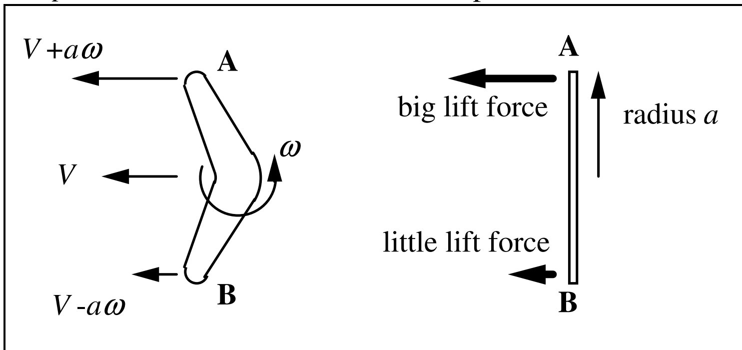
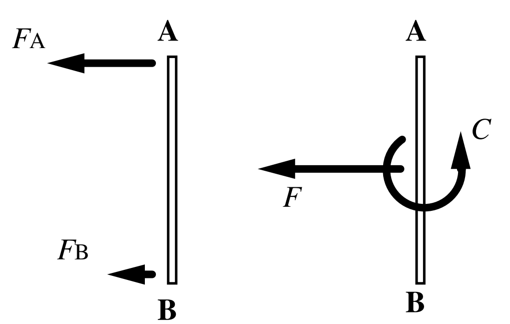
A boomerang does funny things because it is in fact a gyroscope. Aerodynamic forces generate a twisting moment which cause the 'gyroscope' to *precess* and to move on a circular path.

Let us examine the forces acting on a boomerang of radius a. The centre of the boomerang is moving at a constant forward speed V and the boomerang is spinning with angular velocity ω as shown in the diagram. The 'top' end **A** is moving faster than V with speed $V+a\omega$ and the 'bottom' end **B** is moving slower with speed $V-a\omega$. A wing generates more lift when it is moving faster so point A is generating more lift than point B.



side-on view showing velocities

edge-on view showing lift forces



actual forces

equivalent to a force and a moment

The two forces F_A and F_B can be represented by a single force F and a single couple C. With this simple representation of the forces acting on the boomerang we can give two reasons why it moves on a circular path:

1. A constant centripetal force F produces circular motion with velocity V on a radius R :

$$F = mV^2/R \qquad \text{eq. 1}$$

2. A constant couple C acting on a gyroscope spinning at angular velocity ω causes steady precession at rate Ω :

$$C = J\omega\Omega \qquad \text{eq. 2}$$

with m & J the boomerang's mass & moment of inertia.

If the rate of precession Ω exactly corresponds to the angular velocity of circular motion, then the boomerang stays tangential to the flight path as shown. This gives an equation relating V to Ω

$$V = R\Omega \qquad \text{eq. 3}$$

The aerodynamic lift force L acting on an airfoil of area A moving at speed v in air with density ρ is given by

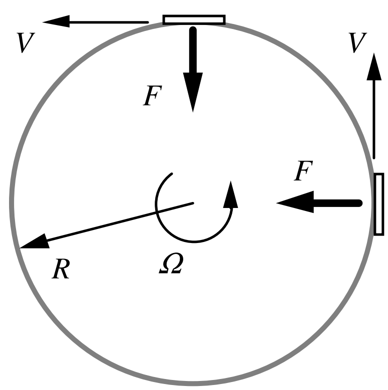
$$L = \tfrac{1}{2}\rho v^2 C_L A \qquad \text{eq. 4}$$

where C_L is defined as the *lift coefficient*. It can be shown[1] by integrating the lift force over the area of a cross-shaped boomerang that the net lift force F and aerodynamic couple C are given by

$$F = \tfrac{1}{4}\rho(V^2+(a\omega)^2)\, C_L A_s \qquad \text{eq. 5}$$

and

$$C = \tfrac{1}{4}\rho V a^2 \omega C_L A_s \qquad \text{eq. 6}$$



boomerang on circular flight path

where $A_s = \pi a^2$ is the swept area of the boomerang, and V, ω and a are the velocity, spin speed and radius of the boomerang as before.

From equations 2, 3 and 6, we find that the radius R of the circular flight path is independent of spin speed ω and forward velocity V, and that it is a constant for a given boomerang:

$$R = \frac{4J}{\rho C_L \pi a^4} \qquad \text{eq. 7}$$

For the case of a cross-shaped boomerang, $J = \tfrac{1}{3}ma^2$ and equations 1, 5 & 7 can be arranged to give

$$a\omega = \sqrt{2}\,V \qquad \text{eq. 8}$$

which defines the 'flick-of-the-wrist' needed to make the boomerang fly properly.

[1] For more information see http://www.eng.cam.ac.uk/~hemh/boomerangs.htm