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THE RICOCHET OF SPINNING AND NON-SPINNING SPHERICAL PROJECTILES, MAINLY FROM WATER. PART II: AN OUTLINE OF THEORY AND WARLIKE APPLICATIONS

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Summary—The Birkhoff *et al.* theory for the ricochet of non-spinning spheres off a liquid surface is outlined. An improvement, by including projectile weight and medium resistance, is given for (i) non-spinning projectiles and (ii) using Rayleigh's impact, pressure formula. That the critical angle for ricochet, (i), increases with velocity for media of zero resistance and (ii), for internally resistant materials, leads to a cut-off angle which decreases as speed increases, are both accounted for.

The adaptation of the latter results to include spinning in the use of spherical bombs in World War II is described with particular relevance to Wallis's "Bouncing" Bomb. © 1997 Published by Elsevier Science Ltd.

ELEMENTARY RICOCHET THEORY

Birkhoff et al.*'s analysis: allowing for hydrodynamic lift only*

A theory which predicts the greatest or critical angle of attack at a liquid surface, θ_c for a uniform solid sphere to undergo ricochet was first given by Birkhoff *et al.* At angles in excess of θ_c the sphere would not ricochet and at angles less than it, ricochet would occur. It is my belief that the well known simple expression $\theta_c = 18^\circ\sqrt{\rho/\rho'}$, was first given by Birkhoff *et al.* in an American report, of 1944 (AMP report 42.4M, 1944) : ρ' is the density of the projectile and ρ that of the liquid; this approach was repeated in the paper by Johnson and Reid [*J. Mech. Eng. Sci.* **17** (1975) 71–81].

The equation of motion in the plane of the trajectory at an instant for a sphere of mass M during its encounter and partial immersion in a stationary mass of water is

$$Mv^2\,\mathrm{d}\lambda/\mathrm{d}s = L - Mg\cos\lambda.$$

($\mathrm{d}\lambda/\mathrm{d}s$, is the curvature of the projectile path).

The first term on the left above is the centripetal force on the sphere, L is the impact lift force on the body and Mg its gravitational weight. λ is the inclination of the velocity of the sphere to the horizontal, see Fig. 1.

In this analysis we assume a relatively high speed of ricochet for which the projectile weight may be neglected.

Two assumptions among many others are made, all questionable;

(i) The pressure p on a spherical surface element along its outward drawn normal is $\rho u^2/2$; u is the forward speed of the sphere resolved along the normal.
(ii) The pressure applies only to those parts of the sphere which are immersed below the undisturbed surface of the water. The effect of the splash on the sphere is considered *not* to contribute any pressure.
(iii) Lift L, is then $\int_0^S p\,\mathrm{d}S\cos\phi$. The ($p\mathrm{d}S$) term is resolved vertically and summed for the wetted surface of the sphere, S.

There is no attempt in the Birkhoff report to justify these assumptions rigorously and indeed it is stated that, "it is not supposed that they are true in themselves but merely that they give a reasonable estimate of the forces involved". The assumed pressure distribution is said to be "in the spirit of Newton's original theory but modified by Bernoulli's pressure

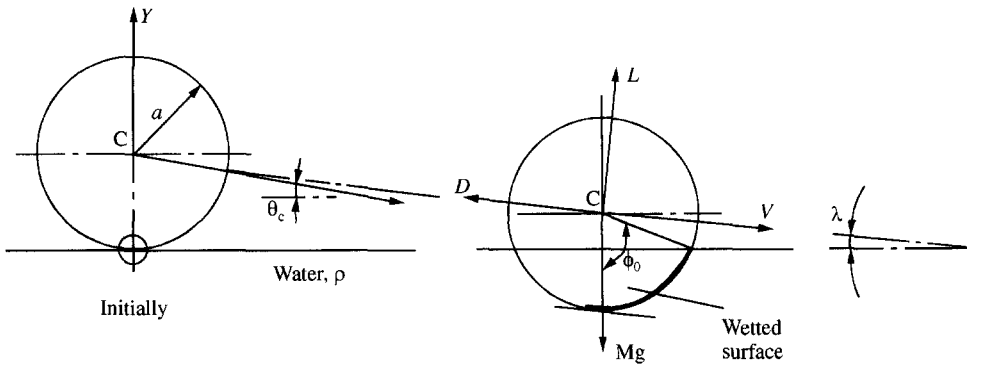


Fig. 1.

formula". *The factor of one-half is therefore introduced arbitrarily.* Note that the pressure distribution over the sphere is assumed independent of its angular velocity and consequently of the effect of spin on ricochet. (Richardson discounted the approach described here: see p. 366 of his paper.)

The critical hydrodynamic angle is that for which the lift, L is finally zero, i.e. when the whole sphere *is just* immersed.

The final result is that, $\theta_c = 18^\circ\sqrt{\rho/\rho'}$ with respect to water; for steel, $\rho/\rho' = 7.8$, and thus $\theta_c \sim 7^\circ$; for aluminum $\theta_c = 9^\circ$ and for ebony is 15°.

Inclusion of the static pressure in the medium through which the projectile advances and projectile weight

We assume a quasi-static flow expression,

$$p = (1/2)\rho v^2 + K,$$

where K is a measure of the static pressure required to deform the material, arising from the internal resistance of the material. (K plays the same role as does R_T, the target plate resistance in Tate's analysis for hypervelocity impact penetration.)

Introducing the expression for p from above and the weight of the projectile into the equation of motion, we find on proceeding to the solution, that,

$$\theta_c^2 = \frac{1}{10}\frac{\rho}{\rho'} + \frac{K\rho'}{\rho\bar{v}^2} - \frac{4ag}{\bar{v}^2},$$

where $\bar{v}$ is the mean speed of passage through the medium. If $K/\rho\bar{v}^2 g \ll 1$, then the medium behaves as if it were a fluid. The last term of the last equation on the right is the reciprocal of the Froude number. Evidently, θ_c increases as mean speed $\bar{v}$ increases.

If projectile weight is negligible, then θ_c reduces as $\bar{v}$ increases.

A 1976 paper in the *Int. J. Mech. Sci.* (**18,** 279–284) gave experimental results pertaining to the latter circumstances, see Fig. 2.

THE WALLIS "BOUNCING" BOMB

Early considerations

Early in 1941, in World War II, Dr. Barnes Wallis proposed an attack on targets previously considered invulnerable to air bombs of even the largest size, though a British Admiralty committee as early as 1938 had studied various European dams as targets for attack. In short, he drafted a paper suggesting an attack on the German Möhne and Eder dams in the Ruhr and this subsequently took place in May 1943. The Möhne dam—a "gravity" or predominantly earthen dam, masonry-faced—was to be the primary target; it was a strong structure 130 ft high, 112 ft thick at its base, tapering to 25 ft at its top.

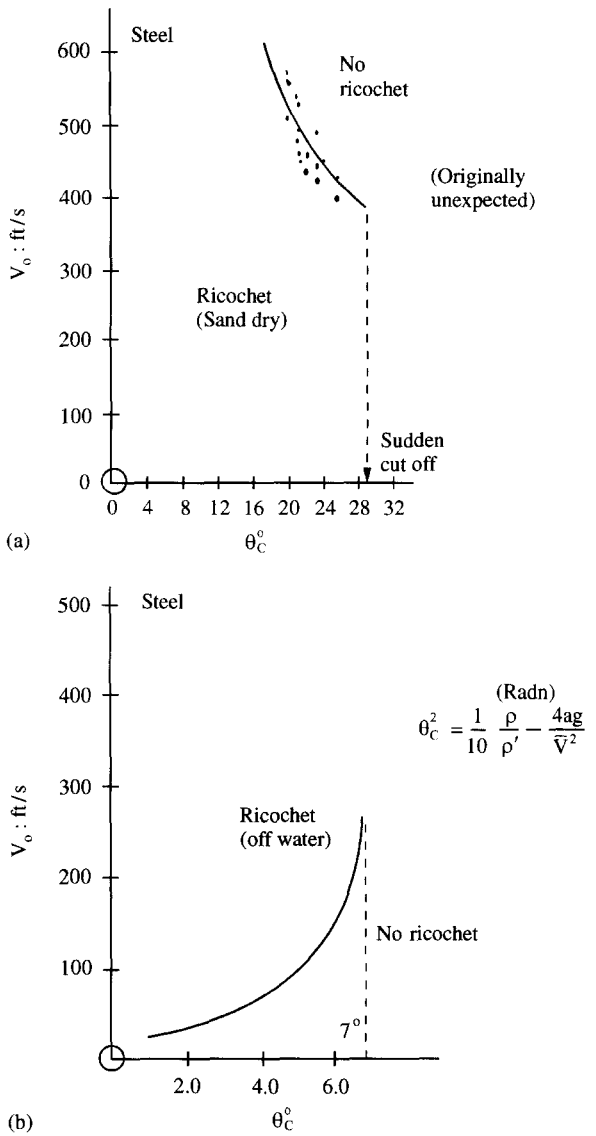


Fig. 2.

Breaching this dam would, it was considered, release water essential for local industry, deprive the local population of drinking water and electricity, flood valleys, impede transport and generally make a large neighbouring area unusable. In aiming to breach the dam it was expected that it would succeed in seriously damaging its foundations and so present civil engineers with the long and difficult job of repair. It was early estimated that a solid charge of 6500 lbs of explosive would be required at a depth of 30 ft below the full water mark. This would then release from the dam, 70% of its water-holding capacity. (Earth dams become notoriously weak and unstable once water finds its way into the earth.)

The delivery of the bomb

The delivery of a bomb to a precise depth on the dam face by normal high-level air attack was so difficult that it was thought to render the dam invulnerable. Thus another approach had to be suggested and there came to mind the thought that a torpedo-like projectile might do the task. However, the defenders of the dam had already thought of this, for they had arranged to have large double boom defences lying on the surface of the water from each of

which hung a web of the heaviest anti-torpedo netting ahead of the dam face. As well, it was realised that the mechanisms and the structure of bombs, impinging from a great height, would have been so severely damaged as to destroy them; if only for this reason it was therefore required to subject them to a *relatively* gentle impact, from very low level bombing.

The need to surmount the nets led to Wallis proposing spherical ricocheting bombs dropped from low flying aircraft. Having small vertical speed and large horizontal speed, the aim would be to have the bombs ricochet or skip over the nets to impinge on the face of the dam. This ricocheting spherical bomb was, early on, referred to as a Spherical Torpedo. See Fig. 3.

The critical angle of ricochet was known to be $18°/(\rho/\rho')^{1/2}$, where (ρ/ρ') is the effective specific gravity of the bomb. However, it was necessary to increase the angle of approach and reflection much above this predicted figure if success was to be achieved. Wallis proceeded to explore the effect of spin in the ricochet process and it was soon shown that spin causing increased lift generally increased ability to ricochet on water to much above that achieved by non-spinning spheres.

Sir George Edwards, formerly chairman of British Aircraft Corporation, in the Christopher Hinton Lecture of 1982, p. 9, wrote, "from what I knew of a cricket ball I persuaded him (Wallis) much against his will into putting back-spin on these bombs. It is a fact that there was a German aeroplane that dropped a comparable round bomb that was not spun and therefore did not bounce and was not used". (This remark may have been hindsight.)

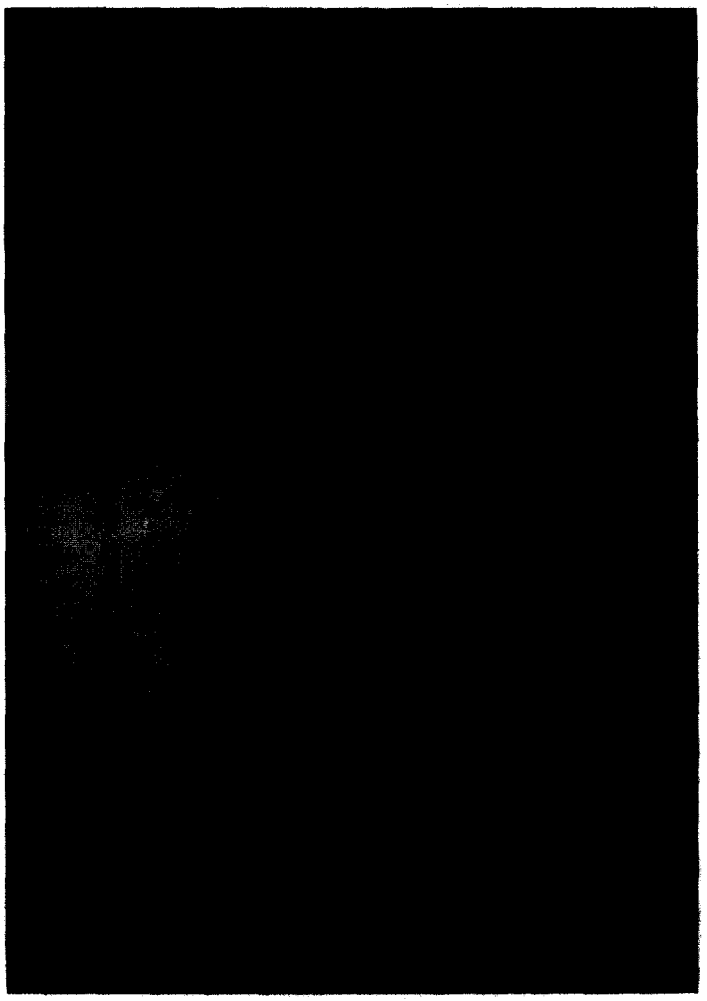


Fig. 3. Early spherical bomb.

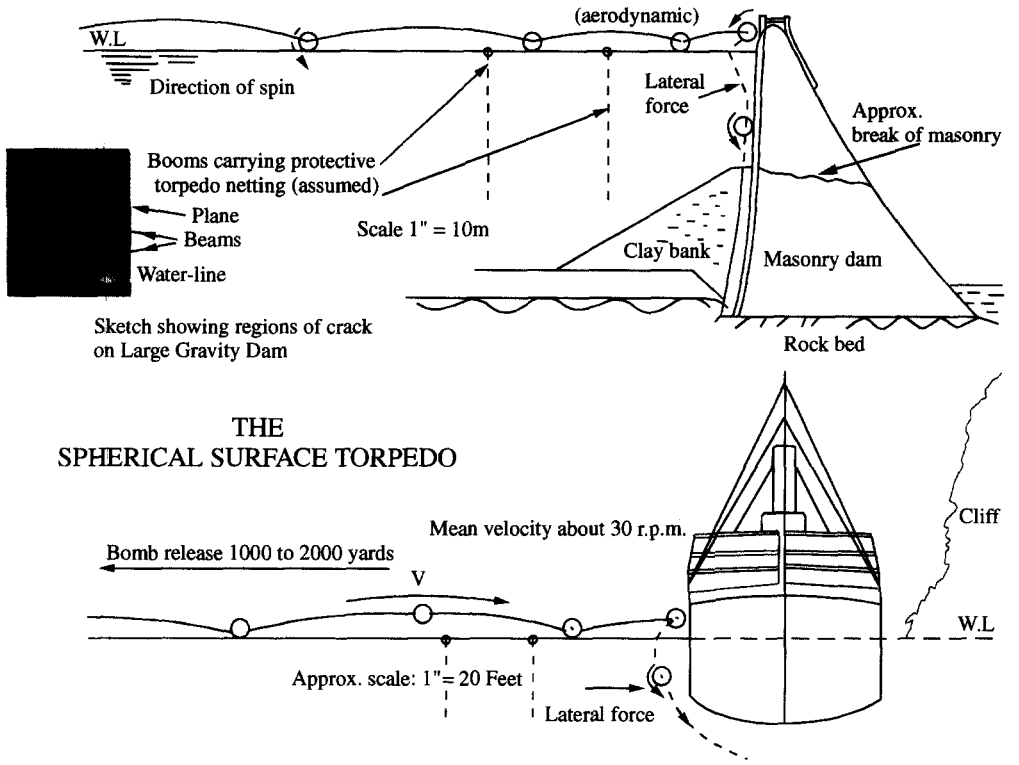


Fig. 4.

Wallis mentioned in his 1942/1943 report that Benjamin Robins in his book, *The New Principles of Gunnery* published in 1742, that spin imposed on smooth bore musket shot or cannon balls caused lift or alternatively an inflight incurvation, in air. In air, this phenomenon is known as the Magnus Effect. The back-spinning golf ball, has a longer range because of this and with a football in soccer, a difficult to deal-with "screw" kick results. In relation to ricochet it did indeed increase "ability to ricochet on water".

Thus it was proposed by Wallis to attack all the dams with back-spinning spherical bombs. The angle of the dam face at the top was 45°, so that once impact occurred, the rotating sphere would bounce backwards as well as upwards and later would fall, the backward spin causing it to move alongside and into contact with the dam face. In a figure in Wallis's report he showed the expected path of a spinning spherical bomb. As used to attack a ship, the spin aids the sinking of the bomb in its passage to arrive at a location beneath and close to the hull. See Fig. 4.

The bomb was to be exploded on the dam face using a hydrostatic pistol but the entrainment of a large bubble of air, generated on plunging through the surface, apparently made for an uncertain depth of explosion. However, the air bubble was no longer carried down when adequate spin was introduced, and thus rendered the depth at which explosion should occur, controllable.

A.R. Collins on the origin and design of the attack on the dams

Security about many technical aspects of this air attack was able to be maintained into the 1970s by virtue of a "30 year blanket" which the British government may place on files it wishes to remain secret. A long and careful account of the planning of this project was however written up and published in 1982 (Inst. of Civil Engrs.) by A.R. Collins. His account is principally devoted to describing experiments on 115, linearly scaled models of the Möhne dam in order to determine the magnitude of explosive charges needed to achieve

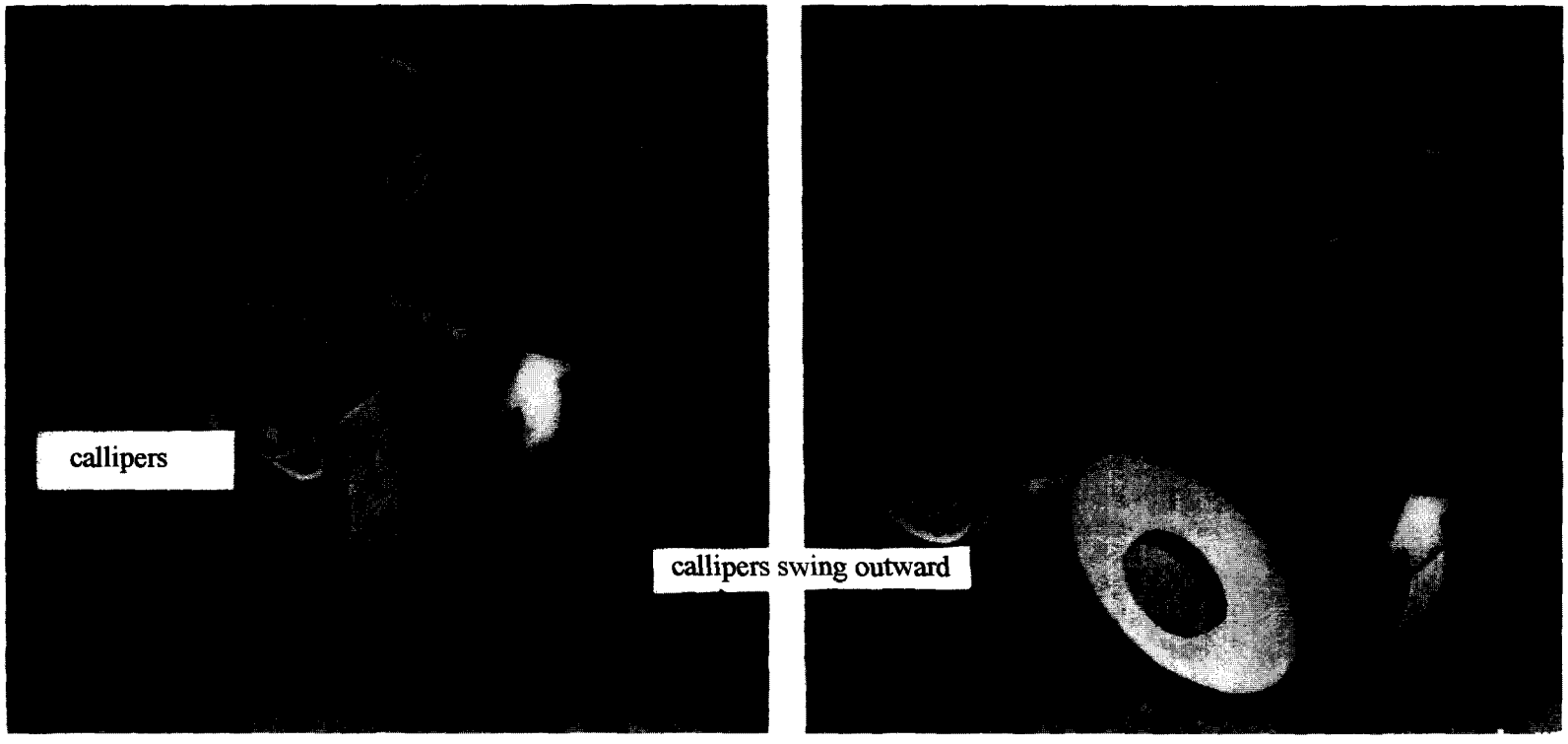


Fig. 5. The story of the 'bouncing bomb'

The bomb originally designed by Dr Barnes Wallis was intended to be spherical; but experiments like the ones described below proved that a spherical bomb strong enough to survive the impact on the water and to break the walls of the dams would be too heavy to carry. Thus it was decided to use a cylindrical bomb, despite its inferior 'bouncing' characteristics. In its operational form, the bomb was carried underneath the fuselage of the Lancaster between two V-shaped callipers, between which the bomb could be revolved before the moment of dropping.

Ten minutes before attack, the aircraft's internal power system began to spin the bomb backwards at 500 rpm. Power to spin the bomb came from a motor in the fuselage, transmitted to the bomb by a belt drive.

The bomb was released by allowing the callipers to spring outwards; if there had been no spin, the bomb would have bounced backwards and exploded away from the wall, instead of at the base, as was essential: it was the 'earthquake' effect of the shock-waves which breached the wall. Non-standard height-finding and bomb-aiming equipment was used to obtain sufficient accuracy; it was far from sophisticated but extremely accurate.

substantial breaches. It is evident from Collins paper that Wallis did not play *an all-important* role in the development of this project and in particular, that very significant contributions were made by, for example, Sir William Glanville, Dr. G. Charlesworth, Dr. A.R. Collins and others of the Road Research Laboratory.

The outcome

It turned out that the notion of using a *spherical* bomb had to be abandoned (at the very end of February, 1943) because even for the low heights of launch projected, the spherical casing would still be unable to withstand the impact loads sustained. (One report is that Wallis made many ground impact tests with spheres made with welded steel pressings spun in a rig and dropped on steel plates.) A nearly cylindrical bomb was developed—actually an oblate spheroid—that turned out to be 50 in. diameter, 60 in. long, weighted 9250 lb. and carried 6600 lb of explosive RDX, with three hydrostatic pistols set to detonate at 30 ft. and with a 90 s. time fuse in case the pistols malfunctioned. It was mounted *across* the bomb bay, held at the centre of its flat, circular ends with callipers and rotated around its axis at 500 rpm. The callipers opened outwards, see Fig. 5, to release the bomb when flying at 220 mph at 60 ft. above the water and 425 yds. from the dam parapet; the 25 yards was a tolerance—a distance covered in 1/4 s. Achieving just 60 ft. height and the 425 yds. range was achieved with the remarkably unsophisticated equipment of beams which converged, from the front and rear of the plane, on the water surface.

The specific gravity of these bombs was 2.17 and thus the critical angle for ricochet with no spin applied, would have been about 12°. The rate of applied backspin was 52 rad/s so that a $\omega/v \simeq 1/3$, the effect of which was to raise the critical angle to ca. 16° and thereby enhance the chance of successful ricochet. This spin would also have increased the rebound velocity and hence the range of individual ricochets.

Losses in this raid were heavy—eight aircraft were lost and 53 out of 113 air crew—about average losses in bombing raids at that time. The subsequent development of larger planes to carry very heavy bombs and the high accuracy given by the American gyroscopic high altitude bomb sight became the means of achieving great penetration of massive concrete targets or depths of soil: this rendered ricochet bombs obsolete.

The Möhne dam breach effected was 250 ft. wide and 50 ft deep and that in the Eder, 200 × 80 ft—both very close to what was predicted from the small scale testing. The Sorpe dam too was damaged but not breached. See Fig. 6.

The formula of Lord Rayleigh (1842–1919)

In a fascinating paper, "*On the Resistance of Fluids*", originally published in 1876 (and No. 42 in his collected Scientific Papers), on p. 291, Rayleigh writes and proves that, "if an elongated blade be held vertically in an horizontal stream so that the angle between the plane of the blade and the stream is β, the mean pressure, p, is

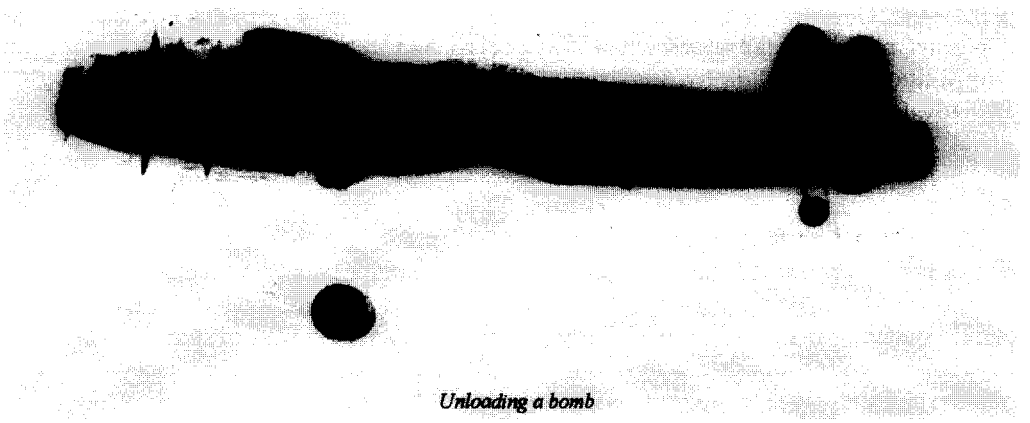
$$p = \{\pi \cos\beta/(4 + \pi \cos\beta)\}\, \rho v^2 \simeq (\pi/5)\cos\beta\, \rho v^2,$$

varying as $\cos\beta$ and not as $\cos^2\beta$; the latter formula was derived for steady-state, uniform two-dimensional motion.

Hutchings used Rayleigh's expression in his paper *to take account of spin.* The assumptions of the theory are however far from fulfilled.

The critical angle of ricochet when rotation is included using Rayleigh's pressure expression

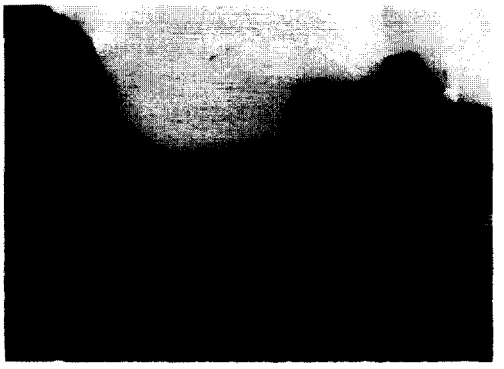
Hutchings, [*Int. J. Mech. Sci.* **18** (1976) 243] made use of Rayleigh's equation to determine the critical angle of ricochet for a sphere when its rate of rotation about a horizontal axis perpendicular to its line of translation, ω, was allowed for. It will be recalled that this assumes that a plane laminar is being considered and that there is steady-state plane flow.



Unloading a bomb



Cylindrical bomb



The breach in the Mohne Dam

Fig. 6.

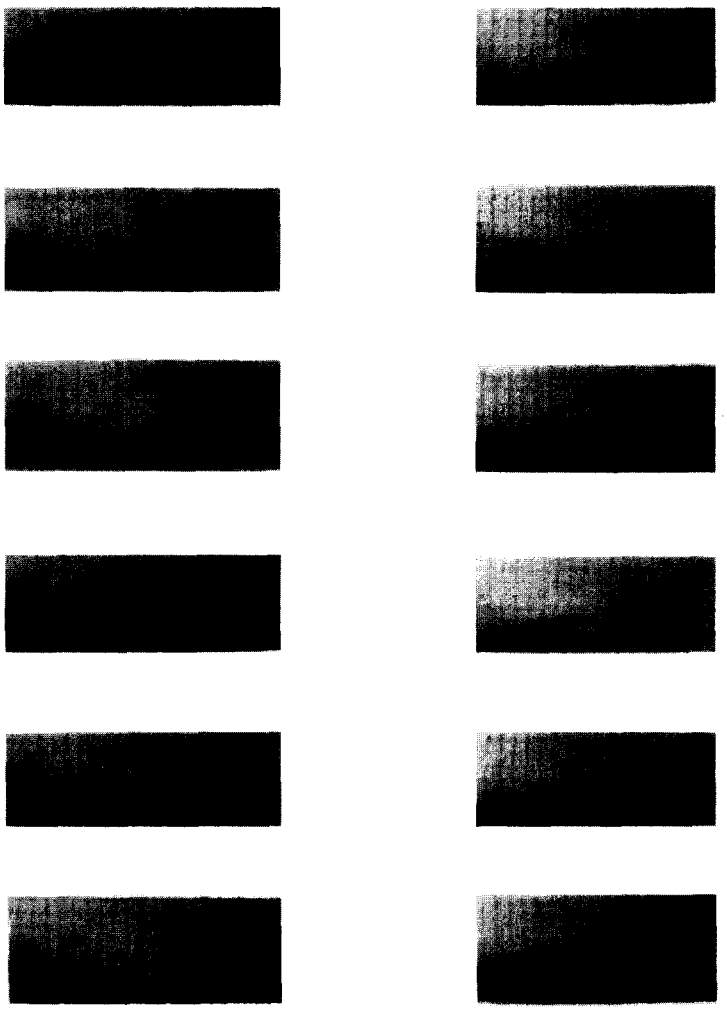
The rotation of the sphere changes the angle at which fluid meets the *rotating* sphere.

In the treatment of ricochet all previous calculators had assumed that there was an impact pressure over the wetted area only and that any pressure on the projectile due to splash was negligible. Hutchings however in part, intended to allow for this latter factor by taking the angular extent of the area on which the pressure acts to be twice the usual wetted angle.

Hutchings then found an expression for the lift which included the rate of rotation of the sphere and used it to find θ_c; it is based on $\rho v^2 \cos\beta$ *not* $\rho(v\cos\beta)^2$. And, incidentally, account was taken of aerodynamic lift. The value as found was $\theta_c^\circ = 18.7°(1 + 0.8\, a\omega/v)/\sqrt{\sigma}$, where σ is the specific gravity of the sphere. For $\omega = 0, \theta_c = 6.6$ (for $\sigma = 7.8$, steel). If $a\omega/v = 1/3, \theta_c = 23°/\sqrt{\sigma}$ or a 27% increase and if $a\omega/v = -1/3, \theta_c$ decreases to $13°/\sqrt{\sigma}$.

'BOUNCING' BOMBS IN THE SECOND WORLD WAR

This is the title of the most comprehensive short account I know of, by I. M. Hutchings, which appeared in *The New Scientist* of 2 March, 1978, pp. 563–567. Describing in detail Wallis's bombs and their continual testing, adaptation and development, it concludes with details about attempts to produce ricochet bombs for the purpose of attacking ships in the Pacific in 1945 but which after two years of development came to nothing. German bombs of similar kind which did not materalize are also described, including the potentially



(Next Column)

Fig. 7.

advantageous "prism" bombs. Two items also treated in the article are

(i) *The highball skip bomb*

A spherical case was designed for this bomb and was once said to possess excellent "bouncing" characteristics; it was strong enough to withstand impact with the water and was intended for use against shipping as well as dams. World War II ended before it was brought into service.

(ii) *The Kurt rocket-assisted skip bomb*

This 850 lb bomb was designed to bounce 450 yds. before sinking, then to be detonated against its target. A ricochet trail unit was added to improve range.

THE HYPERVELOCITY RICOCHET OF LONG RODS

In conclusion, we mention in order to contrast it with the ricochet of *spheres*, that of *long rods*. Long rods of length to diameter ratio at least 10:1 in Plasticine (or American play-dough) were fired at a few hundred m/s against thick plates of identical material at

impact angles greater than 75°. The sequence of photographs [*Int. J. Mech. Sci.* **24** (1982) 425] shows how a rod is bent and unbent during impact as ricochet is secured. Clearly a cavity is formed where mutual contact of rod and substrate is obtained and is the deeper the longer the rod, see Fig. 7 [*Int. J. Mech. Sci.* **24** (7) (1982) 437–455].

The final form of the rod is well seen, emerging bent and concave downwards: this is in good agreement with a computational simulation given by Zukas.

Hypervelocity impact applies especially when the "impact pressure", ρv^2, is several times as high as the yield stress of the materials, Y, say 10, i.e. $\rho v^2 \gg 10Y$. For high Y (metals), v too must be high. By choosing Plasticine (low Y) use can be made of a low value of v—such as is obtained with a (cheap) stud-driver gun.

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