, -Z) & \geq & 0 \\ D_P + D_B & = & 0 \\ r_i & = & \dfrac{\beta_i}{\beta_B + \beta_P}, \ i \in \{B, P\}. \end{array}$$

Solving for D_B gives the reduced representation

$$\max\{Z, m_B(X(\omega)) - m_B(r_B(\omega C_{B+P} - I_{B+P}))\} \leq D_B \leq m_P(r_P(X(\omega)))$$

Similarly to section 3.3.3, we can distinguish three cases depending on the value of Z and the associated exercise decisions.

4. Cooperative real options games in continuous time

In this section, we will show how the model developed in section can be generalized to games with continuous time dynamics and multiple agents. We will focus on cooperative real options games with the following schedule of events:

(1) At time $t = 0$ the agents decide which coalition to form and the coalition decides which contract design to choose.
(2) At time $t = T$ the coalition observes uncertainty and decides on the exercise of any available cooperative options.

(3) Payoff is instantly realized and shared.

Non-cooperative options, if they exist, are exercised immediately after the exercise of the cooperative options. Our goal is to provide guidance on fair and sensible sharing arrangements.

Our setting assumes a European options framework, where optimal timing of options exercise is not an issue. Extending the formulation of a stochastic cooperative game, see Suijs and Borm (1999) , we formally specify a *European cooperative options game* as a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ and a tuple (N, A_S, B_S, X_S), where

- N is a finite set of agents,
- A_S are the sets of contract design choices available to the coalitions $S \subseteq N$ at time $t = 0$, before uncertainty is resolved,
- $B_S(a, \omega)$ are the sets of actions available to the coalitions $S \subseteq N$ at time $t = T$, provided S has chosen design $a \in A_S$ at time $t = 0$ and the event $\omega \in \Omega$ occurs; these are the options available to the coalition,
- $X_S(a, b, \omega)$ are the payoffs to the coalitions S at time $t = T$, provided the coalition has chosen to implement design $a \in A_S$ and has exercised the option $b \in B_S(a, \omega)$ after the event $\omega \in \Omega$ occurred.

4.1. **Cooperative options in complete markets.** We begin with a discussion under a complete markets assumption, which will allow us to use standard arbitrage pricing arguments, see e.g. Hull (2003) for details. We assume that there is a complete arbitrage-free market of traded assets. This is equivalent to the existence and uniqueness of a martingale measure $\mathcal{Q}$ for the market such that any claim $X(T)$ has a no-arbitrage price of

$$x = \mathbf{E}^{\mathcal{Q}}[e^{-\rho T} X(T)],$$

where ρ is the risk-free rate, see Harrison and Pliska (1981). In particular the coalition payoffs $X_S(a)$, following optimal options exercise at $t = T$, can be priced in this way as

$$x_S(a) = \mathbf{E}^{\mathcal{Q}}[e^{-\rho T} \max_{b \in B_S(a,\omega)} X_S(a, b, \omega)].$$

The value $x_S(a)$ is called the risk-neutral valuation of the payoff to coalition S. Given design $a \in A_S$ and state $\omega \in \Omega$, a coalition S will choose the action $b^*_S(a, \omega) \in B_S(a, \omega)$ that maximizes its payoff $X_S(a, b, \omega)$. Since all uncertainty has been resolved this is a deterministic optimization decision, resulting in a contingency plan b^*_S for coalition S.

Under our assumptions, the risk-neutral value x_S is the only value that is consistent with the observed asset prices in the sense that it does not allow for arbitrage profits through trading. The coalition can sign the contract a and at the same time short-sell the associated replicating portfolio. This will result in an immediate payoff of $x_S(a)$ and no future payoff when uncertainty is resolved because the replicating portfolio, suitably re-balanced during the trading period $[0, T]$, will hedge all future payoffs from the contract. The coalition then uses the risk-neutral valuation to choose the design $a_S^* \in A_S$ that maximizes the total risk neutral payoff

$$x_S = \max_{a \in A_S} x_S(a).$$

Given the optimal design choices a_S^* and contingency plans $b_S^*(a_S^*, .)$, the real options game reduces to a deterministic cooperative game

$$\Gamma = (N, \{x_S\}_{S \subseteq N}).$$

Cooperative game theory is primarily concerned with payoff sharing rules $\Phi(\omega) = (\Phi_i(\omega), i \in N)$, where $\Phi_i(\omega)$ specifies the payoff for agent i if state $\omega \in \Omega$ has occurred. The sharing rule has to satisfy

$$\sum_{i \in N} \Phi_i(\omega) = X_N(a^*, b_N^*(\omega), \omega), \forall \omega \in \Omega.$$

The reduced deterministic game Γ, however, is specified in terms of risk-neutral *values* x_S, not in terms of final coalition payoffs. The focus shifts from payoff allocations $\Phi(\omega)$, specified for each future state $\omega \in \Omega$ to value allocations $\phi = (\phi_i, i \in N)$, where ϕ_i is agent i's share of the total value x_N. A payoff sharing rule $\Phi_i(\omega)$ for agent i is compatible with a value allocation ϕ_i if

$$\phi_i = \mathbf{E}^{\mathcal{Q}}[e^{-\rho T} \Phi_i(\omega)].$$

In view of the complete markets assumption, agent i can always achieve the value ϕ_i associated with a payoff sharing rule Φ_i by shorting the replicating portfolio for Φ_i, which will result in an immediate payoff ϕ_i and no future payoffs. The discussion of sensible sharing arrangements has moved from the space of functions $\Phi : \Omega \to R^N$ to the space of vectors $\phi \in R^N$.

The complete markets assumption is frequently made in the literature, see e.g. Burnetas and Ritchken (2005), Dixit and Pindyck (1994), although it is rather restrictive. The above analysis remains conceptually correct in a situation without hedging opportunities, provided the agents are risk-neutral and the following conditions apply:

- The martingale measure $\mathcal{Q}$ is replaced by the, possibly subjective, probability measure $\mathcal{P}$ of the underlying probability space and agents have homogenous beliefs about this measure[7].
- The agents agree on a suitable discount rate ρ.

Notice that in the absence of a complete market, the payoff sharing rule $\Phi(\omega)$, $\omega \in \Omega$, needs to be specified in the contract, because agents cannot hedge their risks through shorting a replicating portfolio. However, if all agents are risk neutral then they will be indifferent between payoff allocations that result in the same value ϕ. For example one agent may receive the stochastic payoff $X_N(a^*, b^*(\omega), \omega)$, taking all the risk, and pay all other

[7]If agents have heterogenous beliefs efficient side betting may be employed, see Feltham and Christensen (2002) page 125-6.

agents their deterministic value share ϕ_i. We will deal with the more interesting risk-averse case in the absence of a market later.

Since the cooperative real options game in a complete market reduces to the deterministic cooperative game Γ we can apply the standard solution concepts of cooperative game theory to the value sharing agreements ϕ, see e.g. Young (1994).

The value-core of the game consists of all allocations $\phi = (\phi_i, i \in N)$ such that no sub-coalition S can receive more value from going alone, i.e.,

$$x_S \leq \phi_S = \sum_{i \in S} \phi_i, \ \forall S \subseteq N. \tag{4.1}$$

All payoffs are allocated to the agents in the grand coalition N, i.e.,

$$x_N = \phi_N. \tag{4.2}$$

The Shapley value ϕ_i of agent i is calculated as the Shapley value of the deterministic reduction Γ:

$$\phi_i = \sum_{S \subseteq N} \frac{|S|!(N - |S| - 1)!}{N!}(x_{S \cup i} - x_S)$$

We close this section with two illustrative examples of what one may call the *Black-Scholes-Shapley value*, which results if the standard Black-Scholes assumptions apply.

Example 1. Consider a game with three agents.

(1) The first agent owns a project with stochastic payoffs with present value X_0. The value follows a geometric Brownian motion with volatility σ^2. The agent will have to sell the project at time T for the then market price X_T.
(2) The second agent can offer an expansion option to the project. This call-like option will cost k_C to set up, has an exercise cost (strike price) of K_C and can only be exercised at time T. The Black-Scholes value of this European call option is

$$C = X_0 \Phi(b_1) - K_C e^{-\rho T} \Phi(b_2),$$

where $\Phi(x)$ is the standard normal cumulative distribution function, $d_1 = \frac{\log(\frac{X_0}{K_C}) + (\rho + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma T$.
(3) The third agent can offer an abandonment option to the first agent. This put-like option will cost k_P to set up, offers a sure payment $K_P < K_C$ if exercised, and can again only be exercised at time T. The Black-Scholes value of this put option is

$$P = K_P e^{-\rho T} \Phi(-d_2) - X_0 \Phi(-d_1).$$

The design set $A_{\{1,2,3\}}$ for the grand coalition contains four designs: No option, include the call option, include the put option, or include both options. Set $A_{\{1\}}$ has only one design: no option, sets $A_{\{2\}}$,$A_{\{3\}}$,$A_{\{2,3\}}$ are empty and sets $A_{\{1,2\}}$ and $A_{\{1,3\}}$ have two obvious elements of no option or including

the option that the respective partner brings to agent 1. The optimal design for this problem is to include an option if its risk-neutral value exceeds its set up cost, i.e., $C \geq k_C$ or $P \geq k_P$, respectively.
The optimal exercise strategy for the options is to exercise the call option if the price $X_T \geq K_C$ and to exercise the put option if $X_T \leq K_P$. The Shapley-value allocation is

$$\phi = (X_0 + \frac{1}{2}(C - k_C)^+ + \frac{1}{2}(P - k_P)^+, \frac{1}{2}(C - k_C)^+, \frac{1}{2}(P - k_P)^+).$$

This allocation is very intuitive; the first agent receives the whole value of his project and half the option value added by each of the other two agents. The other two agents receive half the value of the option they bring to the deal. The following example is less intuitive.

Example 2. Again we consider three agents.

(1) The first agent owns the same project as in Example 1.
(2) The second agent can offer both an expansion and an abandonment option to the project at costs k_C and k_P, strike prices K_C and K_P and values C and P respectively. The Black-Scholes values C and P are given by the respective formulas in the above example.
(3) The third agent can lower the strike price for the call option to $K'_C < K_C$ and can increase the strike price of the put option to $K'_P > K_P$ (with $K'_C > K'_P$), without any additional costs, therefore increasing the value of the two options to C' and P'.

Now the design sets $A_{\{1,2,3\}}$ and $A_{\{1,2\}}$ contain four designs: No options, the call alone, the put alone, and both options. Sets $A_{\{1\}}$ and $A_{\{1,3\}}$ only contain the no-option design, sets $A_{\{2\}}$,$A_{\{3\}}$,$A_{\{2,3\}}$ are empty. As before, the optimal design is to include an option if its risk-neutral value exceeds its set up cost, i.e., $C \geq k_C$ or $P \geq k_P$ or $C' \geq k_C$ or $P' \geq k_P$. Assuming it is optimal to include all options, the Shapley-value allocation is

$$\phi_1 = X_0 + \frac{1}{6}(C + P) + \frac{1}{3}(C' + P') - \frac{1}{2}(k_C + k_P)$$
$$\phi_2 = \frac{1}{6}(C + P) + \frac{1}{3}(C' + P') - \frac{1}{2}(k_C + k_P)$$
$$\phi_3 = \frac{1}{3}(C' + P') - \frac{1}{3}(C + P).$$

There is no straight-forward intuition for this Shapley-value allocation.

4.2. **Unilateral options in complete markets.** We will now add the possibility that one agent, say agent i, owns a unilateral option. The timing is now as follows:

(1) At time $t = 0$ the agents decide which coalition to form and which contract design to choose.
(2) At time $t = T$ the coalition observes uncertainty and decides on the exercise of any available options.

(3) Agent i decides on the exercise of her option unilaterally, immediately after the exercise decision for the cooperative options.
(4) Payoff is instantly realized and shared.

To simplify notation we will ignore discounting for the remainder of this section.

Suppose agent i has the option to leave the partnership for an agreed deterministic payoff Z once uncertainty is resolved. Such an option will only be exercised in states ω where Z exceeds the payoff $\Phi_i(\omega)$ the agent would receive in the partnership after the resolution of uncertainty and the exercise of the cooperative options.

We will make a few observations, which we state in the form of propositions.

Proposition 4.1. *The unilateral option does not change the total value of the contract for any coalition.*

Proof. Assume that Φ_i is the option owner's agreed share of the payoff before the unilateral option is exercised. The value of the contract to the option owner is then $\phi_i = \mathbf{E}^{\mathcal{Q}}[\max\{\Phi_i(\omega), Z\}]$ while the sum of the value to everyone else in a coalition S with agent i is the residual value $\phi_{S-i} = \mathbf{E}^{\mathcal{Q}}[X_S(\omega) - \max\{\Phi_i(\omega), Z\}]$. Here we have used the fact that $\Phi(\omega)$ satisfies $\sum_{i \in S} \Phi_i(\omega) = X_S(\omega)$. The total value to the coalition S is therefore

$$\phi_S = \phi_i + \phi_{S-i} = \mathbf{E}^{\mathcal{Q}}(X_S(\omega)) = x_S,$$

which is independent of the option. Q.E.D.

Proposition 4.2. *The value share of the option owner satisfies $\phi_i \geq Z$, independently of the payoff sharing arrangement.*

Proof. Let $\Phi_i(\omega)$ be the payoff sharing rule for the agent who owns the non-cooperative option. Then her value will satisfy

$$\phi_i = \mathbf{E}^{Q}[\max\{\Phi(\omega), Z\}] \geq \mathbf{E}^{Q}[Z] = Z.$$

Q.E.D.

Proposition 4.3. *The opt-out option changes the value-core to*

$$\begin{aligned} \max\{x_i, Z\} &\leq \phi_i \\ x_S &\leq \phi_j = \sum_{j \in S} \phi_j \quad \forall S \subseteq N,\ S \neq \{i\} \\ x_N &= \phi_N, \end{aligned}$$

where i is the owner of the non-cooperative option.

Proof. Recall the conditions (4.1) of the value core of the game without the unilateral option. The first inequality in the proposition follows immediately from Proposition 4.2 and the inequality (4.1) for $S = \{i\}$. The constraint in (4.1) for coalitions $S : i \notin S, S \subseteq N$ are not affected by the presence of the non-cooperative option since the owner of the option is not a member of these coalitions. Due to Proposition 4.1, the core conditions (4.1) for coalitions $S : i \in S, S \neq \{i\}, S \subseteq N$ are also not affected. Q.E.D.

Proposition 4.3 has some interesting consequences. At first glance it seems difficult to assess what the added bargaining value of the opt-out option is for its owner. The option adds value for its owner, provided there exists states $\omega \in \Omega$ with positive probability such that the payoff $\Phi(\omega) < Z$ which, however, requires an agreed payoff sharing rule in the first place. Proposition 4.3 shows that the value core remains independent of the payoff sharing rule. A comparison with the value core without the unilateral option (4.1) shows that the option adds bargaining value to its owner agent i only if $Z > x_i$.
Traditional real options wisdom has it that an option always has a non-negative value for its owner. Proposition 4.3, however, shows that this is not necessarily the case for non-cooperative options in a partnership. If the value of the option is more than the agent's contribution to the coalition ($Z > x_N - x_{N-i}$) then the core becomes empty. The option destroys the partners' incentive to forge a deal and consequently, the unilateral opt-out option will not be realized.
The foregoing results for unilateral options can be extended to opt-out options owned by coalitions $S \subseteq N$. If $\mathcal{O}$ is the collection of coalitions S that have non-cooperative opt-out options with payoff Z_S then the conditions for the value core are

$$\max\{x_S, Z_S\} \leq \phi_S = \sum_{i \in S} \phi_i, \ \forall S \in \mathcal{O} \tag{4.3}$$

$$x_S \leq \phi_S = \sum_{i \in S} \phi_i, \ \forall S \subseteq N, S \notin \mathcal{O}. \tag{4.4}$$

Proposition 4.4. *For every value allocation in the core there is a payoff allocation such that the unilateral option is never exercised.*

Proof. Let $\phi = (\phi_j, \ j \in N)$ be an allocation in the value core and $\Phi = (\Phi_j, \ j \in N)$ be an associated payoff allocation. Then $\epsilon = \phi_i - Z \geq 0$ in view of Proposition 4.3. If agent i owns the option and we define the new payoff allocation

$$\begin{aligned} \tilde{\Phi}_i(\omega) &= Z + \epsilon \\ \tilde{\Phi}_j(\omega) &= \Phi_j(\omega) + \frac{\Phi_i(\omega) - Z - \epsilon}{|N| - 1}, \ j \neq i, \end{aligned}$$

then the corresponding value allocation satisfies

$$\begin{aligned} \tilde{\phi}_i &= \mathbf{E}^Q[\tilde{\Phi}_i(\omega)] = \phi_i \\ \tilde{\phi}_j &= \mathbf{E}^Q[\tilde{\Phi}_j(\omega)] \\ &= \mathbf{E}^Q[\Phi_j(\omega) + \tfrac{\Phi_i(\omega) - Z - \epsilon}{|N|-1}] \\ &= \mathbf{E}^Q[\Phi_j(\omega)] + \tfrac{\mathbf{E}^Q[\Phi_i(\omega)] - Z - \epsilon}{|N|-1} \\ &= \mathbf{E}^Q[\Phi_j(\omega)] + \tfrac{\phi_i - Z - \epsilon}{|N|-1} \\ &= \phi_j. \end{aligned}$$

Finally,

$$\sum_{k\in N}\tilde{\Phi}_k(\omega)=\tilde{\Phi}_i+\sum_{j\neq i}\tilde{\Phi}_j=\sum_{k\in N}\Phi(\omega)=X_N(\omega).$$

Hence the allocation $\tilde{\Phi}$ is in the payoff core and corresponds to a value allocation ϕ. The unilateral option will not be exercised because $\tilde{\Phi}_i(\omega)\geq Z$ for all ω. Q.E.D.

The condition $\phi_i \geq \max\{x_i, Z\}$ in Proposition 4.3 shows that the non-cooperative option provides a second participation benchmark, in addition to the go-alone value x_i, for its owner. The threat of exercising the option is just as credible as the threat of breaking away from a coalition because the go-alone value is larger. Proposition 4.4 is interesting in this regard because it shows that non-cooperative options are powerful bargaining tools, even though the agents may be able to avoid their exercise through a suitable payoff sharing arrangement. This is reminiscent to the fact that the threat of leaving the coalition is a powerful tool that is not exercised, provided the agent's share is in the value core.

An interesting case, not covered by the discussion above, occurs if a unilateral option depends on an exogenous uncertainty, i.e., an uncertainty that adds value to the option owner but is not part of the contract. In such cases the options owner may well exercise the non-cooperative option sub-optimally with regard to the deal but optimally with regard to her overall objective. For example a biotech company may opt out of a co-development contract because some other project in their portfolio, not covered by the contract, is very promising and it makes sense for them to redirect the funds Z from the contract to the external project. In such circumstances it is possible that the non-cooperative option will add value to a coalition as a whole and not only to its owner. A study of such options effects would be interesting but is beyond the scope of this paper.

4.3. **Risk aversion in the absence of hedging opportunities.** We will next consider a situation where risk averse agents negotiate a partnership contract without hedging opportunities through traded assets. We model risk aversion through utility functions u_i, which we assume to have hyperbolic absolute risk aversion (HARA) in order to be able to use the fact that linear contracts are efficient, see Pratt (2000). Prominent examples of HARA utility functions are exponential utilities $u_i(x)=a_i\exp(b_ix),\ b_i,a_i<0$ (constant absolute risk aversion), power law utilities $u_i(x)=a_ix^{b_i},\ a_i>0,\ 0<b_i<1$ (constant relative risk aversion) and, the limiting case as $b_i\to 0$, logarithmic utilities $u_i(x)=a_i\log(x)$.

Agent i's certainty equivalent of a stochastic payoff X at time T is the amount $m_i(X)$ that would make the agent indifferent between receiving $m_i(X)$ for sure at time $t=0$ or receiving the stochastic payoff X at time $t=T$. Formally,

$$m_i(X)=u_i^{-1}(\mathbf{E}[u_i(e^{-\rho T}X)]),$$

where ρ is the risk-free discount rate.

We assume the timing of the cooperative game is as before: Agents agree on the coalition and an options design at $t = 0$. Then, after uncertainty is resolved, they decide on options exercise at $t = T$ and receive an instant payoff.

There are two complications compared to the complete markets case:

- In the complete markets case the final payoff sharing rule $\Phi(\omega)$ is irrelevant because of the presence of the replicating portfolio and associated hedging of risks. Sharing happens at the level of risk-neutral values. In the present case, the coalition needs to specify an explicit payoff sharing rule, taking account of the agents' risk-aversion.
- Due to possibly differing risk preferences, the agents in a coalition S might not agree which design $a \in A_S$ they prefer. To overcome this problem, we will assume that a coalition S makes decisions in the interest of maximizing the sum of the certainty equivalents m_i of the agents $i \in S$.

We will make use of the theory of linear contracts and efficient risk sharing Pratt (2000), Wilson (1968) in the development of the model. In a linear contract, each agent i in a coalition S receives a deterministic payoff D_i before uncertainty is resolved and a proportion r_i of the risky payoff $X_S(a, b, \omega)$, i.e., its payoff allocation is of the form

$$\Phi_i(\omega) = D_i + r_i X_S(a, b, \omega).$$

To achieve complete sharing, we assume that the deterministic payoffs D_i and royalties r_i satisfy

$$\sum_{i \in S} D_i = 0, \ \sum_{i \in S} r_i = 1, \ r_i \geq 0 \ \forall i \in S. \tag{4.5}$$

As mentioned above, we will also assume that coalitions act in the interest of maximising the sum of the certainty equivalents of their members. Applying this principle to the admissible contracts (D_i, r_i) will ensure Pareto-efficiency. The corresponding total value for a given design $a \in A_S$ is given by

$$\overline{X}_S(a) = \max_{(D,r)} \sum_{i \in S} m_i(\max_{b \in B_S(a,\omega)} [D_i + r_i X_S(a, b, \omega)]) \tag{4.6}$$

subject to the constraints (4.5).

Note that the risk averse agents will always agree which of the options $b^* \in B_S(a, \omega)$ to choose because all uncertainty is resolved at the time of exercise. Also, this exercise policy is independent of the royalty rate r_i.

The optimization problem (4.6) decouples if

$$m(a + X) = a + m(X), \tag{4.7}$$

which is the case for exponential utilities. Under this assumption the optimization problem becomes

$$\overline{X}_S(a) = \max_{\sum_{i\in S} D_i = 0} \sum_{i\in S} D_i + \max_{\substack{r\geq 0 \\ \sum_{i\in S} r_i = 1}} \sum_{i\in S} m_i(r_i X_S(a, b^*(\omega), \omega)). \tag{4.8}$$

The deterministic shares D_i are inconsequential to this optimization problem. Optimal royalty rates r_i^* are chosen from the simplex and hence exist if the functions $f_i(r) = m_i(rX_S(a, b^*(\omega), \omega))$ are continuous and are unique if these functions are strictly concave, e.g. in the case of exponential utilities. If the functions f_i are differentiable, the optimal rates are characterized by the KKT conditions

$$\begin{array}{rcll} \frac{\partial f_i(r_i)}{\partial r_i} & = & \frac{\partial f_j(r_j)}{\partial r_j} & \text{if } r_i > 0, r_j > 0 \\ \frac{\partial f_i(r_i)}{\partial r_i} & \geq & \frac{\partial f_j(r_j)}{\partial r_j} & \text{if } r_i > 0, r_j = 0 \\ \sum_{i\in S} r_i & = & 1 & \\ r_i & \geq & 0 & \forall i. \end{array}$$

If (4.7) does not hold then the optimal allocations (r_i, D_i) have to be computed simultaneously. With $g_i(D_i, r_i) = m_i(D_i + r_i X_S(a, b^*(\omega), \omega)$ the KKT conditions become

$$\begin{array}{rcll} \frac{\partial g_i(D_i, r_i)}{\partial r_i} & = & \frac{\partial g_j(D_j, r_j)}{\partial r_j} & \text{if } r_i > 0, r_j > 0 \\ \frac{\partial g_i(D_i, r_i)}{\partial r_i} & \geq & \frac{\partial g_j(D_j, r_j)}{\partial r_j} & \text{if } r_i > 0, r_j = 0 \\ \sum_{i\in S} r_i & = & 1 & \\ r_i & \geq & 0 & \forall i \\ \frac{\partial g_i(D_i, r_i)}{\partial D_i} & = & \frac{\partial g_j(D_j, r_j)}{\partial D_j} & \forall i, j \\ \sum_{i\in S} D_i & = & 0. & \end{array}$$

Given the optimal allocation (r_i, D_i) for coalitions S and option designs $a \in A_S$, the coalition can now decide on the best design $a^* \in A_S$. This results in the value $\overline{X}_S = \max_{a\in A_S} [\overline{X}_S(a)]$.

The stochastic game with real options is therefore again reduced to a deterministic cooperative game

$$\Gamma_{CE} = (N, \{\overline{X}_S\}_{S\subseteq N}),$$

assuming that the agents choose the design a^*, exercise the options $b^*(a^*, \omega)$ and each agent takes a proportion $r_i^*(a^*)$ in the risky payoffs. As in the complete markets case, the standard solutions concepts of cooperative game theory apply.

A key assumption for the model is that the agents in a coalition S agree on a measure of total contract value associated with a given payoff sharing rule (D_i, r_i) and a design $a \in A_S$. We have here assumed that this total value is measured by the sum of the certainty equivalents. Similar models can be developed if total contract value is measured by a, possibly suitably weighted, sum of expected utilities.

4.4. **Non-cooperative options and risk aversion.** Let us finally turn to non-cooperative options. We assume that non-cooperative options are exercised instantly after the exercise of the cooperative options, if there are any. Whilst the general principles of the former section are transferable, it now becomes considerably more difficult to analyse options effects. The chief reason is that the unilateral options exercise will typically turn the certainty equivalents of the players into nonsmooth and possibly nonconcave functions. As a consequence, it might no longer be possible to split the risk efficiently in a linear way amongst the agents.

Assume, for example, that agent i can opt out and receive a fixed amount Z after uncertainty is resolved. Her payoff from the project if design $a \in A_S$ is chosen is

$$\Phi_i(\omega) = \max(D_i + r_i X_S(a, b^*(\omega), \omega), Z),$$

where $b^*(\omega)$ captures the contingency plan for the exercise of the cooperative option, prior to the exercise of the non-cooperative opt-out option. The certainty equivalent $m_i(\Phi_i(\omega))$ is now a nonsmooth function of r_i at the points $r_i = \frac{Z - D_i}{X_S(a,b^*,\omega)}$ where it becomes optimal to opt-out. As a result the certainty equivalent of all other players is nonsmooth at these points. The optimal risk-sharing is non-linear because it is contingent on opt-out. The players $j \in S \backslash \{i\}$ will have to agree on risk sharing rules for both cases, that player i stays in or opts.

There is a possible way out of this dilemma - instead of paying a fixed deterministic payments on opt-out, we pay agreed royalties and milestones, i.e., an agreed royalty rate r_i^O on the uncertain payoff plus a fixed payment Z. Deals of this type are commonplace, e.g., in drug licensing. The royalties force the option owner to retain some of the risk, even if she exercised the opt-out option. If we choose r_i^O to be the optimal rate for the case without the non-cooperative option, then optimal risk sharing between the agents is maintained. The payoff to the option owner is now $\max(D_i, Z) + r_i X_S(a, b^*, \omega)$. The option will only be exercised if $Z > D_i$. The core is shifted in favour of the option holder, similarly to the complete markets case.

5. Managerial Implications and Conclusions

We have drawn attention to sources for partnership synergies beyond the traditional economies of scale and scope. Our focus is on synergies in the presence of uncertainty, in particular risk sharing opportunities when partners have different attitudes to risk and the value of flexibility in a partnership.

Our models provide three main managerial insights. First, *partners with divergent risk attitudes gain more synergies from risk sharing in uncertain environments.* Teaming up with a partner with a different risk attitude can be very beneficial in this regard. Risk sharing opportunities increase both the *asset core*, i.e. the synergy set from traditional economies of scale and scope, as well as the *options core*, the set of synergies gained from flexibility. A prominent example where risk sharing synergies are particularly relevant are co-development contracts between pharma majors and smaller biotechnology companies.

Second, *what can a company do to improve its bargaining position?* Improving the bargaining position is equivalent to shifting the synergy set to a region of higher individual payoff or, in the risk-averse case, higher individual utility. This can be accomplished in several ways. Companies can aim to improve their go-alone capability, in a traditional way by improving the efficiency of operations, or by increasing out-of-the-deal opportunities. Alternatively, a company could increase the volatility that underlies its flexibilities, e.g. by improving its pipeline of innovative but high-risk projects. Finally, non-cooperative options, negotiated prior to a deal, can shift the synergy set in favor of the option holder.

Third, *non-cooperative options are a double-edged sword.* On the one hand they are valuable for individual partners because they cut off lower utility parts of the core. However, if partners are too greedy in setting non-cooperative options clauses then this can make the core empty. Partnerships that still go ahead are more likely to fail when the locked-in partner realizes the pitfalls of giving away flexibility.

From an academic point of view, this paper provides a framework for the investigation of partnership deals in the presence of uncertainty and flexibility. The models are illustrative and stylized but provide the basis for interesting insights. There is much scope for future work. We only mention four areas:

(1) We have focused on European options in this paper. It would be interesting to see how the models extend to American options, where the partners have to agree on optimal exercise on the fly rather than at pre-determined decision points.
(2) We have focused on two extreme situations - complete markets on one side and no markets but risk aversion on the other. What if there are partial hedging opportunities? The work of Smith and Nau (1995) on the integration of decision analysis and real options valuation would be an interesting starting point for investigations in this direction.
(3) We have focused on a closed contract world. Most companies have investment opportunities outside of the contract and will therefore exercise their contractual options not necessarily in the interest of the underlying contract project. This can lead to interesting effects, when unilateral options are exercised optimally within a company's portfolio of opportunities but sub-optimally within the contract frame.

(4) We have focused on sequential decisions by the partners. If we allow for simultaneous decisions, then we are in the context of a cooperative game followed by a non-cooperative game - cooperate to compete is the sequencing. Work along these lines should provide an interesting angle to the study of the relationship between cooperation and competition in the strategy field, see Brandenburger and Nalebuff (1996), and would complement recent work by Brandenburger and Stuart (Forthcoming) on biform games.

Last but not least, we believe that there is ample scope for impactful practical work along the lines described in this paper. A cooperative real options framework can be useful in helping companies understand the value effects of contingency clauses and thereby structure more efficient and more robust contracts. We have had some experience with the developed concepts during the negotiations of a complex co-development contract between Cambridge Antibody Technology Plc., a UK-based biotech company, and Astra Zeneca, which involved substantial flexibility[8]. Whilst this experience was encouraging, it also revealed that much work remains to be done to make cooperative real options models widely useful in business practice.

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